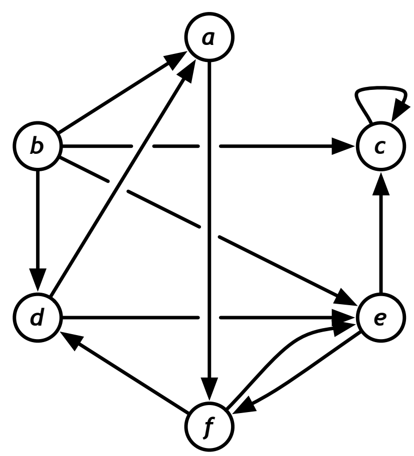
Directed graph (V, E):

- V is an arbitrary set, whose elements are called ***vertices*** (or ***nodes***).
- $E \subseteq V \times V$ is a set of ***edges*** (or ***arcs***).

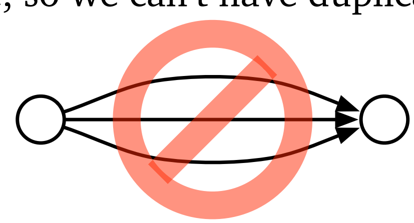
A directed graph *is* a binary relation!



Vertices b and d are the ***endpoints*** of edge (b, d).
Vertices b and d are ***incident*** to edge (b, d).
Vertex b is the ***source*** of edge (b, d).
Vertex d is the ***target*** of edge (b, d).

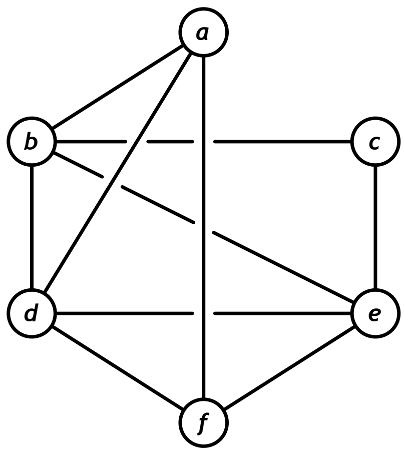
Vertex e has ***indegree*** 3 and ***outdegree*** 1.

E is a *set*, so we can't have duplicate edges.



Undirected graph $(\boldsymbol{V}, \boldsymbol{E})$***:***

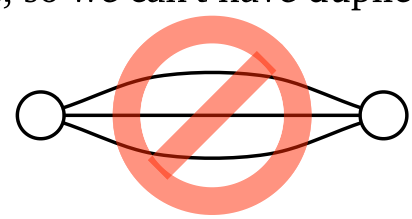
- V is an arbitrary set, whose elements are called ***vertices*** (or ***nodes***).
- E is a set of ***edges***; each edge is a ***set*** of 2 vertices.



Vertices b and d are the ***endpoints*** of edge $\{b, d\}$.
Vertices b and d are ***incident*** to edge $\{b, d\}$.

Vertex a has ***degree*** 3.
$\boldsymbol{\deg(a) = 3}$
Vertex a has 3 ***neighbors***.
Vertex a is ***adjacent*** to 3 other vertices.

E is a *set*, so we can't have duplicate edges.



Every edge is a *set* of size 2, so we can't have loops.

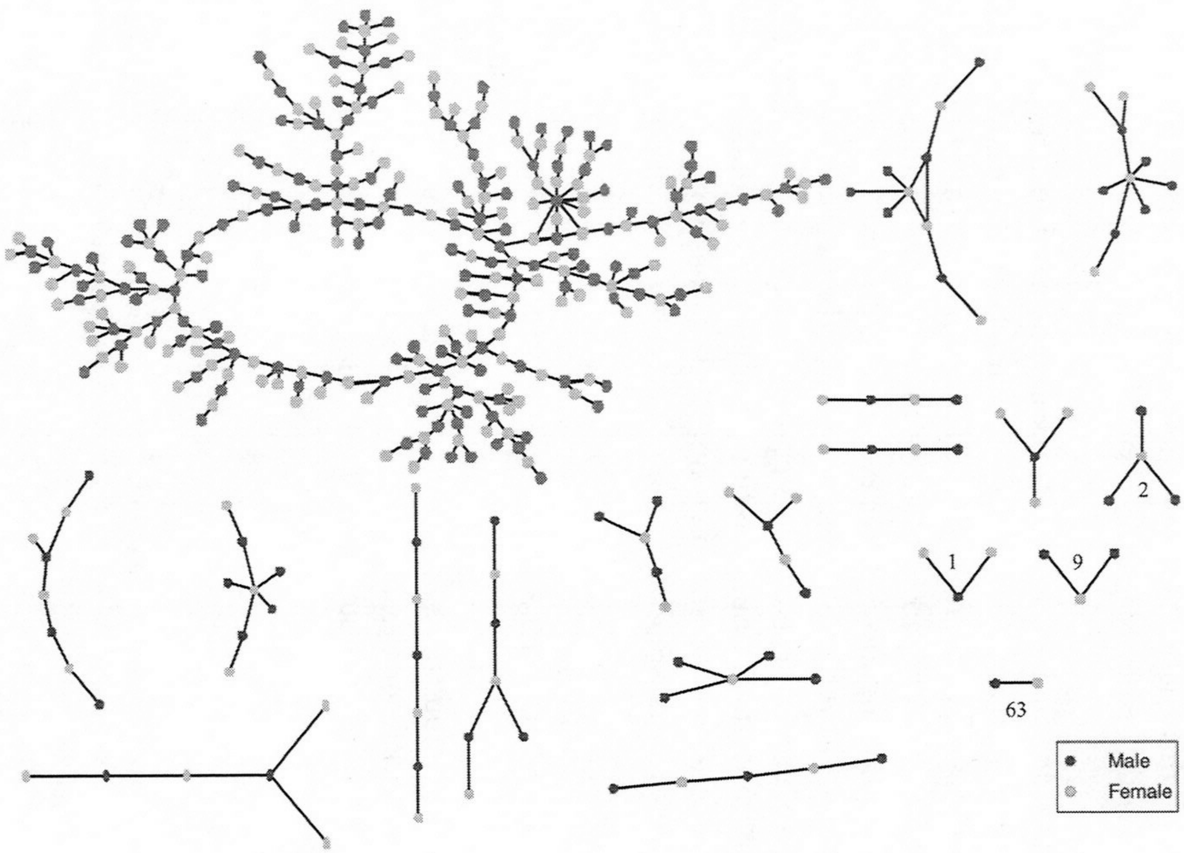
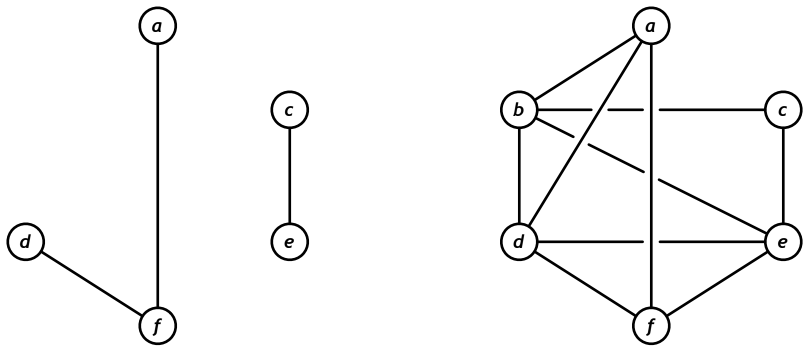


