# EECE 571M/491M

## Introduction to Hybrid Systems and Control

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*http://coursees.ece.ubc.ca/571m*

*moishi@ece.ubc.ca*



# Practical Information

- Course website:
  - `www.ece.ubc.ca/~elec571m`
- Office hours:
  - Monday 1:00-3:00pm, Kaiser 3111 (tentative)

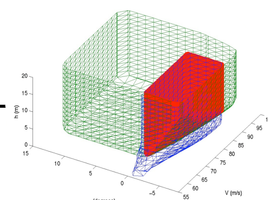


# Your Instructor

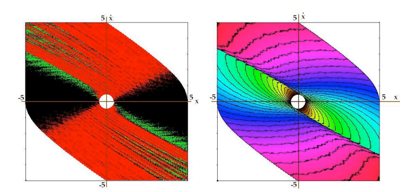
- B.S., Princeton University, 1998
- M.S., Ph.D., Stanford University, 2000 and 2004
- Truman Postdoctoral Fellow, Sandia National Labs (USA)
- Assistant Prof. in ECE at UBC since Aug. 2006
- Visiting researcher at NASA Ames, Honeywell
- Policy experience at US National Academies; US Nat'l Ecological Observatory Network (NEON)



# Research



- Hybrid control systems
- Aircraft flight management systems
- Biological modeling and control
- User-interfaces for hybrid systems
- Research website: *http://www.ece.ubc.ca/~moishi*



## Practical Information

- Textbook
  - B. Friedland, "Control system design: An introduction to state-space methods"
- References
  - Nonlinear continuous systems and control
    - Khalil, "Nonlinear systems"
  - Discrete event systems
    - Cassandras and Lafortune, "Introduction to Discrete Event Systems"
  - Switched systems
    - Liberzon, "Switching in Systems and Control"



## Practical Information

- Lecture presentations
  - Available online on course website
- Additional notes
  - used with permission of C. Tomlin, Stanford University
  - are from a draft of the monograph *Hybrid Systems*, by J. Lygeros, S. Sastry, C. Tomlin.
- Additional handouts
  - Excerpts as necessary



## Practical Information

- Tentative dates:
  - Midterm 1: February 15, 2008 (Fri)
  - Midterm 2: March 28, 2008 (Fri)
  - Final report: April 18, 2008 (Fri)
  - Final presentation: April 7-11, 2008 (last week of classes)
- Grading:
  - Problem Sets: 35%
  - Midterms: 30%
  - Final project and presentation: 35%

*Let me know of conflicts ASAP!*



## Required Background

- Linear algebra
  - Matrix algebra
  - Eigenvalues, eigenvectors
- Differential equations
  - Linear state-space models
- Discrete Math
  - Basic logic operators

# What are hybrid systems?

- Systems with both
  - Continuous processes
  - Discrete processes
- For example
  - Embedded systems
  - Physical processes with supervisory mode-logic
  - Physical processes with discontinuities
  - Hierarchical systems
  - Some biological processes



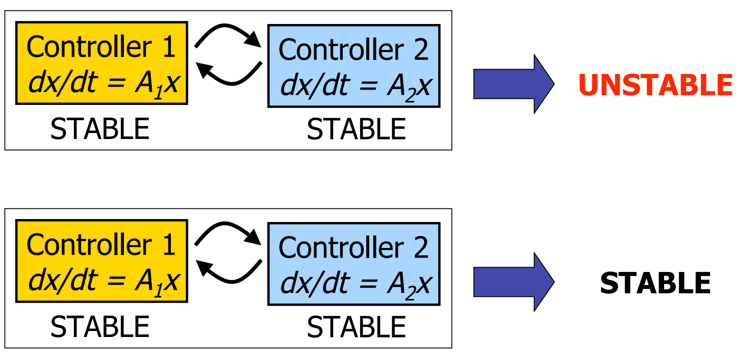
# Motivation

- Hybrid systems are pervasive
- However, techniques from
  - standard control theory (for continuous systems)
  - standard discrete event systems

  don't generalize to hybrid systems
- New techniques have been developed to analyze and control hybrid systems
- We will study some of these techniques in this course
- This is an active area of controls research

