Uncertainty and Robustness for SISO Systems

ELEC 571L – Robust Multivariable Control

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Outline

- Nature of uncertainty (models and signals).
- Physical sources of model uncertainty.
- Mathematical descriptions of model uncertainty.
- Robust stability.
- Robust performance.

Summary of Robustness in Control Systems

- **Determine the nominal model *G(s)* and uncertainty set *G(s)* $\in \Pi$**
- **Design controller, *K(s)*.**
- **Check robust stability (if not RS, return to 2).**
- **Check robust performance (if not RP, return to 2).**

Some controller synthesis techniques (such as H_∞) automate steps 2-4.

Signal Uncertainty
versus
Model Uncertainty

Signal Uncertainty versus Model Uncertainty

- Suppose we are given an open-loop system model:

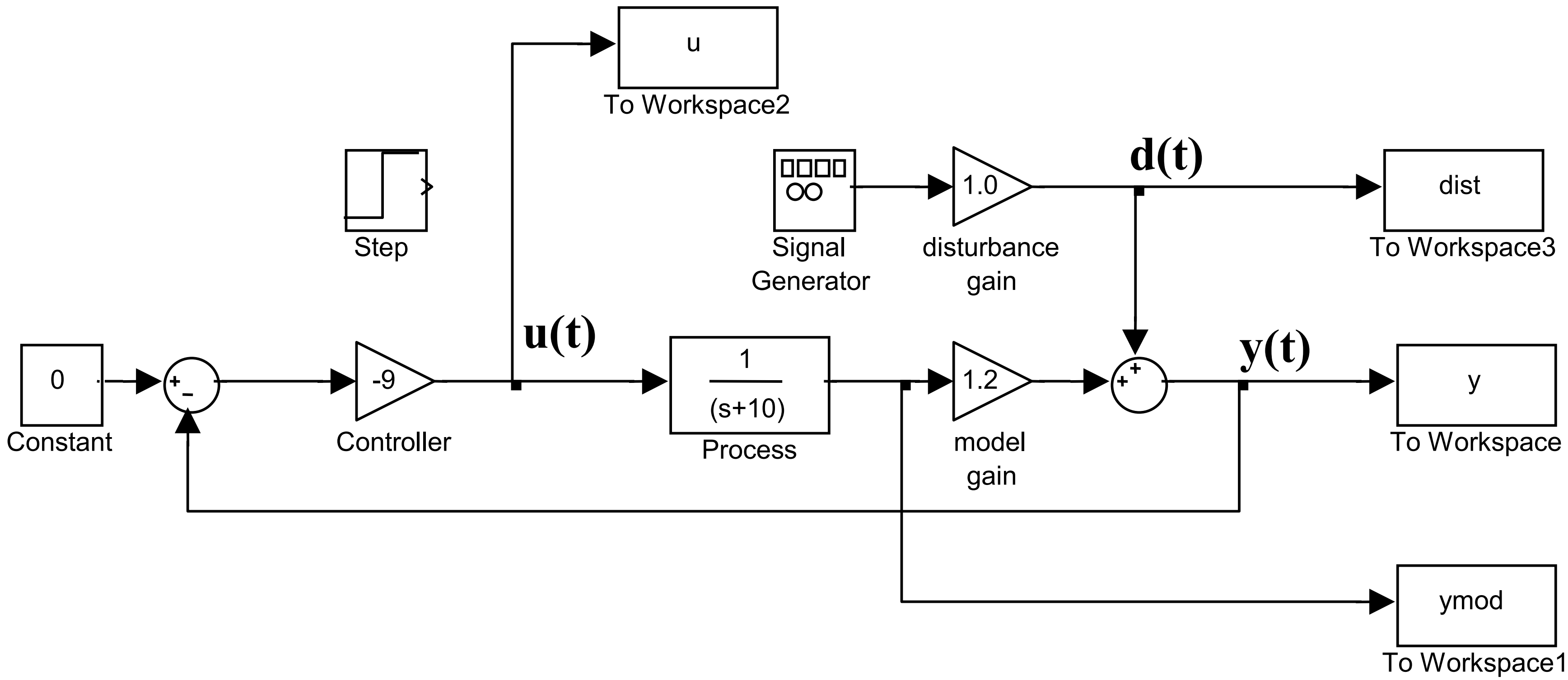
$$g(s) = 1 / (s+10)$$

- and a constant controller:

$$k(s) = -9$$

- These make a stable closed-loop with a single pole at s = -1.