FIG. 2.—The direct relationship structure at Jefferson High

Subgraphs:

- (V, E) is a ***subgraph*** of (V', E') iff $V \subseteq V'$ and $E \subseteq E'$.



- A ***walk*** is a sequence of vertices $v_0, v_1, v_2, \ldots, v_n$ such that adjacent pairs are connected by edges $\{v_{i-1}, v_i\}$ or (v_{i-1}, v_i).

  The first vertex v_0 and the last vertex v_n are the ***endpoints*** of the walk.

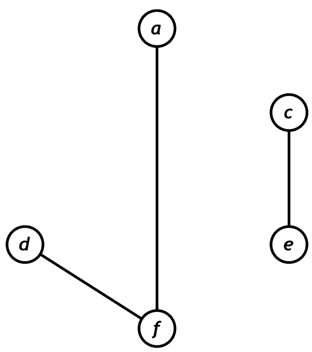
  The ***length*** of a walk is the number of edges.

- A ***path*** is a walk with no repeated vertices.

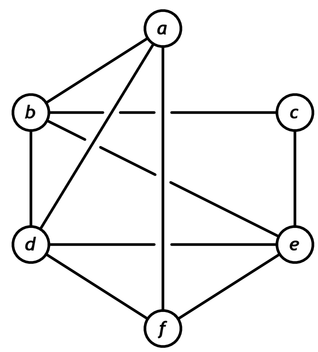
- A ***cycle*** is a walk that starts and ends at the same vertex, but otherwise has no repeated vertices.

Undirected graphs:

- An undirected graph is ***connected*** if it contains a walk between *every* pair of vertices.
- An undirected graph is ***acyclic*** (or a ***forest***) if no subgraph is a cycle.

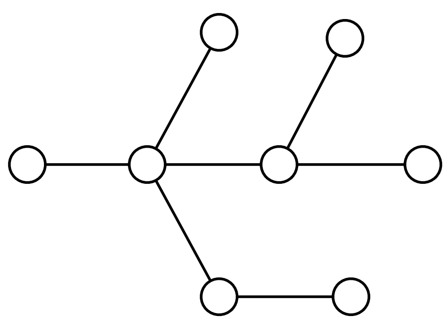


acyclic but not connected
(no walk from a to c)



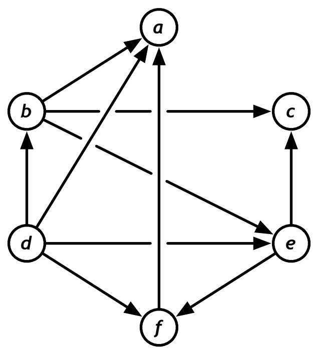
connected but not acyclic
(a, b, d, f, a is a cycle)

- A ***tree*** is a connected, acyclic, undirected graph.

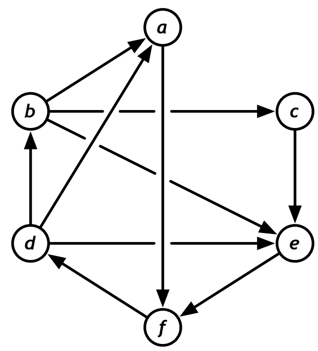


Directed graphs:

- A directed graph is ***(weakly) connected*** if the underlying undirected graph is connected.
- A directed graph is ***strongly connected*** if the there is a directed walk from any vertex to any other vertex.
- A directed graph is ***acyclic*** (or a ***dag***) if no subgraph is a directed cycle. Strongly connected digraphs *cannot* be acyclic.



acyclic and connected
but not strongly connected



connected but not acyclic
(many directed cycles)