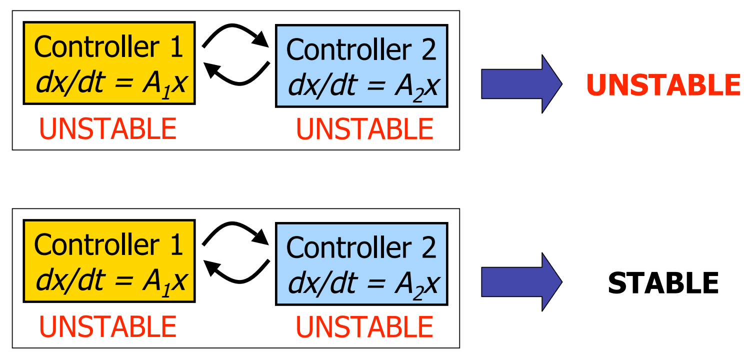


# Motivation: Switching



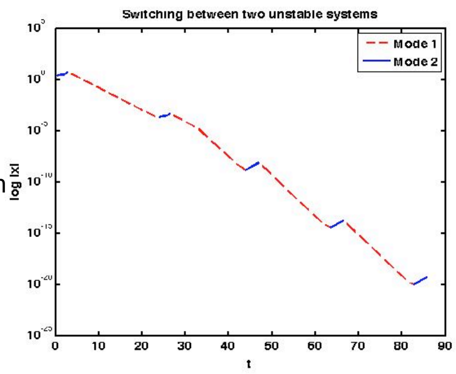



# Motivation: Switching



## Motivation: Switching

- Mode 1: unstable saddle
- Mode 2: unstable focus
- Without switching, the system in either mode is UNSTABLE
- However, the hybrid system with switching on specific surfaces is STABLE






## Motivation: Switching

- When will switching de-stabilize a system?
- When can switching stabilize an unstable system?

- Switching can be
  - Constrained by time
  - Constrained by hypersurfaces (conditions on the state-space)
  - Driven by control inputs
  - Driven by disturbances

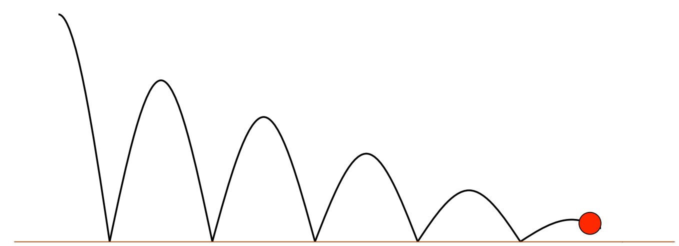


## Motivation

- In addition to stability, other control concepts must also be reconsidered
  - Robustness
  - Optimality
  - Safety (Reachability)
  - Estimation
  - Controllability and observability
  - and others…



