ELEC 343, Assignment 2:
Problems form the Textbook: SP2.3-3, Problem(s) 4, 7, 13, 14, 18, and 19.

SP2.3-3. $W_f = \frac{2}{3} a(x) i^3$ and $W_c = \frac{1}{3} a(x) i^3$

$$\Delta W_f = W_{f(3A)} - W_{f(2A)} = (\frac{2}{3})(1)(3)^3 - (\frac{2}{3})(1)(2)^3 = \frac{38}{3} \text{ J}$$

$$\Delta W_c = W_{c(3A)} - W_{c(2A)} = (\frac{1}{3})(1)(3)^3 - (\frac{1}{3})(1)(2)^3 = \frac{19}{3} \text{ J}$$

4. (a) $W_f = \int i \, d\lambda$

With $dx = 0$,

$$d\lambda = (3xi^2 + 1)\, di; \text{ thus,}$$

$$W_f(i,x) = \int (i)(3xi^2 + 1)\, di = \int_0^i (3x\xi^3 + \xi)\, d\xi = \frac{3}{4} x i^4 + \frac{1}{2} i^2$$

$$W_c = \int \lambda\, di = \int (xi^3 + i)\, di = \int_0^i (x\xi^3 + \xi)\, d\xi = \frac{1}{4} x i^4 + \frac{1}{2} i^2$$

(b) With $dx = 0$,

$$d\lambda = (-2xi + \sin x)\, di; \text{ thus,}$$

$$W_f(i,x) = \int i(-2xi + \sin x)\, di = \int_0^i \xi(-2x\xi + \sin x)\, d\xi = -\frac{2}{3} x i^3 + \frac{1}{2} i^2 \sin x$$

$$W_c = \int (-xi^2 + i \sin x)\, di = \int_0^i (-x\xi^2 + \xi \sin x)\, d\xi = -\frac{1}{3} x i^3 + \frac{1}{2} i^2 \sin x$$

7. (a) $\Delta W_f = 02A0 - 01A0 = 0210$

(b) $\Delta W_c = 0C20 - 0B10$

(c) $\Delta W_e = 0$

(d) $\Delta W_m = \Delta W_f - \Delta W_e = \Delta W_f = 0210$

13. $W_{eS} = \frac{1}{2} l i^2 = 0$, since $l = 0$.

$W_f = \frac{1}{2} L(x) i^2$

For $x = 2.5$ mm,

$$L(x) = \frac{k}{x} = \frac{6.283 \times 10^{-5}}{2.5 \times 10^{-3}} = 0.0251 \text{ H}$$

Thus, with $i = 0.5$ A

$$W_f = \frac{1}{2}(0.0251)(0.5)^2 = 3.14 \text{ mJ}$$

$$W_{mS} = K \int_{x_0}^{x} (\xi - x_0)\, d\xi = \frac{1}{2} K (x - x_0)^2$$

$$= \frac{1}{2} 2667 (2.5 \times 10^{-3} - 3.0 \times 10^{-3})^2 = 0.333 \text{ mJ}$$

14. $v = r i + e_f$, from which $i = \frac{v - e_f}{r}$; thus,

$$W_{eL} = \int r i^2\, dt = \int r \left(\frac{v - e_f}{r}\right)^2 dt$$

$$= \frac{v^2}{r} \int dt - \frac{2v}{r} \int e_f\, dt + \frac{1}{r} \int e_f^2\, dt$$

Now, $W_E = W_{eL} + W_e$ and

$$W_E = \int v i\, dt = \int v \left(\frac{v - e_f}{r}\right) dt$$

$$= \frac{v^2}{r} \int dt - \frac{v}{r} \int e_f\, dt$$

This is the energy supplied from source. Therefore, W_e, which is the energy from the coupling field, is

$$W_e = W_E - W_{eL} = \frac{v}{r} \int e_f\, dt - \frac{1}{r} \int e_f^2\, dt$$

18. (a) $L_{ab} = -L_{sr}\cos(\theta_r + \frac{\pi}{6})$

To arrive at this expression let $\theta_r = -\frac{\pi}{6}$, whereupon the mutual coupling is a negative maximum.

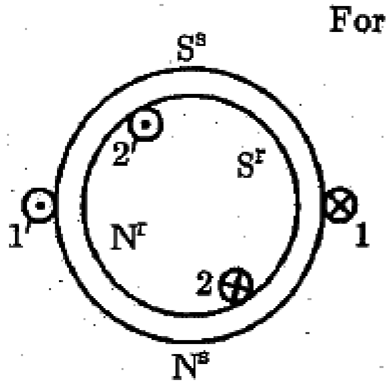
(b) $W_c = W_f = \frac{1}{2}L_{aa}\, i_a^2 + L_{ab}\, i_a i_b + \frac{1}{2}L_{bb}\, i^2$

Since L_{aa} and L_{bb} are constants,

$$T_e = \frac{\partial W_c}{\partial \theta_r} = -i_a\, i_b\, L_{sr}\, \frac{\partial \cos(\theta_r + \frac{\pi}{6})}{\partial \theta_r} = i_a\, i_b\, L_{sr}\sin(\theta_r - \frac{\pi}{6})$$

19. (a) $L_{12} = L_{sr}\sin\theta_r$

(b) For positive i_1 and negative i_2.



(c) $T_e = \frac{\partial W_c}{\partial \theta_r}$, since L_{11} and L_{22} are constants.

$$T_e = i_1\, i_2\, L_{sr}\, \frac{\partial \sin\theta_r}{\partial \theta_r} = i_1\, i_2\, L_{sr}\cos\theta_r$$