EECE 491M, Spring 2008

Lecture 12

Static Reference Inputs

Dr. Meeko Oishi

Electrical and Computer Engineering

University of British Columbia, BC

http://courses.ece.ubc.ca/~elec491m

moishi@ece.ubc.ca

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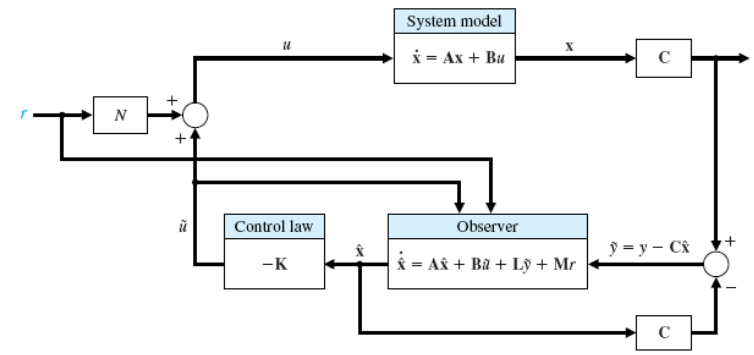
Reference inputs

- Output-based command following



$$(\tilde{u} = u - Nr)$$

Reference inputs

- For command following, additional control components may be needed
- The control *u = -Kx+r* can lead to steady-state errors when tracking a reference input *r*.
- **Solution**: Pre-multiply *r* by carefully chosen matrix *N*

Case 1: Full-state feedback

$$u = -Kx + Nr$$

Case 2: Full-state feedback with full-order observer

$$u = -K\hat{x} + Nr$$

$$\dot{\hat{x}} = A\hat{x} + B(u - Nr) + L(y - C\hat{x}) + Mr$$

Reference inputs: Case 1

Full-state feedback regulation of steady-state output y_{ss} to desired steady-state value r_{ss}

- Standard procedure:
  - To track a constant desired position x_{ss} (ss="steady-state") with control u_{ss}, note that

$$0 = Ax_{ss} + Bu_{ss}$$
$$y_{ss} = Cx_{ss} + Du_{ss}$$

  - and substituting $x_{ss}=N_x r_{ss}$, $u_{ss}=N_u r_{ss}$, we get

$$0 = AN_x r_{ss} + BN_u r_{ss}$$
$$r_{ss} = CN_x r_{ss} + DN_u r_{ss}$$

  - when $y_{ss}=r_{ss}$ in the steady-state

Reference inputs: Case 1

- In matrix form,

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} r_{ss} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} N_x \\ N_u \end{bmatrix} r_{ss}$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- and therefore the full-state regulating input is

$$u = -Kx + Nr, \quad N = N_u + KN_x$$
$$= -Kx + \left(N_u + KN_x\right) r$$

to ensure **no** steady-state error in tracking *r.*

Reference inputs: Case 2

Command following with full-state feedback and full-order observer

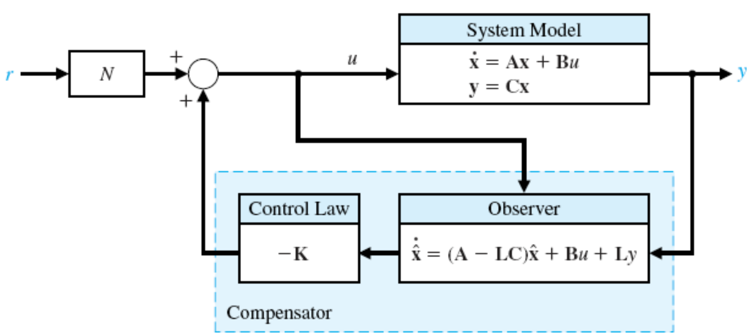
- General form:

$$u = -K\hat{x} + Nr$$
$$\dot{\hat{x}} = A\hat{x} + B(u - Nr) + L(y - C\hat{x}) + Mr$$

- There are a variety of ways to choose *M*, *N*
  - Autonomous estimator: Select *M* and *N* so that the state estimator error equation is independent of *r*
  - Tracking-error estimator: Select *M* and *N* so that only the **tracking** error ($e_r = r - y$) is used in the control
  - And others…

Reference inputs: Case 2

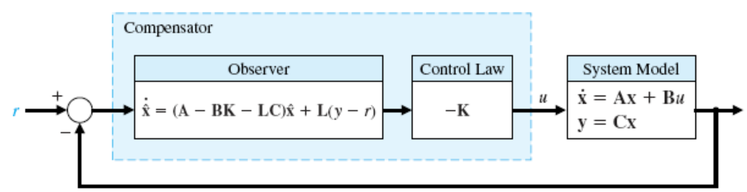
- Autonomous estimator: Select *M* and *N* so that the state estimator error equation is independent of *r*



