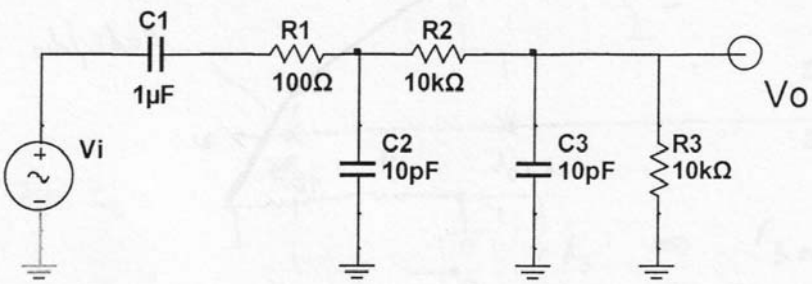
1. [ 50 marks] Compute the transfer function of the following circuit, using the open circuit and short circuit time constant method. State explicitly the values of $\omega_{L3dB}$ and $\omega_{H3db}$. Draw the amplitude and phase Bode plots of the transfer function, $H(s)=\frac{V_o(s)}{V_i(s)}$

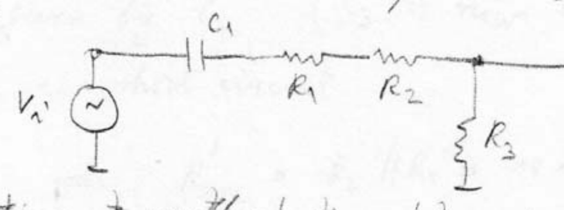



From circuit topology: $C_1$ imposes a zero at $\omega = 0$, while $C_2$, $C_3$ will short-circuit the signal path as $\omega \to \infty$ $\Rightarrow$ $C_1$ will be a component relevant for LF transfer function, while $C_2$, $C_3$ will count for the HF part

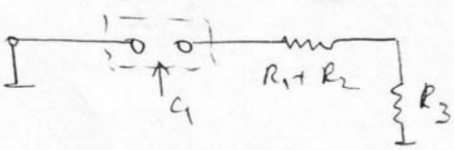
$$H(s) = \underset{\underset{C_1}{\uparrow}}{H_{LF}(s)} \cdot H_M \cdot \underset{\underset{C_2,\ C_3}{\uparrow}}{H_{HF}(s)}$$

This is further confirmed by their values: $C_1 = 1\mu F \gg C_2 = C_3 = 10pF$

The LF circuit reduces to:
($C_2$, $C_3$ are open circuited)




We apply the short circuit time ct method to determine $\omega_{LH}$




$\Rightarrow$ $R_{1SC} = R_1 + R_2 + R_3 = 20.1 k\Omega$

$\tau_{1SC} = R_{1SC} \cdot C_1 = 20.1 \cdot 10^3 \cdot 10^{-6} = 20.1 \cdot 10^{-3}\,s$

$\Downarrow$

$\omega_{LH} = \frac{1}{\tau_{1SC}} = \frac{1}{20.1 \cdot 10^{-3}}$ rad/s $\approx \frac{1000}{20} \approx 50$ rad/s

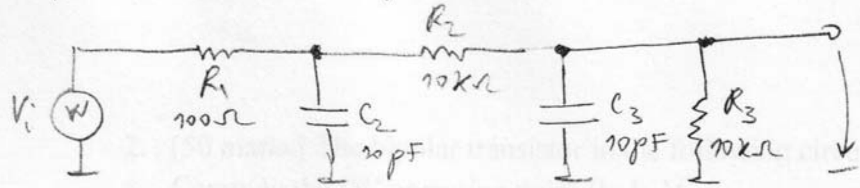
$$H_{LF} = \frac{s}{s+\omega_{LH}} = \frac{s}{s+50}$$

The circuit in midband:


$V_o(s) = \frac{R_3}{R_1+R_2+R_3} V_i'(s) \Rightarrow$

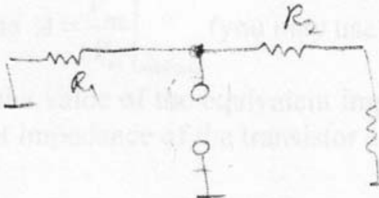
$$\Rightarrow H_M(s) = \frac{R_3}{R_1+R_2+R_3} = \frac{10 k\Omega}{20.1 k\Omega} \approx 0.5$$

▶ For the HF part, the circuit reduces to:




