The 16 binary boolean functions

p	q	T	$p \vee q$	$p \leftarrow q$	p	$p \rightarrow q$	q	$p \leftrightarrow q$	$p \wedge q$
T	T	T	T	T	T	T	T	T	T
T	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F

p	q	$p \uparrow q$	$p \oplus q$	$\neg q$	$p \nrightarrow q$	$\neg p$	$p \nleftarrow q$	$p \downarrow q$	F
T	T	F	F	F	F	F	F	F	F
T	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F

Common logical equivalences

(for formulas using only ¬, ∨, ∧)

Double negation: $\neg\neg p \iff p$

$p \wedge T \iff p$	Identity	$p \vee F \iff p$
$p \wedge F \iff F$	Domination	$p \vee T \iff T$
$p \wedge p \iff p$	Idempotence	$p \vee p \iff p$
$p \wedge \neg p \iff F$	Negation	$p \vee \neg p \iff T$
$p \wedge (p \vee q) \iff p$	Absorption	$p \vee (p \wedge q) \iff p$
$p \wedge q \iff q \wedge p$	Commutativity	$p \vee q \iff q \vee p$
$(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$	Associativity	$(p \vee q) \vee r \iff p \vee (q \vee r)$
$p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$	Distribution	$p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$
$\neg(p \wedge q) \iff \neg p \vee \neg q$	De Morgan's laws	$\neg(p \vee q) \iff \neg p \wedge \neg q$

Common logical equivalences

(for formulas using →)

Definition: $p \rightarrow q \iff \neg p \vee q$

$p \rightarrow F \iff \neg p$	Identity	$T \rightarrow p \iff p$
$p \rightarrow T \iff T$	Domination	$F \rightarrow p \iff T$
$p \rightarrow \neg p \iff \neg p$	Idempotence	$\neg p \rightarrow p \iff p$
$p \rightarrow p \iff T$	Negation	$\neg p \rightarrow \neg p \iff T$
$p \wedge (\neg p \rightarrow q) \iff p$	Absorption	$\neg p \rightarrow (p \wedge q) \iff p$
$(\neg p \rightarrow q) \wedge p \iff p$	Absorption	$(p \rightarrow q) \rightarrow p \iff p$
$p \rightarrow \neg q \iff q \rightarrow \neg p$	Commutativity	$\neg p \rightarrow q \iff \neg q \rightarrow p$
$(p \wedge q) \rightarrow r \iff p \rightarrow (q \rightarrow r)$	Associativity	$(p \rightarrow q) \vee r \iff p \rightarrow (q \vee r)$
$(p \rightarrow q) \wedge r \iff (p \rightarrow r) \rightarrow (q \wedge r)$	Distribution	$p \rightarrow (q \wedge r) \iff (p \rightarrow q) \wedge (p \rightarrow r)$
$p \wedge (q \rightarrow r) \iff (p \rightarrow q) \rightarrow (p \wedge r)$	Distribution	$(p \vee q) \rightarrow r \iff (p \rightarrow r) \wedge (q \rightarrow r)$
$\neg(p \wedge q) \iff p \rightarrow \neg q$	De Morgan's laws	$\neg(p \rightarrow q) \iff p \wedge \neg q$

Common inference rules

Simplification	$p \wedge q \implies p$
Biconditional Elimination	$p \leftrightarrow q \implies p \to q$
Addition	$p \implies p \vee q$
Trivial Proof	$q \implies p \to q$
Vacuous Proof	$\neg p \implies p \to q$
Modus Ponens	$p \wedge (p \to q) \implies q$
Modus Tollens	$\neg q \wedge (p \to q) \implies \neg p$
Disjunctive Syllogism	$\neg p \wedge (p \vee q) \implies q$
Proof by Contradiction	$\neg p \to F \implies p$
Reductio Ad Absurdum	$(p \to q) \wedge (p \to \neg q) \implies \neg p$
Hypothetical Syllogism	$(p \to q) \wedge (q \to r) \implies p \to r$
Separation of Cases	$(p \vee q) \wedge (p \to r) \wedge (q \to r) \implies r$
Biconditional Introduction	$(p \to q) \wedge (q \to p) \implies p \leftrightarrow q$

Existential Instantiation	$\exists x \in X : P(x) \implies$ Fix $a \in X$ such that $P(a)$
Universal Instantiation	$a \in X \wedge \forall x \in X : P(x) \implies P(a)$
Existential Generalization	$a \in X \wedge P(a) \implies \exists x \in X : P(x)$
Universal Generalization	$P(a)$ for **arbitrary** $a \in X \implies \forall x \in X : P(x)$

Axiom 1. *Everyone sane is my friend.*

Axiom 2. *Chickens live underwater.*

Axiom 3. *There is a sane chicken.*

Theorem: *I have a friend who lives underwater.*

Proof:

1	Everyone sane is my friend.	[Axiom 1]
2	Chickens live underwater.	[Axiom 2]
3	There is a sane chicken.	[Axiom 3]
4	Name the sane chicken Carlos.	[∃ instantiation, 3]
5	Carlos is sane.	[Simplification, 4]
6	If Carlos is sane, then he is my friend.	[∀ instantiation, 1]
7	Carlos is my friend.	[Modus ponens, 5 and 6]
8	Carlos is a chicken.	[Simplification, 4]
9	If Carlos is a chicken, then he lives underwater.	[∀ instantiation, 2]
10	Carlos lives underwater.	[Modus ponens, 9 and 10]
11	Carlos is my friend and lives underwater.	[Conjunction, 7 and 10]
12	I have a friend who lives underwater.	[∃ generalization, 12]

□

Axiom 1. $\forall x\,(S(x) \to F(x))$

Axiom 2. $\forall x\,(C(x) \to U(x))$

Axiom 3. $\exists x\,(C(x) \land S(x))$

Theorem: $\exists x\,(F(x) \land U(x))$

Proof:

$\boxed{1}$	$\forall x\,(S(x) \to F(x))$	[Axiom 1]
$\boxed{2}$	$\forall x\,(C(x) \to U(x))$	[Axiom 2]
$\boxed{3}$	$\exists x\,(C(x) \land S(x))$	[Axiom 3]
$\boxed{4}$	$C(\textit{Carlos}) \land S(\textit{Carlos})$	[$\exists$ instantiation, $\boxed{3}$]
$\boxed{5}$	$S(\textit{Carlos})$	[Simplification, $\boxed{4}$]
$\boxed{6}$	$D(\textit{Carlos}) \to F(\textit{Carlos})$	[$\forall$ instantiation, $\boxed{1}$]
$\boxed{7}$	$F(\textit{Carlos})$	[Modus ponens, $\boxed{5}$ and $\boxed{6}$]
$\boxed{8}$	$C(\textit{Carlos})$	[Simplification, $\boxed{4}$]
$\boxed{9}$	$C(\textit{Carlos}) \to U(\textit{Carlos})$	[$\forall$ instantiation, $\boxed{2}$]
$\boxed{10}$	$U(\textit{Carlos})$	[Modus ponens, $\boxed{9}$ and $\boxed{10}$]
$\boxed{11}$	$F(\textit{Carlos}) \land U(\textit{Carlos})$	[Conjunction, $\boxed{7}$ and $\boxed{10}$]
$\boxed{12}$	$\exists x\,(F(x) \land U(x))$	[$\exists$ generalization, $\boxed{12}$]

$\square$