## Example: Bouncing ball

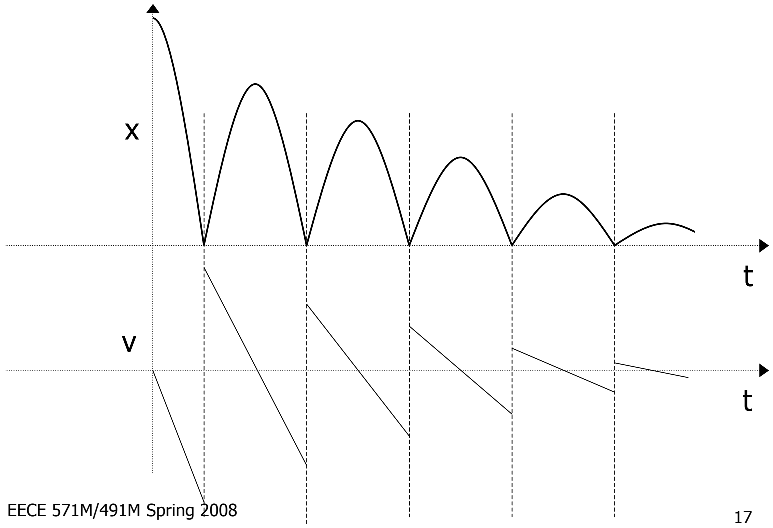


- While the ball is in the air

  $m\, d^2x/dt^2 = -mg$
- Upon impact with the ground

  $v_{new} = -c\, v_{old}, \quad 0 < c < 1$

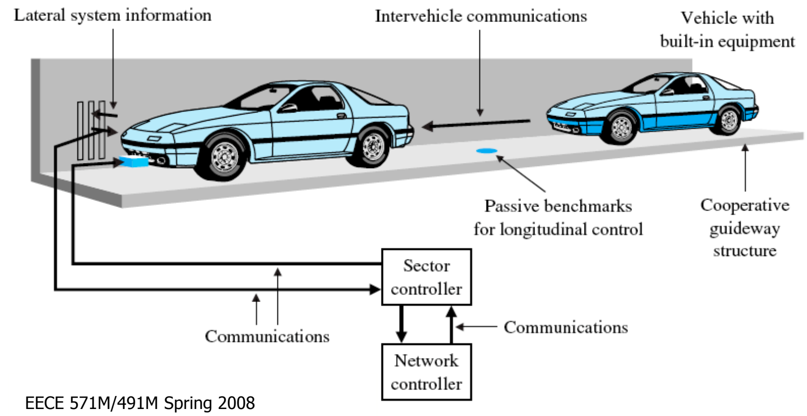
# Example: Bouncing ball






# Example: Automated highway

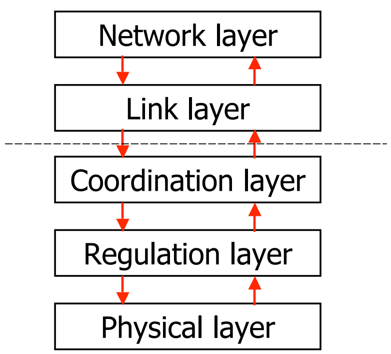
- Automated fleets of vehicles

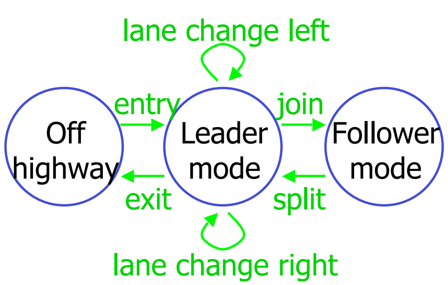





# Example: Automated highway

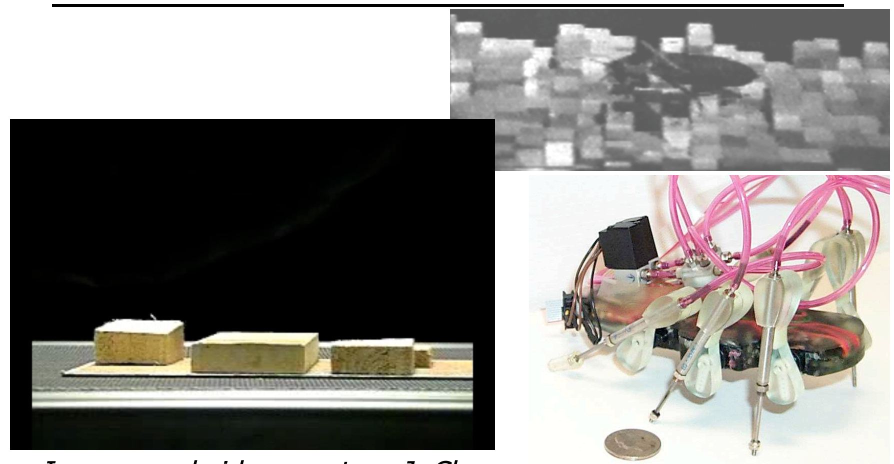
- Fleet control
- Individual car control