We apply the open circuit time constant method

# For $C_2$:




$\Rightarrow R_{2OC} = (R_2 + R_3) \,//\, R_1$

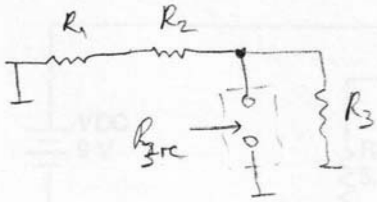
$= 20k\Omega \,//\, 100\Omega \approx 100\Omega$

$\Downarrow$

$\tau_{2OC} = R_{2OC} \cdot C_2 = 100 \cdot 10 \cdot 10^{-12} s$

$\tau_{2OC} = 10^{-9} s = 1 ns$

# For $C_3$:




$\Rightarrow R_{3OC} = R_3 \,//\, (R_1 + R_2) = 10k\Omega \,//\, 10.1k\Omega$

$R_{3OC} \approx 5k\Omega$

$\Downarrow$

$\tau_{3OC} = R_{3OC} \cdot C_3 = 5 \cdot 10^3 \cdot 10 \cdot 10^{-12} s$

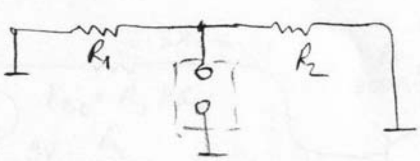
$\tau_{3OC} = 50 \cdot 10^{-9} s = 50 ns$

$$\omega_H = \frac{1}{\tau_{2OC} + \tau_{3OC}} = \frac{1}{10^{-9} + 50 \cdot 10^{-9}} s^{-1} = \frac{10^9}{51} rad/s \approx \frac{100\cancel{0}}{5\cancel{0}} \cdot 10^6 rad/s$$

$\omega_H \approx 20 \cdot 10^6 rad/s$

$\tau_{2OC} << \tau_{3OC} \Rightarrow$ $C_3$ is the responsible for the dominant pole $\omega_{H,3dB}$

· It remains to compute the pole given by $C_2$ ($C_3$ is now in the lower frequency region, and it will be treated as short circuit




$\Rightarrow R'_{2OC} = R_2 \,//\, R_1 = 10k\Omega \,//\, 100\Omega \approx 100\Omega$