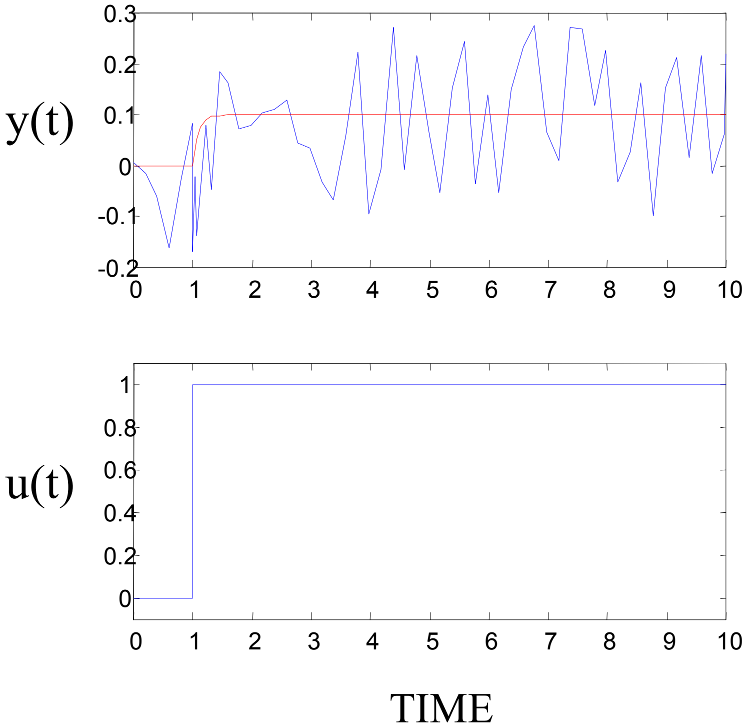
Simulink Model of the Closed-Loop*



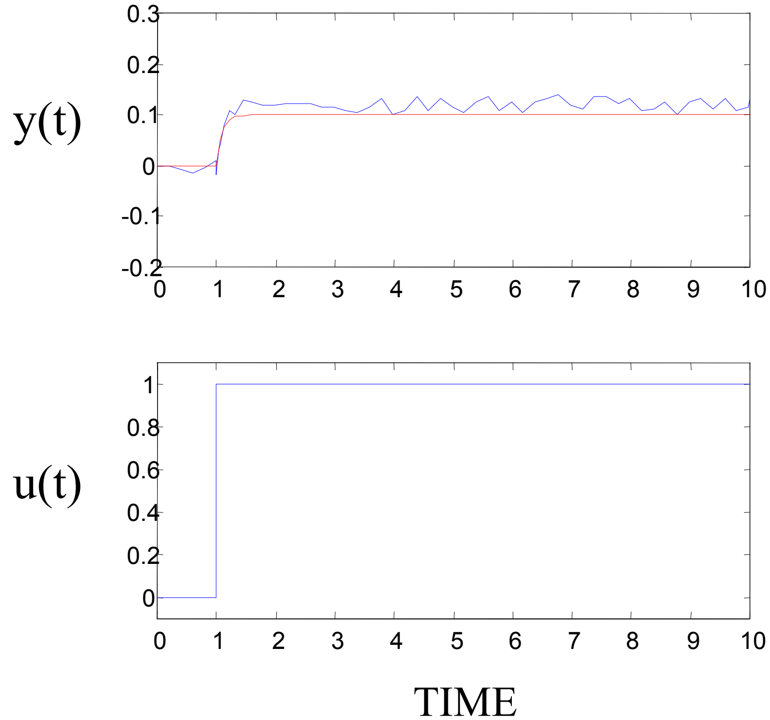
***Configured for System 2.**

Modelled and Measured Responses

System 1: open-loop



System 2: open-loop



- **Given that we designed a stabilizing controller for the nominal model (smooth line in red), can we say which of these two systems will perform better in closed-loop?**