Reference inputs: Case 2

- Tracking-error estimator: Select M and N so that only the **tracking** error ($e_r = r - y$) is used in the control

$$N = 0, \quad M = -L$$



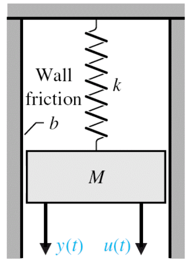
Example: Spring-Mass-Damper

- Case 1: No observer

$$\begin{bmatrix}N_x\\N_u\end{bmatrix}=\begin{bmatrix}A&B\\C&D\end{bmatrix}^{-1}\begin{bmatrix}0\\1\end{bmatrix}$$

$$=\begin{bmatrix}0&1&0\\-\frac{k}{M}&-\frac{b}{M}&1\\1&0&0\end{bmatrix}^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$=\begin{bmatrix}0&0&1\\1&0&0\\\frac{b}{M}&1&\frac{k}{M}\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}1\\0\\\frac{k}{M}\end{bmatrix}$$

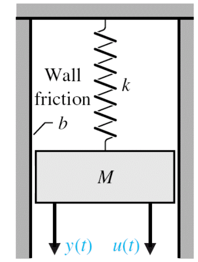


Example: Spring-Mass-Damper

- Case 1: No observer

$$N_x=\begin{bmatrix}0\\1\end{bmatrix}, N_u=\frac{k}{M}$$

$$u=-Kx+Nr$$
$$=-Kx+\left(N_u+KN_x\right)r$$
$$=-\left[\omega_n^2-\frac{k}{M}\quad 2\zeta\omega_n-\frac{b}{M}\right]x+\left(\frac{k}{M}+\left[\omega_n^2-\frac{k}{M}\quad 2\zeta\omega_n-\frac{b}{M}\right]\begin{bmatrix}1\\0\end{bmatrix}\right)r$$
$$=-\left[\omega_n^2-\frac{k}{M}\quad 2\zeta\omega_n-\frac{b}{M}\right]x+\boxed{\omega_n^2 r}$$



Example: Spring-Mass-Damper

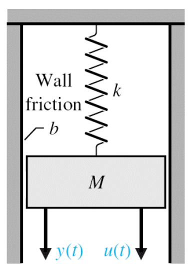
- Case 2: With observer,
- Autonomous estimation (reference input appears in controller but not in estimator)

$$\boxed{N=\omega_n^2,}\quad u=-K\hat{x}+Nr$$

$$\boxed{M=BN=\begin{bmatrix}0\\\omega_n^2\end{bmatrix}}\text{ to make}$$

$$\dot{e}=(A-LC)e$$

- So the estimation dynamics do not depend on the reference command



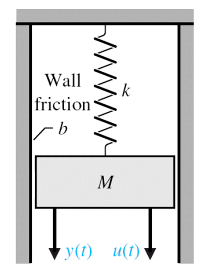
Example: Spring-Mass-Damper

- Case 3: With observer,
- Tracking estimation (reference input r appears in estimator but not in controller)

$$\boxed{\begin{matrix}N=0\\M=-L\end{matrix}}\begin{matrix}u=-K\hat{x}\\\text{to make control depend on }\hat{x}\text{ only}\end{matrix}$$

$$\dot{\hat{x}}=(A-BK-LC)\hat{x}+L(y-r)$$

- Comparable to tracking with standard classical control (based on deviations between actual and desired output)



Other possibilities

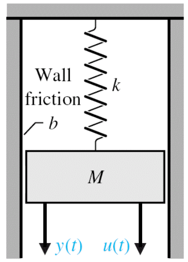
- Kalman filter
  - Observer gain which minimizes effect of noise from measurements
- Linear Quadratic Regulator (LQR)
  - Controller which optimizes a cost function (e.g. minimize fuel usage, error magnitude, control authority, etc.)
- Reduced state-control
  - When only part of x needs to be controlled
- Reduced-order observer
  - When only part of x needs to be estimated
- And **many** others…

Example: Spring-Mass-Damper

- Because the system is controllable and observable, the closed-loop poles of the error dynamics and the system dynamics can be placed arbitrarily.

- However, the further away the closed-loop poles are placed from the open-loop poles, the higher the control effort.

- Additionally, excessively high observer gains can lead to amplification of noise inherent to the output measurements.



Summary

- **Separation principle** allows independent design of the controller and observer
- Generally design controller first, then observer to be faster than the controller
- Reference inputs
  - To track a reference input, the regulating controller/observer must be modified
  - Construct N in $u = -Kx+Nr$ to remove steady-state error with full-state feedback
  - More complicated options in observer design depend on control problem at hand.