$\Downarrow$

$\tau'_{2OC} = R'_{2OC} C_2 = 10^{-9} s$

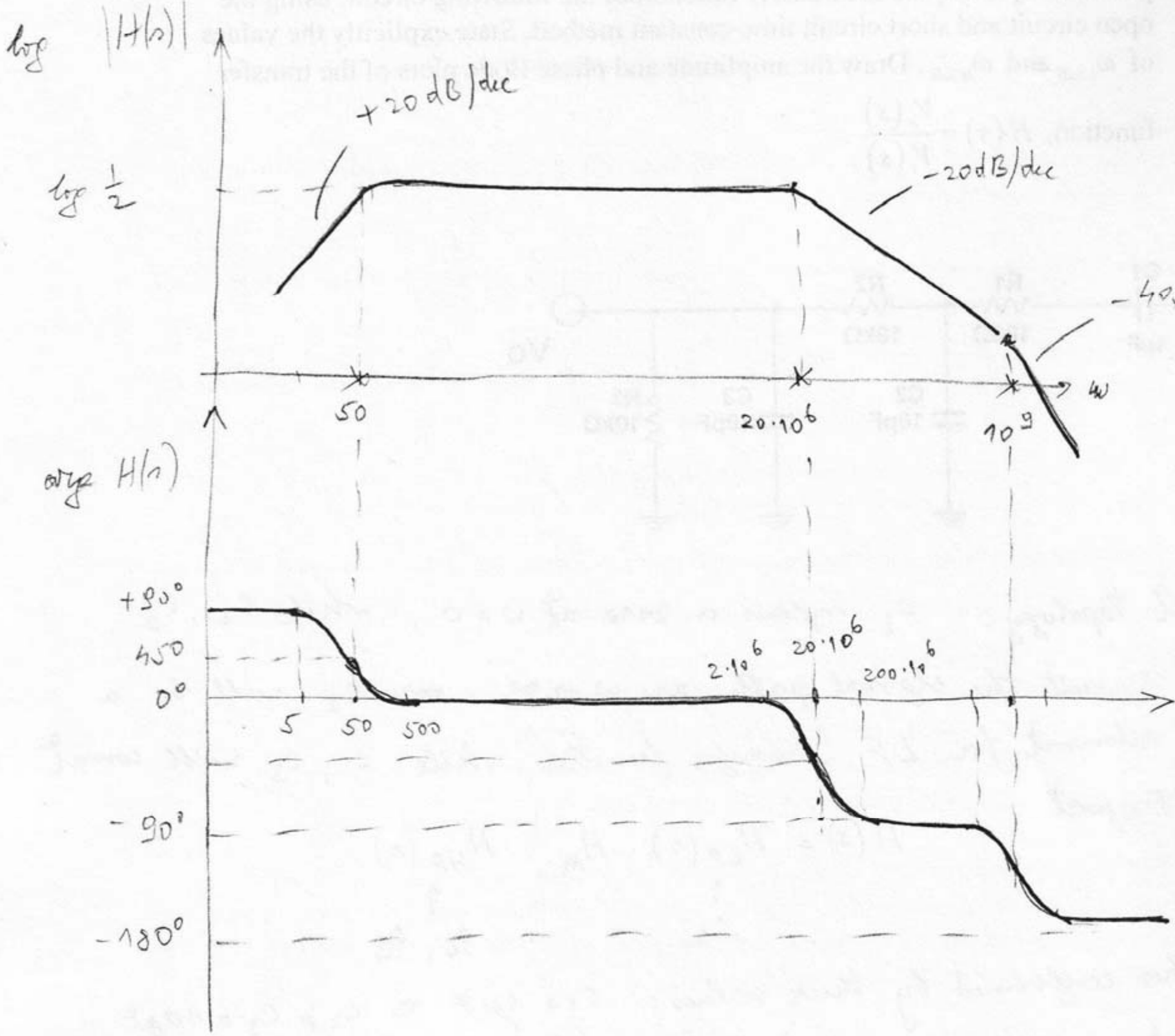
$\Downarrow$

$\omega_2 = \frac{1}{\tau'_{2OC}} = 10^9 rad/s$

The final transfer function is:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s}{s+50} \cdot \frac{1}{2} \cdot \frac{1}{\left(1 + \frac{s}{20 \cdot 10^6}\right)\left(1 + \frac{s}{10^9}\right)}$$

▸ The Bode plot of $$H(s) = \frac{s}{s+50} \cdot \frac{1}{2} \cdot \frac{1}{\left(1+\frac{s}{20\cdot 10^6}\right)\left(1+\frac{s}{10^9}\right)}$$

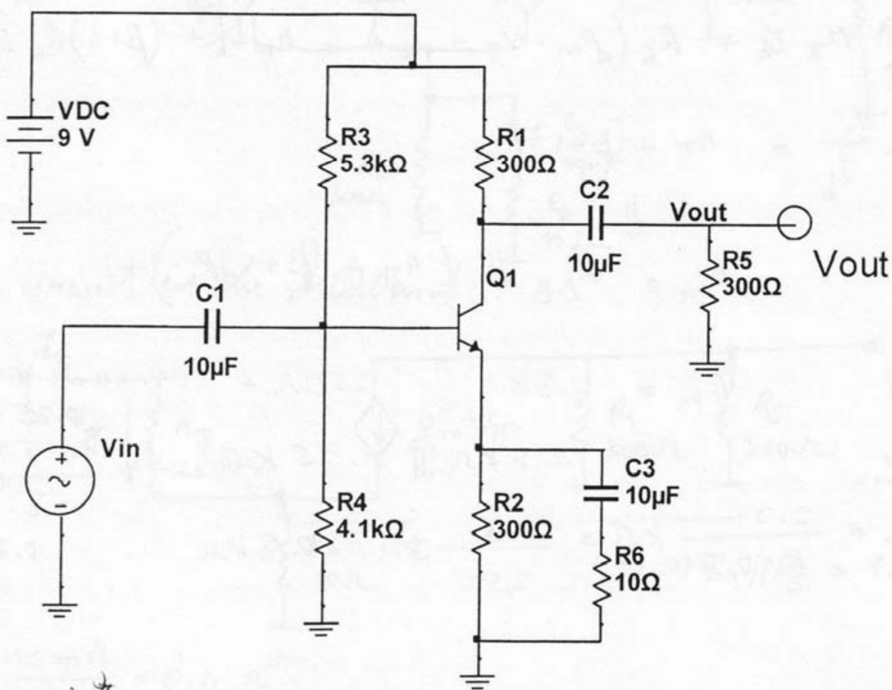


2. [50 marks] The bipolar transistor in the following circuit has $\beta = 100$.

