# Quiz 3

November-20-11
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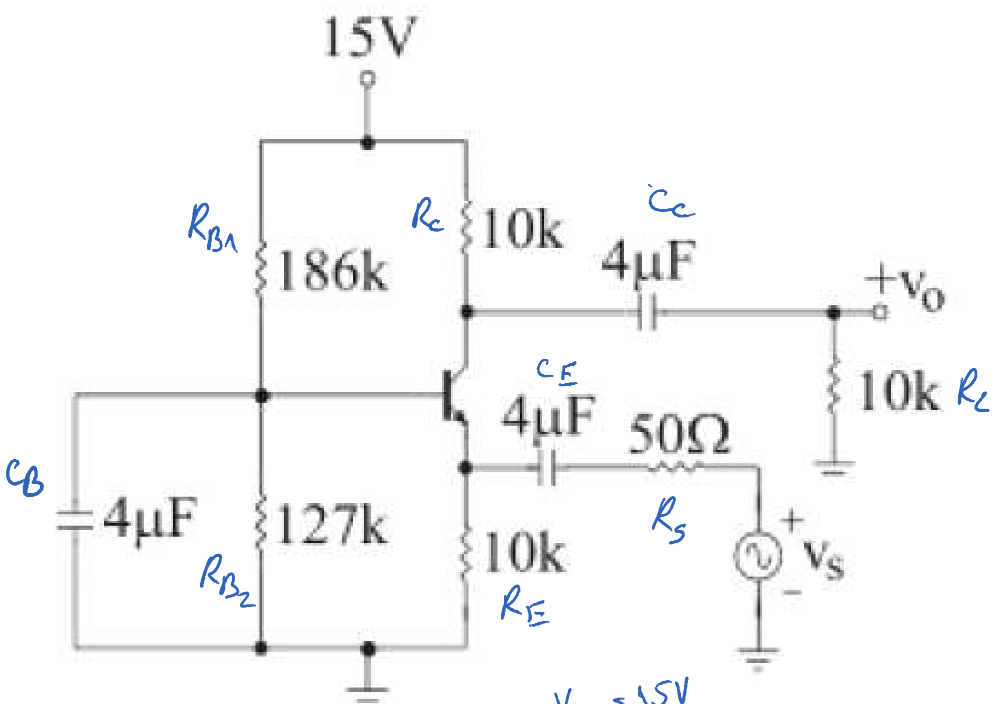
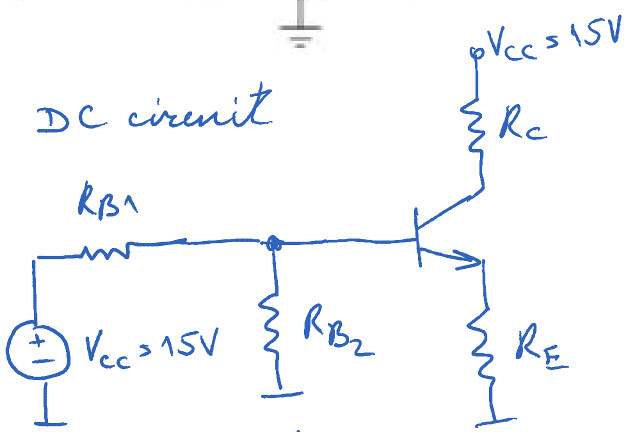
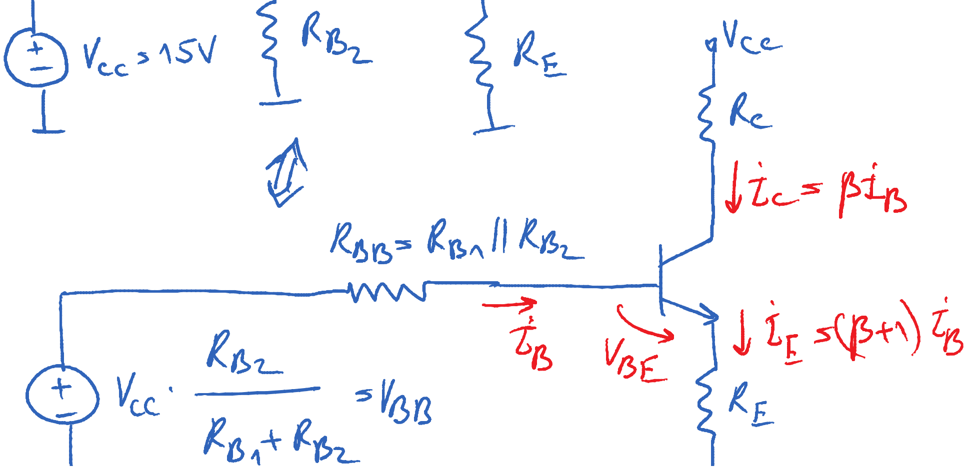
1. For the circuit shown in the figure, the transistor has $\beta = 100$.
   a. Find $I_B, I_C, V_C, V_B, g_m, r_\pi$
   b. Draw the low frequency circuit, the midband circuit and the high frequency circuit
   c. Find the midband gain (AM) and the corner frequencies $\omega_{3dBL}, \omega_{3dBH}$

