
EECE 481

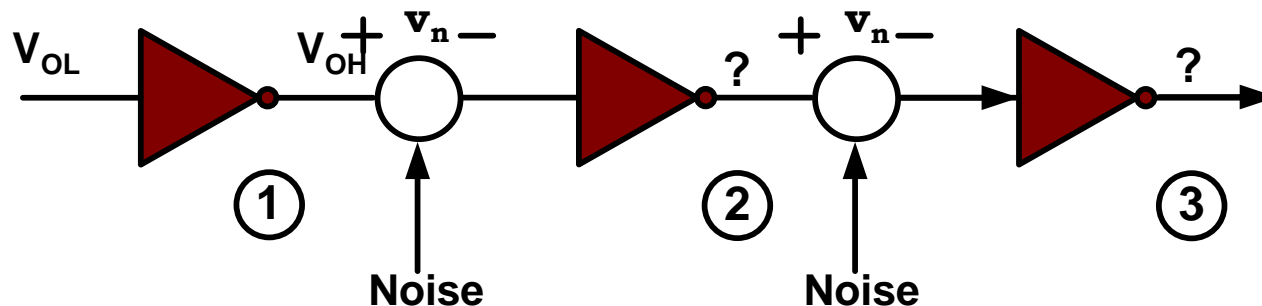
Design of Resistive-Load Inverter (Noise-margin-centric approach) Lecture 5

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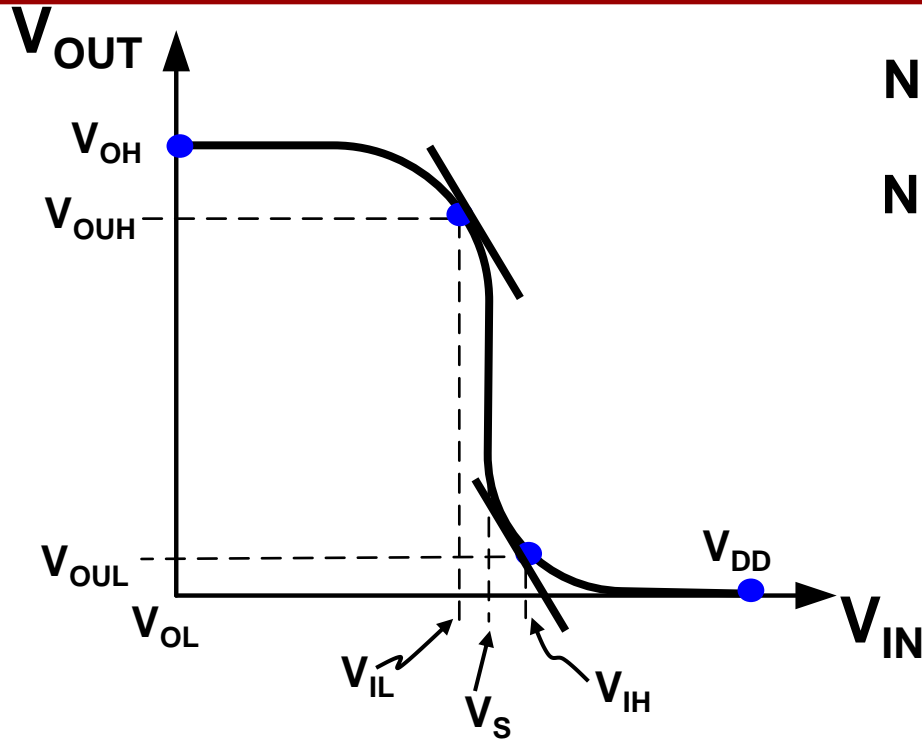
Multi Stage Noise Margin (MSNM) - Review

- Noise actually occurs between every gate and not at a single stage, as we have assumed so far