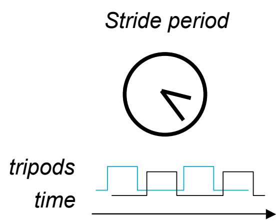


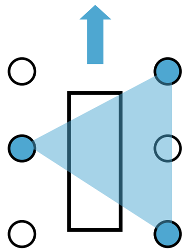

# Example: Robotic hexapod

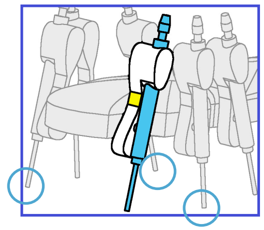


*Images and video courtesy J. Cham*

# Example: Robotic hexapod





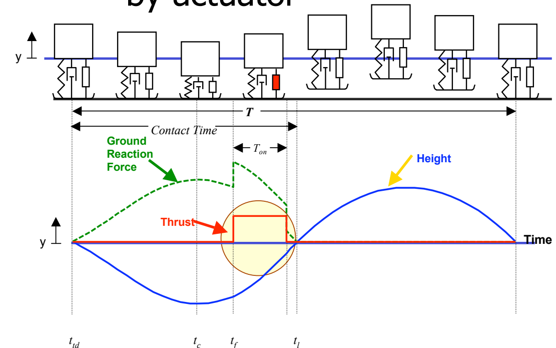
- Open-loop, clock-driven motor pattern
- Alternates thrust in leg tripods at a certain "stride period"

*Images and video courtesy J. Cham*



# Example: Robotic hexapod

- Find timing of thrust to maximize work done by actuator

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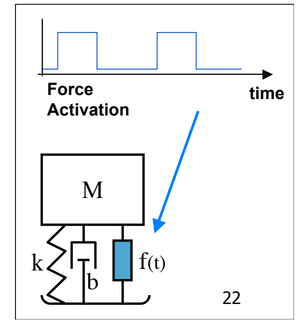




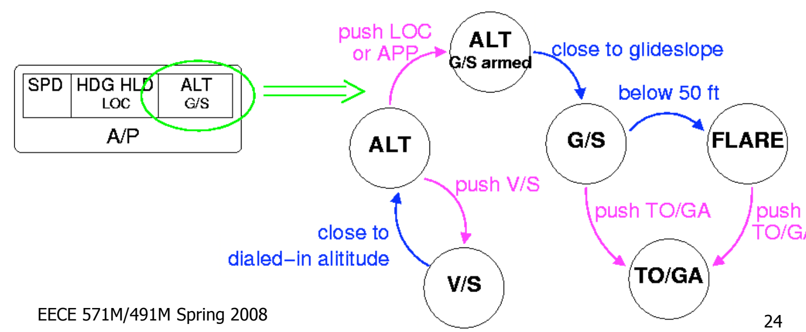

# Example: Aircraft FMS

- Aircraft Flight Management System
- Flight "modes"
  - Lateral
  - Longitudinal
  - Throttle



# Example: Aircraft FMS

- Hundreds of flight modes on the Boeing 777
- Longitudinal modes during landing/go-around