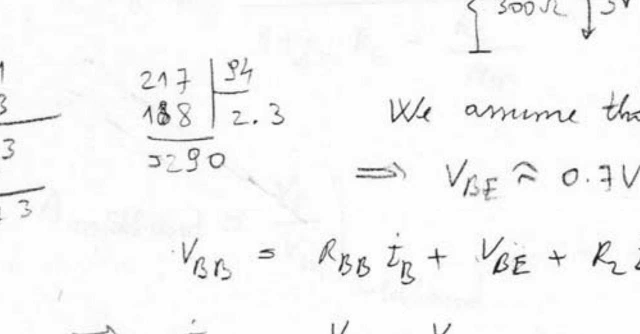
a. Compute the DC operating point ($I_B, I_C, V_{CE}$)

b. Redraw the schematic for the small signal analysis. Compute the midband gain value $A = \left.\frac{V_{out}}{V_{in}}\right|_{midband}$ (you may use $V_T \approx 25mV$)

c. What is the value of the equivalent impedance seen by the voltage source Vin (the input impedance of the transistor amplifier stage) in the midband?




a) For the DC operating point




$V_{BB} = 9V \cdot \frac{R_4}{R_3+R_4} = 9V \cdot \frac{4.1}{4.1+5.3} = 9V \cdot \frac{4.1}{9.4}$

$V_{BB} \approx 3.9V$

$$\begin{array}{r|l} 36.9 & 94 \\ 282 & 3.9 \\ =870 & \\ 846 & \\ 24 & \end{array}$$

$R_3 \| R_4 = \frac{4.1 \cdot 5.3}{9.4} = \frac{21.7}{9.4} k\Omega \approx 2.3 k\Omega$

$$\begin{array}{r} 4.1 \\ 5.3 \\ \hline 123 \\ 205 \\ \hline 21.73 \end{array} \qquad \begin{array}{r|l} 217 & 94 \\ 188 & 2.3 \\ \hline =290 & \end{array}$$

We assume that $Q_1$ is in the active operating region $\Rightarrow$

$\Rightarrow V_{BE} \approx 0.7V$

$$V_{BB} = R_{BB} i_B + V_{BE} + R_2 i_E = V_{BE} + \left(\frac{R_{BB}}{\beta+1} + R_2\right) i_E \Rightarrow$$

$$\Rightarrow i_E = \frac{V_{BB} - V_{BE}}{\frac{R_{BB}}{\beta+1} + R_2} = \frac{3.9V - 0.7V}{\frac{2.3k\Omega}{101} + 300\Omega} \approx \frac{3.2V}{320\Omega} = 10 mA$$

$$I_C = \alpha I_E \approx I_E = 10\,mA$$

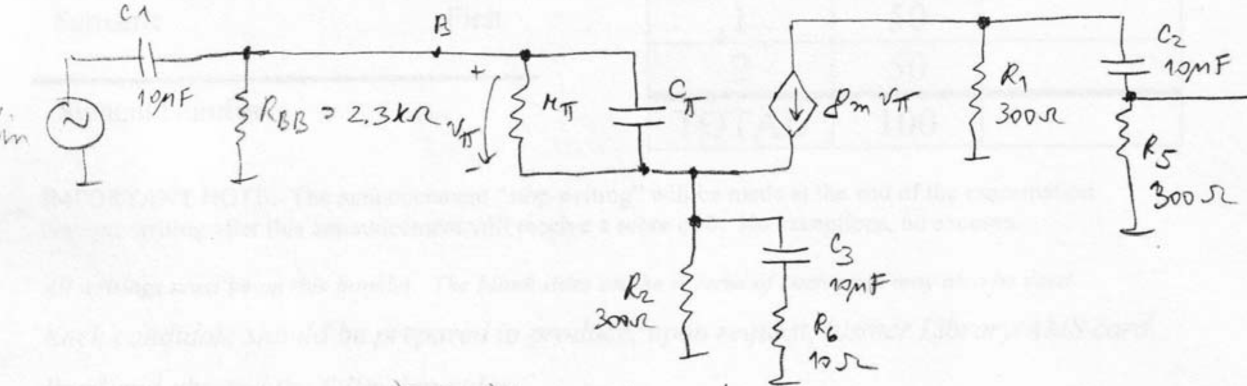
$$I_B = \frac{I_C}{\beta} = \frac{10\,mA}{100} = 100\,\mu A$$