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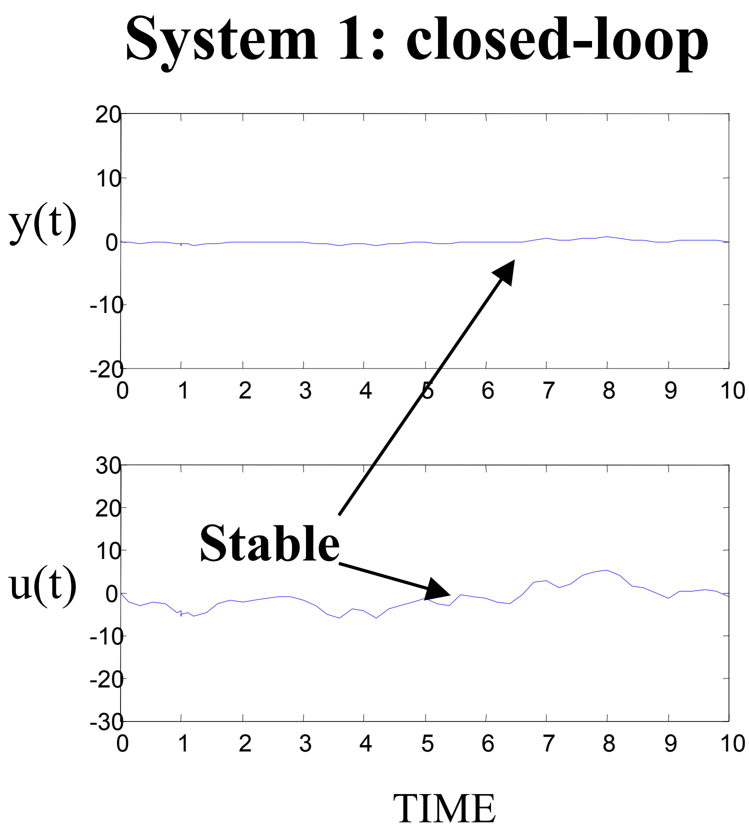
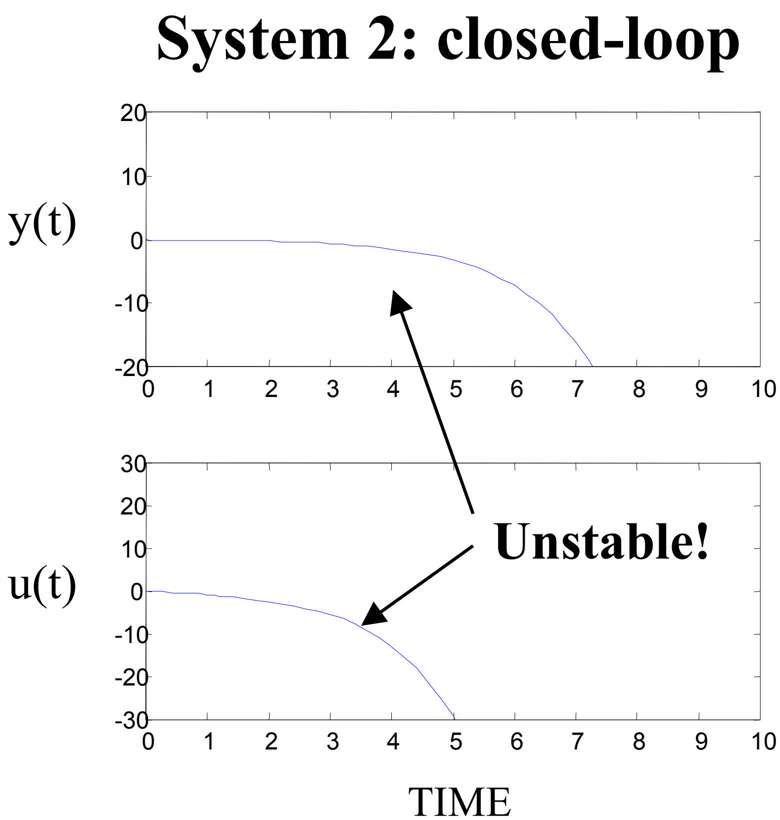
Closed-Loop Responses