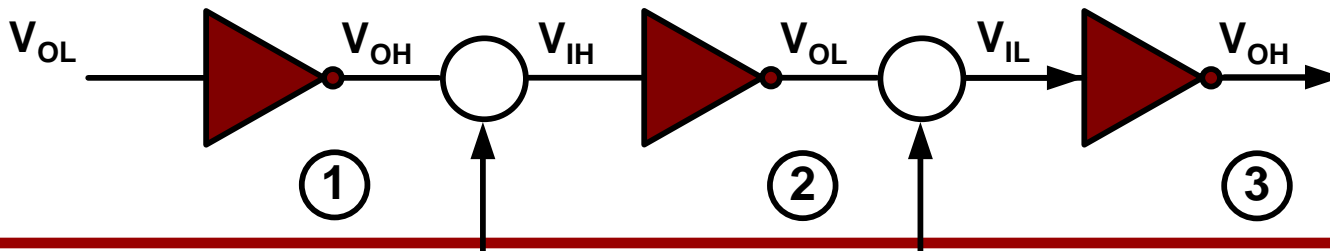
- In the above circuit, we need to determine how much noise we can tolerate before the circuit stops working as expected
- We note that $V_{out} = f(V_{in})$
- With noise $V_{out}' = f(V_{in}) + V_{noise} \times \text{Gain} + \text{Higher-order terms}$
- If the $\text{Gain} < 1$, then noise is attenuated; otherwise it is amplified
- IDEA: Develop a metric based on keeping the $\text{Gain} < 1$

Classical Noise Margin - Review



$$NM_L = V_{IL} - V_{OL}$$

$$NM_H = V_{OH} - V_{IH}$$

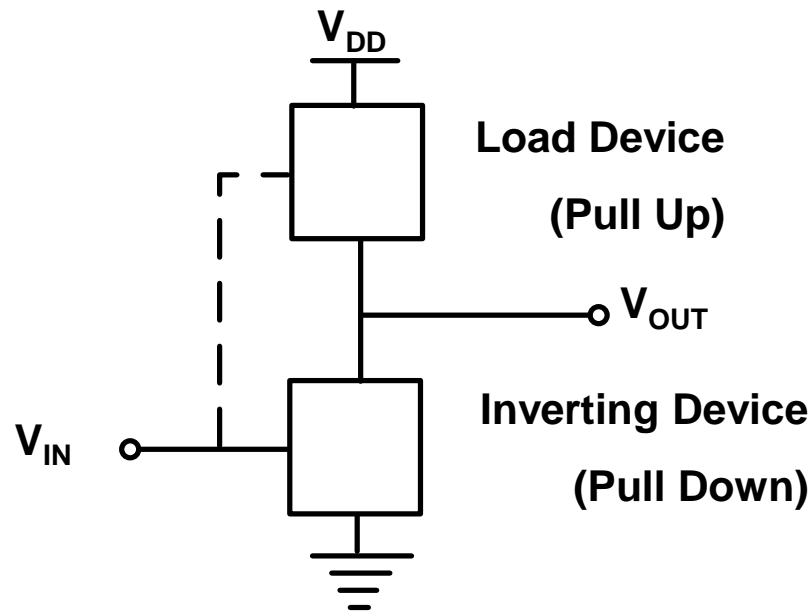


Requirements for a Valid Logic Gate - Review

- Must have a high gain region between two low gain regions
- Gain must be below 1 for low gain regions
- Gain must be greater than 1 for high gain region
- Output must swing from valid low to valid high
 - Low output should be below V_{IL}
 - High output should be above V_{IH}