$$V_C = V_{CC} - R_1 I_C = 9V - 300 \cdot 10 \cdot 10^{-3} V = 6V$$

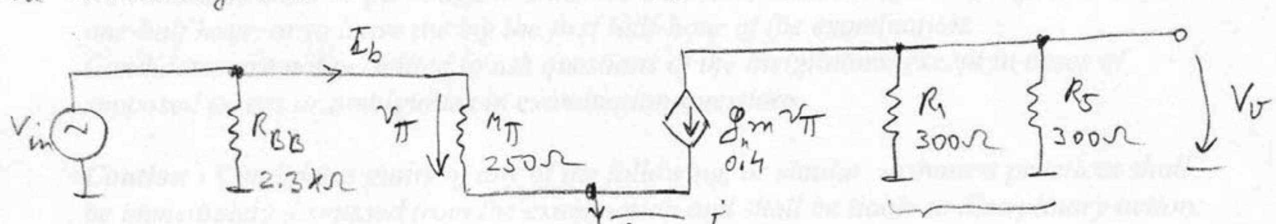
$$\Downarrow$$

$$V_{CE} = V_{CC} - R_1 I_C - R_2 I_E = 3V$$

(b) Small signal analysis circuit:



The small signal circuit in the midband:



$$R_L = R_1 \| R_5 = 150\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{10\,mA}{25\,mV} = 0.4\,\Omega^{-1}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.4}\,\Omega = 250\Omega$$

$$V_{in} = v_\pi + \left(g_m v_\pi + \frac{v_\pi}{r_\pi}\right) R_6 = v_\pi \left(1 + g_m R_6 + \frac{R_6}{r_\pi}\right) \Rightarrow$$

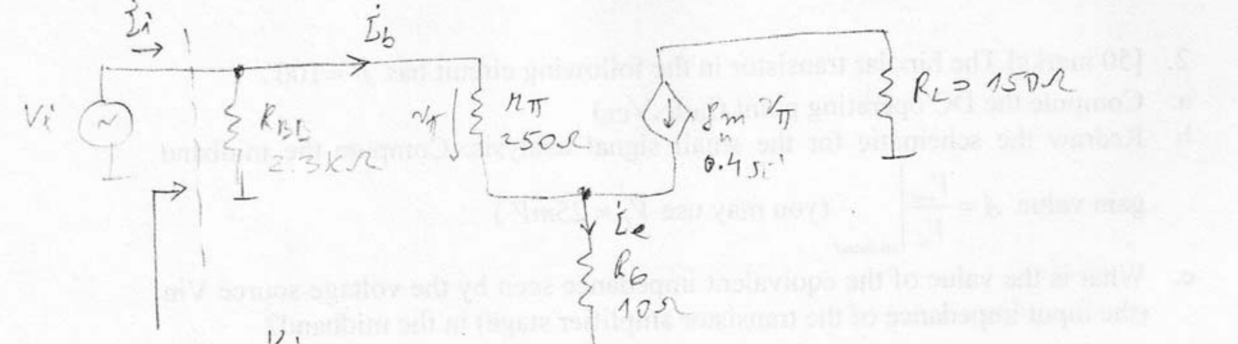
$$\Rightarrow v_\pi = \frac{V_{in}}{1 + g_m R_6 + \frac{R_6}{r_\pi}} \Rightarrow V_o = -g_m v_\pi R_L$$

$$V_o = -\frac{g_m}{1 + g_m R_6 + \frac{R_6}{r_\pi}} \cdot R_L \cdot V_{in}$$

$$A_{midband} = \left.\frac{V_o}{V_{in}}\right|_{midband} = -\frac{g_m R_L}{1 + g_m R_6 + \frac{R_6}{r_\pi}} = -\frac{0.4 \times 150}{1 + 0.4 \cdot 10 + \frac{10}{250}} =$$

$$= -\frac{60}{1 + 4 + 0.04} \approx -\frac{60}{5} = -12$$

c) The equivalent input impedance:




$$R_{in} = \frac{V_i}{i_i} = R_{BB} \parallel \left(\frac{V_i}{i_b}\right)$$

$$V_i = r_\pi i_b + R_6\left(g_m \cdot v_\pi + i_b\right) = r_\pi i_b + (\beta+1) R_6 \, i_b \Rightarrow$$

$$\Rightarrow \frac{V_i}{i_b} = r_\pi + (\beta+1) R_6$$

$$\Downarrow$$

$$R_{in} = R_{BB} \parallel \left(r_\pi + (\beta+1) R_6\right)$$

$$R_{in} = 2.3\,k\Omega \parallel \left(250\Omega + 101 \cdot 10\Omega\right)$$

$$= 2.3\,k\Omega \parallel 1.25\,k\Omega = \frac{\overset{0.25}{1.25} \times 2.3}{\underset{0.71}{3.55}}\,k\Omega$$

$$R_{in} = \frac{2.3}{4 \cdot 0.71}\,k\Omega = \frac{2.3}{2.84}\,k\Omega \approx 0.8\,k\Omega$$