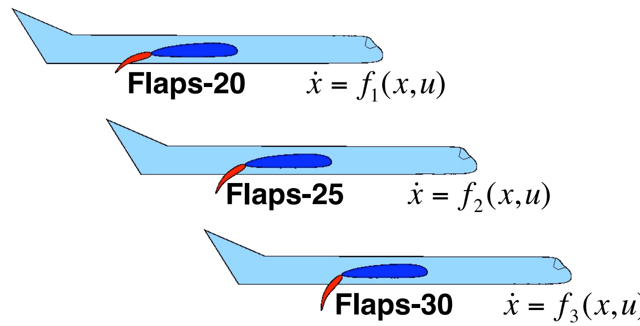


# Example: Aircraft FMS

**Hybrid** due to:
•Aerodynamic configuration
•Autopilot modes



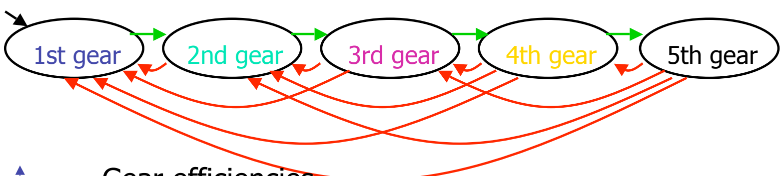

# Example: Aircraft FMS



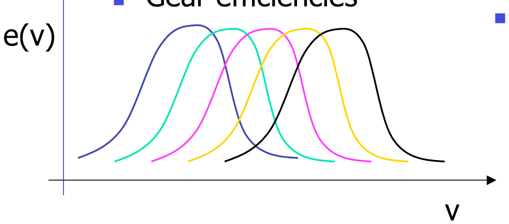



# Example: Car transmission

- Five-gear transmission




- Gear efficiencies



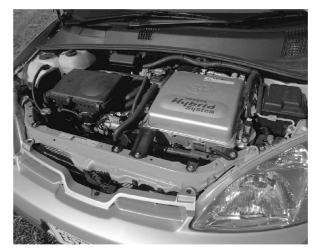

- What is optimal switching amongst gears to reach 100 kmh within time T?



# Example: Car components

Hybrid engine

- What switching scheme between electric and gas engines will maximize efficiency and performance?

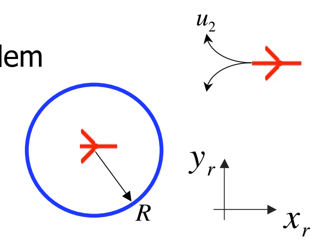


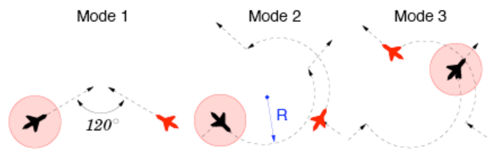
ABS

- Which frequencies of on/off brake pulses allows tire traction while decelerating the car as quickly as possible?

# Example: Collision avoidance

- Kinematics of two vehicles
- "Homicidal chauffeur" problem
- Aircraft move along known trajectories and initiate avoidance maneuvers simultaneously






# Example: Tryptophan, *E. Coli*

Metabolic reactions

Analog circuit abstraction

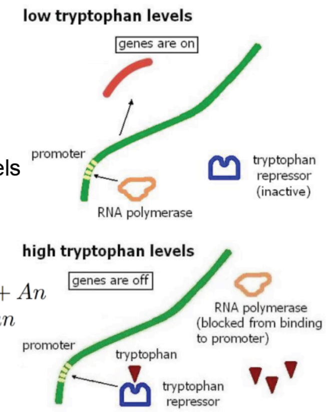
- Accumulation: Capacitor
- Degradation: Resistor

Circuit logic

- Encodes on/off switches (e.g., genes triggered by levels of protein concentrations)

Stoichiometric reactions

$$\begin{aligned} Chor + Gln &\rightarrow Glu + Pyr + An \\ An + Prpp &\rightarrow Ppi + Npran \\ Npran &\rightarrow Cpad5p \\ Cpad5p &\rightarrow Co2 + Igp \\ Igp + Ser &\rightarrow T3p1 + Trp \end{aligned}$$