The hybrid-model has the following parameters: $C_\pi = 5pF, C_\mu = 1pF, r_o = \infty$




a) DC circuit



$V_{CC} \cdot \dfrac{\quad}{R_{B1}+R_{B2}} = V_{BB}$

$$R_{BB} = \frac{R_{B1} R_{B2}}{R_{B1}+R_{B2}} = \frac{186 \cdot 127}{186+127} k\Omega = 75.5 \, k\Omega$$

$$V_{BB} = V_{CC} \cdot \frac{R_{B2}}{R_{B1}+R_{B2}} = 15V \cdot \frac{127}{127+186} = 6V$$

$$V_{BB} = R_{BB} I_B + V_{BE} + R_E I_E$$

$$V_{BB} = R_{BB} I_B + V_{BE} + R_E (\beta+1) I_B$$

- Assume BJT operates in the active region (to be checked at the end) $\Rightarrow V_{BE} = V_{BE,on} \cong 0.7V$

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB} + (\beta+1) R_E} = \frac{6V - 0.7V}{75.5k\Omega + 101 \cdot 10k\Omega}$$

$\boxed{I_B = 4.9 \mu A} \Rightarrow I_C = \beta I_B = 490 \mu A$

$I_E = (\beta+1) I_B = 495 \mu A$

$$V_B = V_{BB} - R_{BB} I_B = 6V - 75.5 \cdot 10^3 \cdot 4.9 \cdot 10^{-6} V$$

$$V_B = 5.63 V$$

$$V_C = V_{CC} - R_C I_C = 15V - 10^4 \cdot 490 \cdot 10^{-6} V = 10.1 V$$

( $V_{CB} > 0$, $V_{BE} > 0$ $\Rightarrow$ this confirms the operation in active mode)

- Small signal parameters: $r_\pi = \dfrac{V_T}{I_B} = \dfrac{26mV}{4.9\mu A} = 5.3k\Omega$

$$g_m = \frac{I_C}{V_T} = \frac{490\mu A}{26mV} = 18.8 mA/V$$

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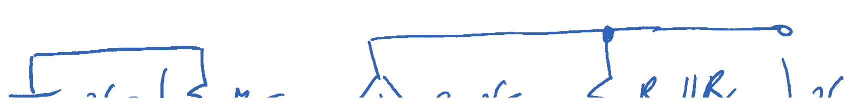
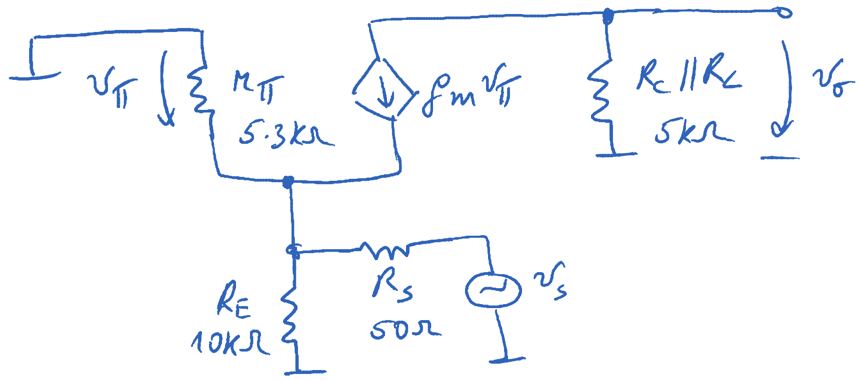
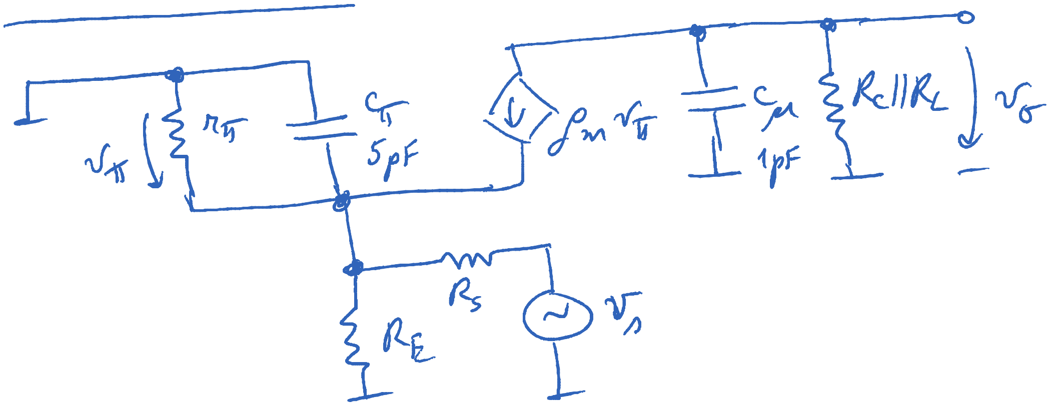
(b) The midband circuit:

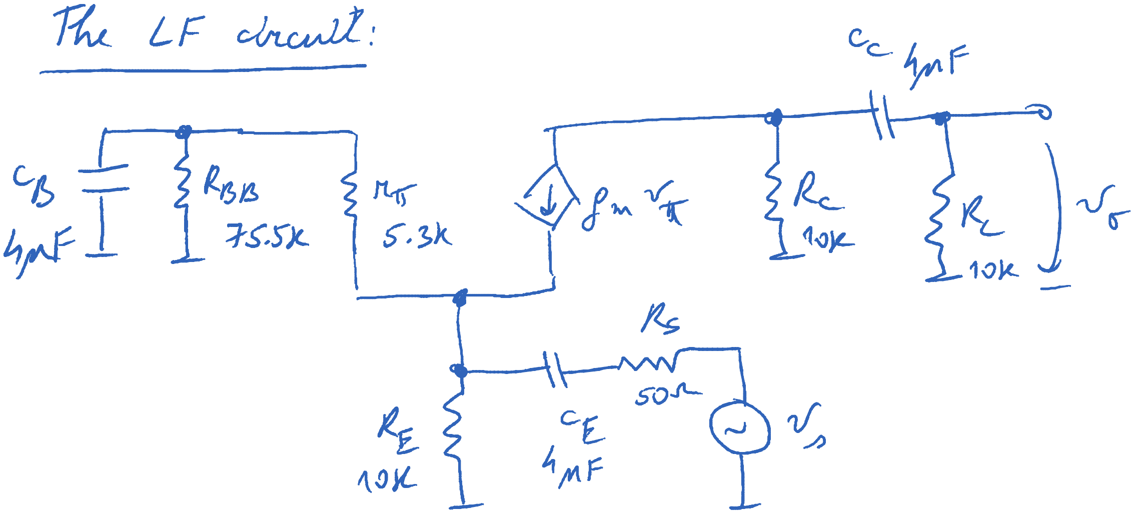




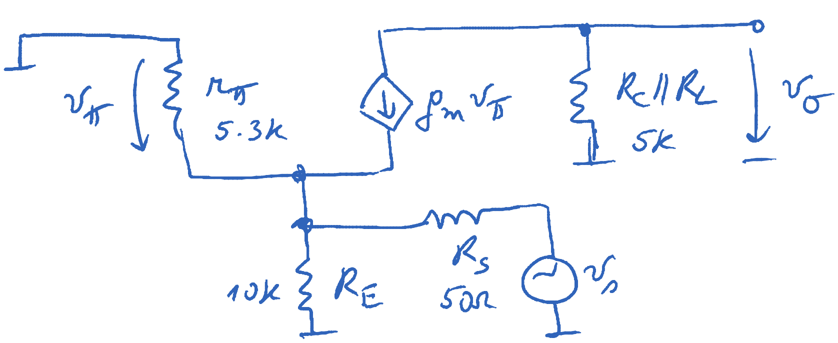

## The HF circuit:




## The LF circuit:




## (c) The midband gain:



$$-v_\pi = \frac{R_E \,//\, \frac{r_\pi}{\beta+1}}{R_s + R_E \,//\, \frac{r_\pi}{\beta+1}} v_s$$