Generic Structure of MOS inverters

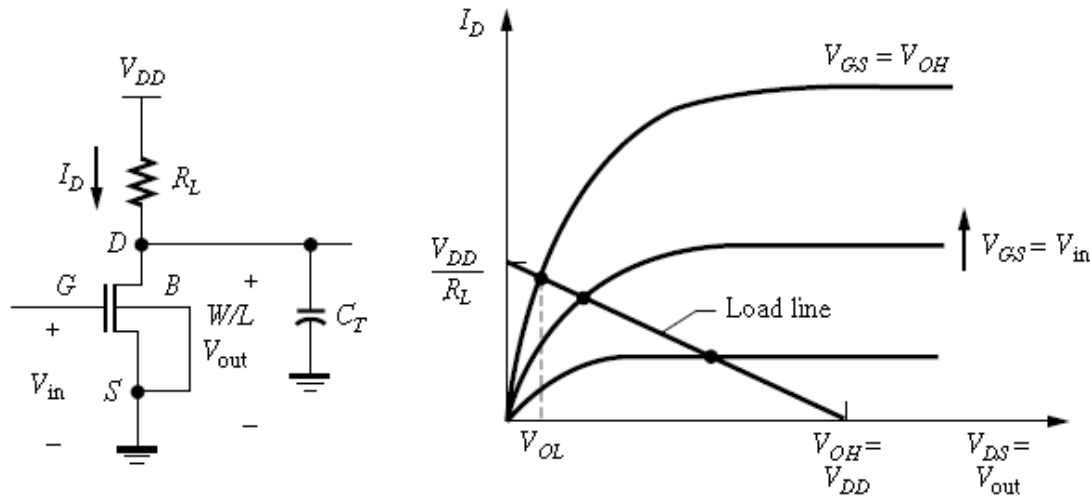
- Basic structure of MOS inverter is shown below: NMOS pulldown device with a variety of possible pullup devices



- We will derive the noise margin parameters for different types of inverters

Resistive-Load Inverter Design

“5 Point Technique”



How to plot VTC from load line representation?

- 1) Find the two extremes of output voltage, e.g. V_{OH} and V_{OL} (be careful with the MOS region of operation for each derivation)
- 2) Express the V_{out} - V_{in} relationship (usually the current equations of the devices provides that) and find points where $\delta V_{out}/\delta V_{in} = -1$ (V_{IH} and V_{IL})
- 3) Find the switching point where $V_{in} = V_{out} = V_S$
- 4) Plot VTC using the values of five points and calculate NM as $V_{OH} - V_{IH}$ and $V_{IL} - V_{OL}$

Resistive-Load Inverter Design – cont'd

Let's apply the technique to resistive-load inverter design

1) The extremes

Remember that in inverters an **input** value of V_{OH} (it is the output of **previous** stage) creates an **output** of V_{OL} and vice versa, hence we first find the easier of the two, i.e. the more straightforward one, and then replace it in expression of $I_{OUT}=I_{load}$ to find the other one

- At one extreme the transistor is off hence no current, $V_{OH}=V_{DD}$

- At the other extreme we use $V_{in}=V_{OH}=V_{DD}$ in the expression of $I_{OUT}=I_{load}$ to find the V_{OL}

$$I_R = I_{DS}(\text{lin})$$

Substituting in the expressions for the resistor current and NMOS current:

$$\frac{V_{DD} - V_{OL}}{R_L} = \frac{W_N}{L_N} \frac{\mu_n C_{ox}}{2 \left(1 + \frac{V_{OL}}{E_C L} \right)} [2(V_{OH} - V_T)V_{OL} - V_{OL}^2]$$

We know V_{OL} is very small so for your calculations we can ignore it (Engineering approximations are IMPORTANT!)

Resistive-Load Inverter Design – cont'd

setting $k = (W/L) \mu_n C_{ox}$:

$$\frac{V_{DD} - V_{OL}}{R_L} = \frac{k}{2} [2(V_{OH} - V_T)V_{OL} - V_{OL}^2]$$

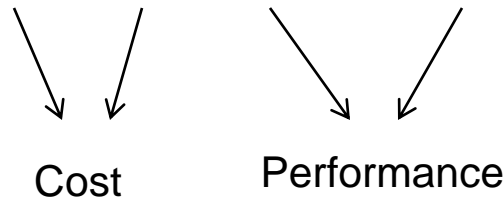
Board Notes

Knowing that $V_{OH} = V_{DD}$ and neglecting V_{OL}^2

$$V_{OL} \approx \frac{V_{DD}}{1 + kR_L(V_{DD} - V_T)}$$

Q: What should V_{OL} ideally be? What are the engineering trade-offs to make it happen?