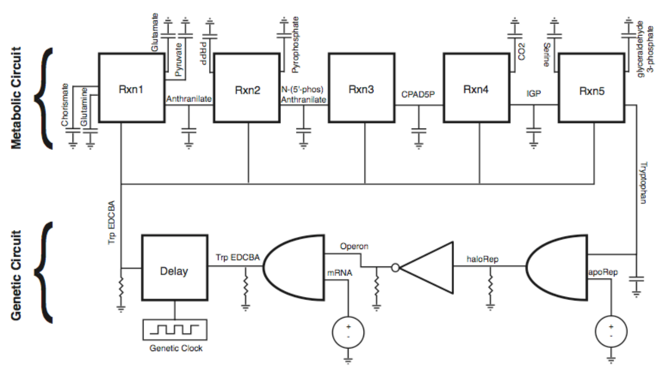



Courtesy E. May



# Example: Tryptophan, *E. Coli*

- Metabolic and genetic networks



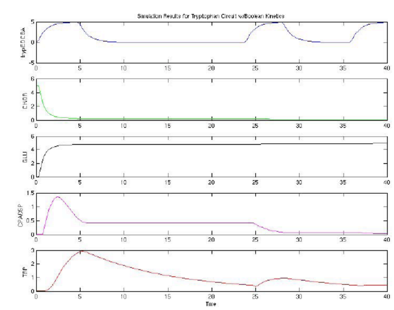

Courtesy E. May



# Example: Tryptophan, *E. Coli*

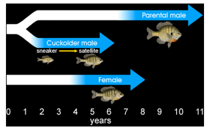
- Metabolic and genetic networks

$$\begin{gathered} chor + gln \rightarrow glu + pyr + an \\ an + prpp \rightarrow ppi + npran \\ npran \rightarrow cpad5p \\ cpad5p \rightarrow co2 + igp \\ igp + ser \rightarrow t3p1 + trp \end{gathered}$$



Courtesy E. May

# Example: Coalition dynamics



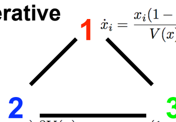

**Competitive**

1: $\dot{p} = \frac{p(1-p)}{V_1(p,q,r)} \frac{\partial V_1(p,q,r)}{\partial p}$

2: $\dot{q} = \frac{q(1-q)}{V_2(p,q,r)} \frac{\partial V_2(p,q,r)}{\partial q}$

3: $\dot{r} = \frac{r(1-r)}{V_3(p,q,r)} \frac{\partial V_3(p,q,r)}{\partial r}$

**Cooperative**




$\dot{x}_i = \frac{x_i(1-x_i)}{V(x)} \frac{\partial V(x)}{\partial x_i}$

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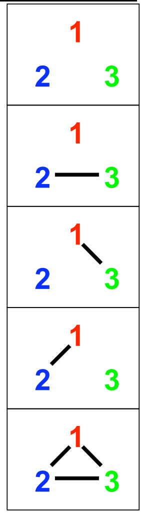


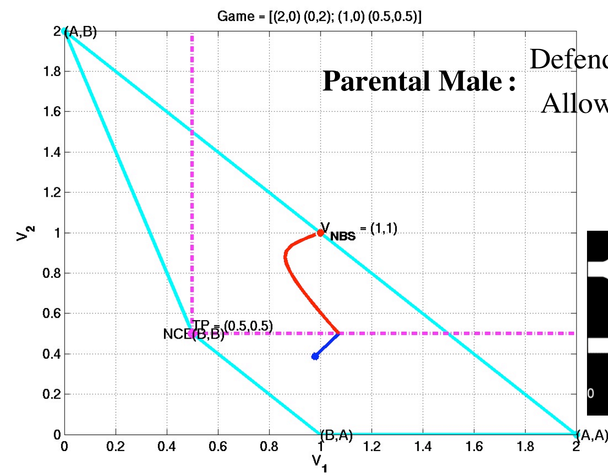


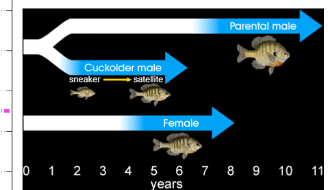

# Example: Coalition dynamics

**Cuckolder Male:**

**Parental Male:**

| | Avoid | Intrude |
|---|---|---|
| Defend | (2,0) | (0,2) |
| Allow | (1,0) | (0.5,0.5) |