$\Downarrow$

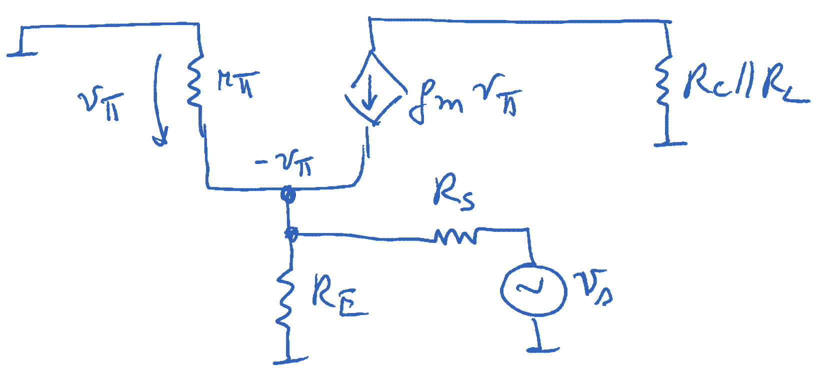
$$v_o = -g_m v_\pi (R_C // R_L) = \frac{g_m (R_C // R_L) \cdot \left(R_E \,//\, \frac{r_\pi}{\beta+1}\right)}{R_s + R_E \,//\, \frac{r_\pi}{\beta+1}} v_s \Rightarrow$$

$$\Rightarrow A_m = \frac{v_o}{v_s} = \frac{g_m \cdot (R_C // R_L)\; R_E \,//\, \frac{r_\pi}{\beta+1}}{R_s + R_E \,//\, \frac{r_\pi}{\beta+1}}$$

$$R_E \,//\, \frac{r_\pi}{\beta+1} = \frac{10 \cdot \frac{5.3}{101}}{10 + \frac{5.3}{101}} k\Omega = 52.2\Omega$$

$$A_m = \frac{18.8 \cdot 10^{-3} \cdot 5 \cdot 10^{3} \cdot 52.2}{50 + 52.2} = 48$$

<u>Note:</u> we could have used superposition (with dependent sources) to determine $v_\pi$ :




$$-v_\pi = \frac{R_E // r_\pi}{R_s + R_E // r_\pi} v_s + g_m v_\pi (r_\pi // R_E // R_s) \Leftrightarrow$$

$$\Leftrightarrow -v_\pi (1 + g_m R_s // R_E // r_\pi) = \frac{R_E // r_\pi}{R_s + R_E // r_\pi} v_s \Rightarrow$$

$$\Rightarrow v_\pi = -\frac{R_E // r_\pi}{R_s + R_E // r_\pi} \cdot \frac{1}{\quad} v_s$$

$$\Longrightarrow v_\pi = -\frac{R_E \| r_\pi}{R_s + R_E \| r_\pi} \cdot \frac{1}{1+g_m\left(r_\pi \| R_E \| R_s\right)}\, v_s$$

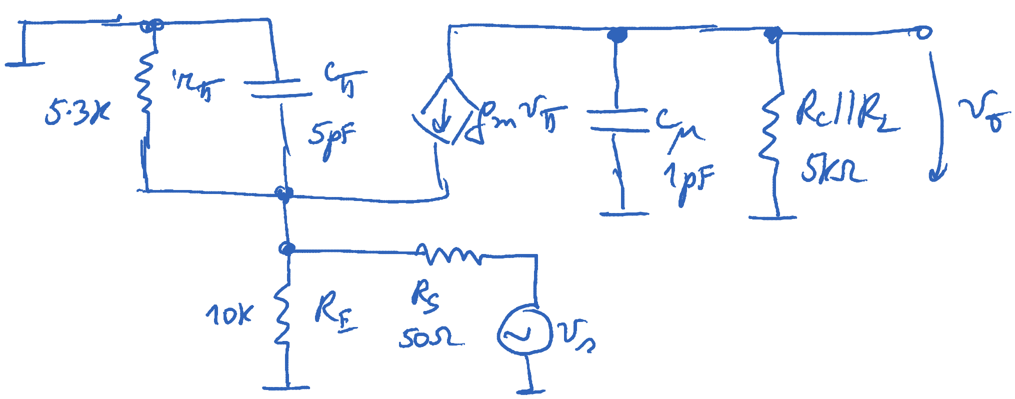
$$-\frac{v_\pi}{v_s} = \frac{\left(\frac{1}{R_E}+\frac{1}{r_\pi}\right)^{-1}}{R_s + \frac{1}{\frac{1}{R_E}+\frac{1}{r_\pi}}} \cdot \frac{1}{1+g_m \frac{1}{\frac{1}{r_\pi}+\frac{1}{R_E}+\frac{1}{R_s}}} =$$