- **System 2 looked better in open-loop, but is far worse in closed-loop.**
- **Why did this happen?**

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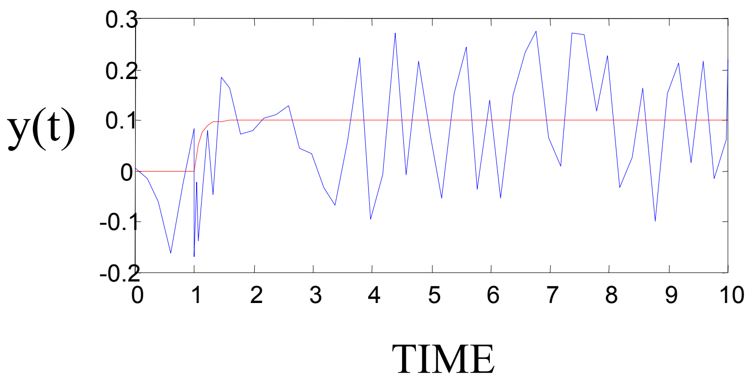
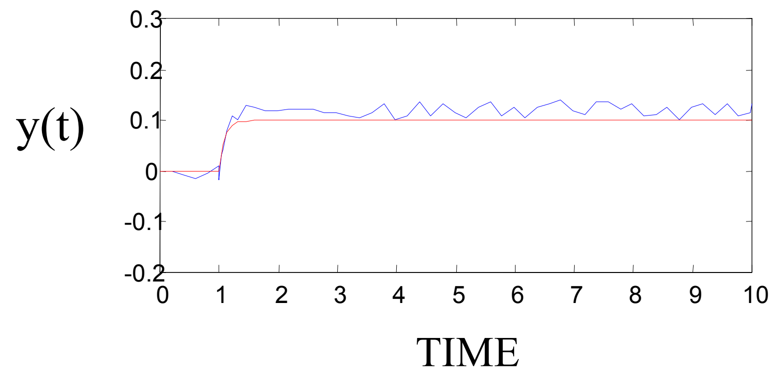
Explanation

System 1: open-loop



System 2: open-loop



y(t) = g(s) u(t) + *10* d(t)

then k(s) = -9, ⇒

closed-loop pole = -1

y(t) = *1.2* g(s) u(t) + d(t)

then k(s) = -9, ⇒

closed-loop pole = +0.8

A 20% change in the gain of the process model destabilized the closed-loop.

Bounded additive disturbances have no effect on system stability.

Moral of the story

- **Uncertainty is always present in both signals and models.**
- **Perturbations due to model uncertainty can destabilize a system. Bounded signal uncertainty will not.**
- **Feedback can *create* infinite signals. Be careful with it!**
- **Common sense "analysis" of model uncertainty can be misleading.**

Physical Sources of Model Uncertainty

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Sources of Model Uncertainty

- Parameters in the linear model
  - identified from noisy input/output data
  - calculated from physical modelling
- Nonlinearities in actuators and sensors
  - actuator/sensor saturation
  - actuator/sensor failure
  - hardware deterioration over time
  - physical systems are inherently nonlinear

Sources of Model Uncertainty

- At high frequencies, we know almost nothing about the process:
  - control and model identification concentrate on low frequencies.
- Deliberate simplification of the model
  - it is easier to design controllers for simple processes than for complicated processes.