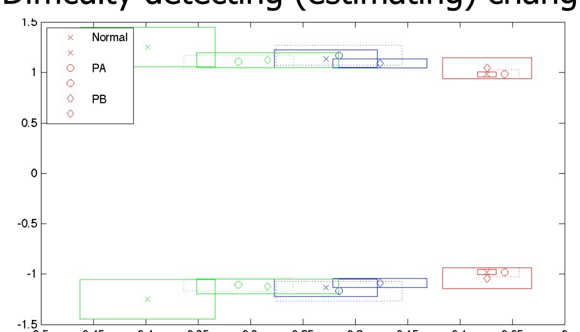

# Example: Feedback in PD

- Problems with movement due to chemical imbalance
- Faulty feedback signal in the brain
- Failures in multi-tasking
  - Reaching
  - Balancing
- Fast switching between tasks



# Example: Feedback in PD

- Problems with movement due to chemical imbalance
- Difficulty detecting (estimating) changes



## Further examples…

- Transportation
  - Automobiles
  - Civil jet aircraft (autopilots; air traffic control)
  - Traffic signals, highway monitoring
- Robotics
  - Dextrous manipulation, haptics
  - Research submarines
  - Biomedical devices
  - Smart wheelchairs



## Further examples…

- Biological systems
  - Cell regulation mechanisms
  - Population dynamics
- Economic systems
  - Fiscal policies
- Biomedical systems
  - Circadian clock
  - Prostheses
  - Mitigation of Parkinson's disease symptoms



## Challenges in Hybrid Control

- Cooperative control of multiple autonomous vehicles
- Control of embedded systems
- Verification / Reachability
- Estimation and control of stochastic systems
- Computational tools for control synthesis
- High-dimensional systems (scalability of computational methods)
- Biological modeling and control
- Accurate simulation
- Incorporating the human in the loop



## Course Content

- Mathematical models
  - Continuous-time models (linear)
  - Discrete event systems (finite state machines, timed automata)
  - Hybrid system formulations
- Basic stability concepts
  - Review of continuous system stability
  - Phase-plane analysis for 2D nonlinear systems
  - Hybrid equilibrium
  - Multiple Lyapunov functions

# Course Content

- Switched stability
  - Systems with state-based switching
  - Common Lyapunov function
- LMIs to prove stability of linear systems
  - Global quadratic Lypaunov function
  - Piecewise quadratic Lyapunv functions
- Observability / Controllability of linear hybrid systems
- Optimality
- Estimation



# Goals of the Course

- Identify when hybrid control is useful
- Categorization and familiarity with different classes of hybrid systems
- Analyze and predict common behaviors of hybrid systems with linear continuous dynamics
- Read research papers in hybrid control
- Solve hybrid control design problems
- Use relevant computational tools
- Recognize difficult hybrid control problems and when computation will be difficult



# Summary

- Logistics
  - Check course website for most up-to-date information, including room location change starting next week
  - Notify me ASAP of any conflicts with midterm or presentation dates
- Introductory material
  - Hybrid systems are pervasive
  - Hybrid systems require specialized tools for analysis and design
  - Wide range of applications, engineered and natural, can be modeled as hybrid systems
  - Hybrid systems draw upon ideas from both continuous control theory and discrete event systems



# Summary

- Upcoming topics (this week and next)
  - Further examples
  - Modeling formalism
    - Continuous systems
    - Discrete event systems
    - Hybrid systems
  - Linear vs. nonlinear systems
- Readings
  - Tomlin Lecture Notes 1 and 2
  - Khalil excerpt