$$= \frac{1}{1+R_s\left(\frac{1}{R_E}+\frac{1}{r_\pi}\right)} \cdot \frac{\frac{1}{r_\pi}+\frac{1}{R_E}+\frac{1}{R_s}}{g_m+\frac{1}{r_\pi}+\frac{1}{R_E}+\frac{1}{R_s}} =$$

$$= \frac{\frac{1}{R_s}}{\cancel{\frac{1}{R_s}+\frac{1}{R_E}+\frac{1}{r_\pi}}} \cdot \frac{\cancel{\frac{1}{R_s}+\frac{1}{R_E}+\frac{1}{r_\pi}}}{\underbrace{\left(\frac{\beta+1}{r_\pi}+\frac{1}{R_E}\right)}_{\left(R_E \| \frac{r_\pi}{\beta+1}\right)^{-1}}+\frac{1}{R_s}} = \frac{1}{1+R_s\cdot\left(R_E \| \frac{r_\pi}{\beta+1}\right)^{-1}}$$

$$-\frac{v_\pi}{v_s} = \frac{R_E \| \frac{r_\pi}{\beta+1}}{R_s + R_E \| \frac{r_\pi}{\beta+1}} \longleftarrow \text{the same expression as before}$$

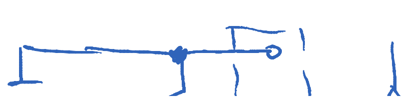
- For $\omega_H$ – we take the HF circuit:

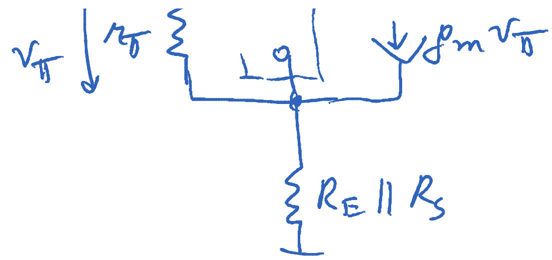



The method of OC:

For $C_\mu$: $\tau_{\mu,OC} = C_\mu \cdot (R_C \| R_L) = 10^{-12} \cdot 5 \cdot 10^{3} = 5\,\text{ns}$

For $C_\pi$:






$$R_{\pi,oc} = R_E \| R_S \| \frac{r_\pi}{\beta+1} \approx 50\Omega \| 53\Omega \approx 25.2\Omega$$

$$\tau_{\pi,oc} = \left(R_E \| R_S \| \frac{r_\pi}{\beta+1}\right) C_\pi = 25 \cdot 5 \cdot 10^{-12} = 125\,ps$$

$$\omega_H = \frac{1}{\tau_{\pi oc} + \tau_{\mu oc}} = \frac{1}{5 \cdot 10^{-9} + 0.125 \cdot 10^{-9}}\ rad/s$$

