

ELEC 343, Assignment 5, Synchronous Motors:

Do Study Problem: SP7.3-2, SP7.4-1, SP7.4-3, do the examples 7B and 7C (absolutely vital), SP7.7-4  
Textbook Chapter 7 Problem(s) - 3, 9, 10, and 11.

$$\text{SP7.3-2. } v_{as} = r_s i_{as} + L_{asas} \frac{di_{as}}{dt} + \frac{d(L_{asfd} i_{fd})}{dt}$$

With  $i_{as} = 0$ ,

$$v_{as} = L_{asfd} \frac{di_{fd}}{dt} + i_{fd} \frac{dL_{asfd}}{dt}$$

With  $i_{fd}$  a constant, 
$$v_{as} = i_{fd} \frac{\partial L_{asfd}}{\partial \theta_r} \frac{d\theta_r}{dt} = i_{fd} \omega_r L_{sfd} \cos \theta_r$$

Since  $\omega_r = \frac{d\theta_r}{dt}$ , we have 
$$V_{as} = (1)(10)(0.1) \cos 10t = \cos 10t \text{ V}$$

$$v_{bs} = i_{fd} \frac{\partial L_{bsfd}}{\partial \theta_r} \frac{d\theta_r}{dt} = i_{fd} \omega_r L_{sfd} \sin \theta_r$$

Thus, 
$$V_{bs} = (1)(10)(0.1) \sin 10t = \sin 10t \text{ V}$$

**SP7.4-1.** Note that when  $L_{md} = L_{mq}$  the term with  $(L_{md} - L_{mq})$  disappears. Also note that the currents within the [ ] are stator currents. This is the reluctance torque.

**SP7.4-3.** The terms containing  $i_{fd}$ , which also multiplies only the stator currents.

**SP7.7-4.** From (7.7-30) with  $E_{afd}$  terms eliminated,

$$T_e = -\left(\frac{P}{2}\right)\left(\frac{1}{\omega_e}\right)\left[\frac{1}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)(\sqrt{2} V_s)^2 \sin 2\delta\right]$$

From Example 7C,

$$X_q = (377)(0.025) = 9.43 \Omega \quad X_d = (377)(0.105) = 39.59 \Omega$$

$$\left(\frac{1}{X_q} - \frac{1}{X_d}\right) = \left(\frac{1}{9.43} - \frac{1}{39.59}\right) = 0.0808$$

$$(\sqrt{2} V_s)^2 = [\sqrt{2} (110)]^2 = 24,200$$

$$T_e = -\left(\frac{2}{2}\right)\left(\frac{1}{377}\right)\left[\left(\frac{1}{2}\right)(0.0808)(24,200) \sin(-60^\circ)\right] = -2.25 \text{ N}\cdot\text{m}$$

**3.** From Fig. 7.5-1,

$$\begin{bmatrix} f_{qs}^r \\ f_{ds}^r \end{bmatrix} = \begin{bmatrix} -\sin \theta_r & \cos \theta_r \\ \cos \theta_r & \sin \theta_r \end{bmatrix} \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix} \quad \text{Thus, } \mathbf{K}_s^r = \begin{bmatrix} -\sin \theta_r & \cos \theta_r \\ \cos \theta_r & \sin \theta_r \end{bmatrix}$$

9.  $\sqrt{2} \tilde{I}_{as} e^{-j\delta} = I_{qs}^r - j I_{ds}^r$ . From Example 7B(a),

$$\begin{aligned} \sqrt{2} \tilde{I}_{as} e^{-j\delta} &= \sqrt{2} (52.5) \angle -30^\circ e^{-j(-23.4^\circ)} \\ &= 74.2 \angle -6.6^\circ = 73.8 - j8.53 \text{ A} \end{aligned}$$

$$I_{qs}^r = 73.8 \text{ A}; \quad I_{ds}^r = 8.53 \text{ A}$$

$$P_{in} = 40 \times 10^3 \text{ W}$$

$$2 |\tilde{I}_{as}|^2 r_s = (2)(52.5)^2(0.3) = 1.654 \text{ kW}$$

$$T_e = \frac{P_{out}}{\omega_{rm}} = \frac{40,000 - 1654}{(377)(\frac{2}{6})} = 305 \text{ N}\cdot\text{m}$$

This is essentially the torque for all parts since only the  $|\tilde{I}_{as}|^2 r_s$  terms change in (b) and its magnitude is insignificant compared to the total power input.

From Example 7B(b),

$$\begin{aligned} \sqrt{2} \tilde{I}_{as} e^{-j\delta} &= \sqrt{2} (45.4 \angle 0^\circ) e^{-j(-19.9^\circ)} \\ &= 64.2 \angle 19.9^\circ = 60.4 + j21.9 \text{ A} \end{aligned}$$

$$I_{qs}^s = 60.4 \text{ A}; \quad I_{ds}^s = -21.9 \text{ A}$$

From example 7B(c),

$$\begin{aligned} \sqrt{2} \tilde{I}_{as} e^{-j\delta} &= \sqrt{2} (52.5 \angle 30^\circ) e^{-j(-17.4^\circ)} \\ &= 74.2 \angle 47.4^\circ = 50.3 + j54.7 \text{ A} \end{aligned}$$

$$I_{qs}^s = 50.3 \text{ A}; \quad I_{ds}^s = -54.7 \text{ A}$$

10. From Example 6C,  $\tilde{V}_{as} = 110 \angle 0^\circ$  and  $\tilde{I}_{as} = 4.55 \angle -51.6^\circ$

$$\begin{aligned}\tilde{E}_a &= \tilde{V}_{as} - (r_s + jX_q)\tilde{I}_{as} \\ &= 110 \angle 0^\circ - [1 + j(377)(0.005 + 0.02)](4.55 \angle -51.6^\circ) \\ &= 110 \angle 0^\circ - 43.1 \angle 32.3^\circ = 77.1 \angle -17.4^\circ \text{ V}\end{aligned}$$

Note that, in Example 6C,  $\delta$  was found to be  $-17.4^\circ$  which checks with the above calculation. The phasor diagram is



11.  $\tilde{V}_{as} = (r_s + jX_q)\tilde{I}_{as} + \tilde{E}_a$

For open circuit conditions,  $\tilde{V}_{as} = \tilde{E}_a$ ; also,

$$\tilde{E}_a = \frac{1}{\sqrt{2}} [(X_d - X_q)I_{ds} + X_{md} I'_{fd}] e^{j\delta}$$

With  $\tilde{I}_{as} = 0$  and  $\tilde{V}_{as} = 440 \angle 0^\circ$ ,

$$\tilde{E}_a = \frac{1}{\sqrt{2}} X_{md} I'_{fd} \angle 0^\circ = 440 \angle 0^\circ$$

$$I'_{fd} = \frac{440 \sqrt{2}}{(377)(13.7 \times 10^{-3})} = 120.5 \text{ A}$$

$$V'_{fd} = r'_{fd} I'_{fd} = (0.13)(120.5) = 15.7 \text{ V}$$

The value of  $I'_{fd}$  in Fig. 6.8-1 appears to be slightly less than 125 A; however, the scale of the graph does not allow us to check this value exactly.