Problem Set #5

By now you should be familiar with the $M\Delta$ structure for separating your uncertainty from your nominal models. The following is an exercise[1] to give you an opportunity to practice the manipulations required to produce such a structure.

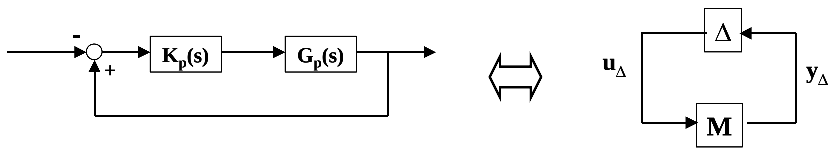

Figure 1: Physical configuration and $M\Delta$ configuration.

The nominal process model and controller are given by,

$$G(s) = [I + Q(s)]^{-1} \cdot P(s), \qquad K(s) = [I + D(s)]^{-1} \cdot C(s) \tag{1}$$

where each transfer matrix $P(s), Q(s), C(s), D(s) \in \mathcal{C}^{n\times n}$.

The use of positive feedback in Figure 1 does not limit the generality of the control system. The use of positive feedback is often used[2] in robust control simply because to make the transfer matrix blocks easier to work with.

Now suppose that we have uncertainty in *both* the process model and the controller, such that the perturbed system is given by,

$$G_p(s) = [I + Q_p(s)]^{-1} \cdot P_p(s) \qquad K_p(s) = [I + D_p(s)]^{-1} \cdot C_p(s) \tag{2}$$

where each transfer matrix is perturbed,

$$\begin{aligned} P_p(s) &= P(s) + \Delta_P(s) \\ Q_p(s) &= Q(s) + \Delta_Q(s) \\ C_p(s) &= C(s) + \Delta_C(s) \\ D_p(s) &= D(s) + \Delta_D(s) \end{aligned} \tag{3}$$

Your job is to write the system in (1)–(3) using an $M\Delta$ structure.

[1] You may be happy to know that this problem occurred (at least) once during the design of an industrial controller for a paper machine, and is not simply a mathematical exercise!

[2] Your textbook appears to be an exception as it makes an effort to preserve notation familiar to most control engineers.