$$\omega_H = 195\ rad/s \Rightarrow f_H = \frac{\omega_H}{2\pi} = 31\,MHz$$

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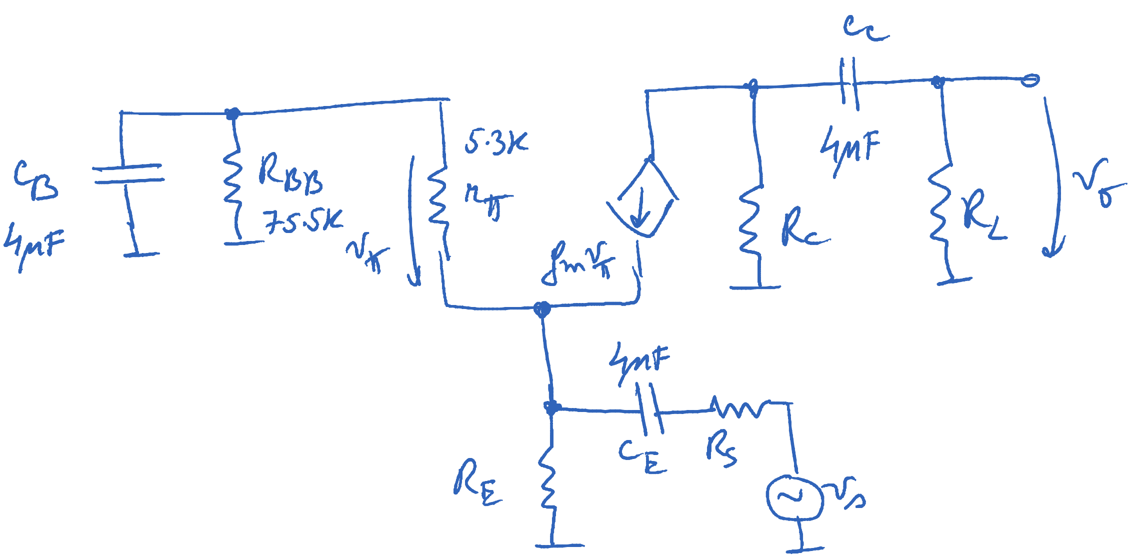
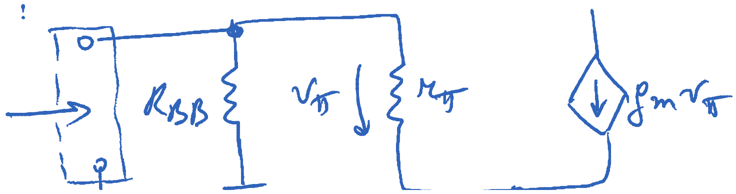
For $\omega_{3dBL} = \omega_L$ → the LF circuit:

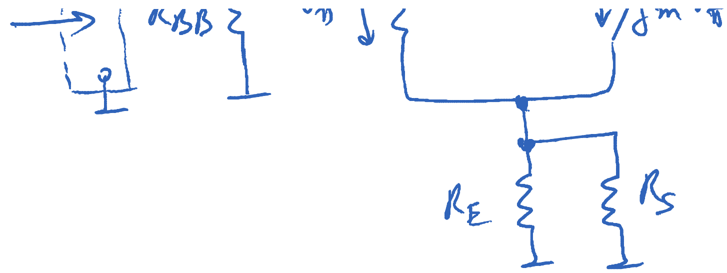



The method of sc:

$C_C$ : $\tau_{C,sc} = C_C \cdot (R_C + R_L) = 4 \cdot 10^{-6} \cdot 10 \cdot 10^{3}\,s = 40\,ms$

For $C_B$ :



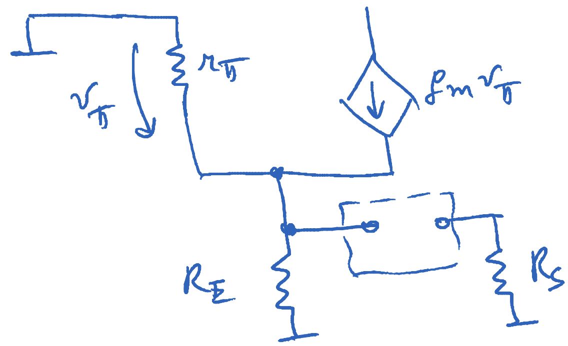



$$R_{B,x} = R_{BB} \,//\, \left(r_\pi + (\beta+1)\, R_E // R_S\right)$$

$$= 75.5\,k\Omega \,//\, \left(5.3\,k\Omega + 101 \cdot 50\,\Omega\right) = 9.1\,k\Omega$$

$$\tau_{B,x} = 9.1 \cdot 10^{3} \cdot 4 \cdot 10^{-6}\,s = 36.4\,ms$$

For $C_E$:




$$R_{E,x} = \left(R_E \,//\, \frac{r_\pi}{\beta+1}\right) + R_S = 52.2\,\Omega + 50\,\Omega = 102.2\,\Omega$$

$$\tau_{E,x} = 102.2 \cdot 4 \cdot 10^{-6}\,s = 408.8 \cdot 10^{-6}\,s$$

$$\omega_L = \frac{1}{\tau_{Ex}} + \frac{1}{\tau_{Bx}} + \frac{1}{\tau_{Cx}} = \frac{1}{409 \cdot 10^{-6}} + \frac{1}{36 \cdot 10^{-3}} + \frac{1}{40 \cdot 10^{-3}}$$

$$\omega_L \approx \frac{1}{\tau_{Ex}} = 2446\,rad/s \Rightarrow f_L = 389\,Hz$$