Note that we always trade-off **power**, **area**, **timing** and **noise margin** versus one another



Resistive-Load Inverter Design – cont'd

2. Now, let's derive V_{IL}

Note that these values are now input voltages corresponding to $\delta V_{out}/\delta V_{in} = -1$

Step 1: detect the region of MOS operation

For V_{IL} the input is still considered low and output voltage is near V_{DD} (large V_{DS})

$$I_R = I_{DS}(\text{sat})$$

$$\frac{V_{DD} - V_{out}}{R_L} = \frac{W\mu_{sat}C_{ox}(V_{in} - V_T)^2}{(V_{in} - V_T) + E_C L}$$

Board Notes

Use engineering intuition again: $V_{in} - V_T$ is small

Recalling that $v_{sat} = \mu E_C / 2$

Setting $k = (W/L)\mu_n C_{OX}$

Now differentiate with respect to V_{in} , set $\delta V_{out}/\delta V_{in} = -1$, and $V_{in} = V_{IL}$

$$V_{IL} = V_T + \frac{1}{kR_L}$$

Resistive-Load Inverter Design – cont'd

Let's look at NM_L now:

$$V_{IL} = V_T + \frac{1}{kR_L}$$

$$V_{OL} \approx \frac{V_{DD}}{1 + kR_L(V_{DD} - V_T)}$$

Q: Do you see the problem with noise margin optimization? (change k and R_L to get a higher NM_L)

Q: Which one would be a higher priority?

[Board Notes](#)

Resistive-Load Inverter Design – cont'd

2. Now, let's derive V_{IH}

Step 1: detect the region of MOS operation

For V_{IH} the output voltage is near 0V, small V_{DS} , Linear region of operation

$$\frac{W}{L} \frac{\mu_n C_{ox}}{\left(1 + \frac{V_{out}}{E_C L}\right)} \left[(V_{in} - V_T) V_{out} - \frac{V_{out}^2}{2} \right] = \frac{V_{DD} - V_{out}}{R_L}$$

Simplification using Engineering intuition

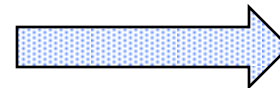
$$k \left[(V_{in} - V_T) V_{out} - \frac{V_{out}^2}{2} \right] \approx \frac{V_{DD} - V_{out}}{R_L}$$

$$V_{IH} = V_T + \sqrt{\frac{8V_{DD}}{3kR_L}} - \frac{1}{kR_L}$$

Differentiation

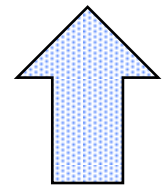
$$k \left[(V_{in} - V_T) \frac{\partial V_{out}}{\partial V_{in}} + V_{out} - V_{out} \frac{\partial V_{out}}{\partial V_{in}} \right] = -\frac{1}{R_L} \frac{\partial V_{out}}{\partial V_{in}}$$

$$\delta V_{out} / \delta V_{in} = -1$$



$$V_{IH} = V_T + 2V_{out} - \frac{1}{kR_L}$$

What is V_{out} here?



Resistive-Load Inverter Design – cont'd

3. In order to complete the derivation and plot the VTC we only need V_S now (switching point)

Q: what is the MOS region of operation at switching point? Why?

$$I_R = I_{DS} \quad \text{and} \quad V_{in} = V_{out} = V_S$$

$$\frac{WU_{sat}C_{ox}(V_S - V_T)^2}{(V_S - V_T) + E_C L} = \frac{V_{DD} - V_S}{R_L}$$

Side discussion: How to solve higher-order equation?

- Deterministic equations
- Iteration technique
- SPICE