Sources of Model Uncertainty

- Uncertainty in the controller (often neglected!)
  - deliberate reduction of controller order for simpler implementation
  - implementation issues, e.g.
    - finite floating point precision in computers
    - error/limit checking in the controller implementation
  - sometimes referred to as fragility (Keel et al, TAC, Aug 97)

Mathematical Descriptions of Model Uncertainty

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Parametric Uncertainty

- **Most intuitive kind of model uncertainty – uncertainty in the parameters of the linear model.**

- **For example, consider a first-order transfer function:**

$$g / (s+a)$$

with parameters

$$g = 1.0 \pm 0.2$$

$$a = 10 \pm 1.3$$

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Parametric Uncertainty

- Write the nominal process model as:

$$g(s) = 1.0 \,/\, (s+10)$$

- All possible models are given by:

$$g_p(s) \in \Pi$$

where the uncertainty set is defined by:

$$\Pi = \left\{ g \,/\, (s+a) : \quad 0.8 \le g \le 1.2, \quad 8.7 \le a \le 11.3 \right\}$$

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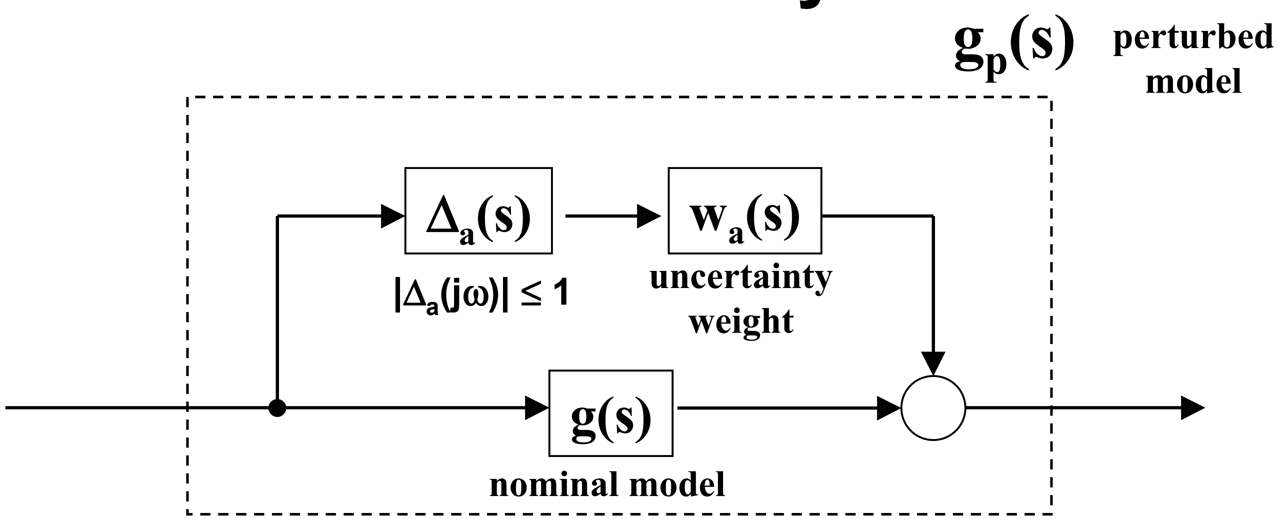
Parametric Uncertainty

- Despite the intuitive nature of parametric uncertainty models, they are not used often.
  - The model structure is set, leaving no room for unmodelled dynamics.
  - You need to be very sure of the model in order to use such a “high-fidelity” uncertainty structure.
  - The mathematics of controller design and analysis is cumbersome in this framework.

Complex Model Perturbations

- Additive model uncertainty



- where $\Delta_a(s)$ is ***<u>any</u>*** stable transfer function that satisfies $|\Delta_a(j\omega)| \leq 1$, for all frequencies ω.

- we then use transfer function $w_a(s)$ to model the physical uncertainty (typically high-pass filter).

A Few Words About Delta

- $\Delta_a(s)$ is different than most transfer functions due to the fact that we assume so little about it.

- We only know that it is stable and $|\Delta_a(j\omega)| \leq 1$.

- For example, *any* of the following are permitted:

$\Delta_a(s) = +1, +0.5,$ etc.

$\Delta_a(s) = a / (s+a)$, for $a>0$

$\Delta_a(s) = -1, -0.5,$ etc.

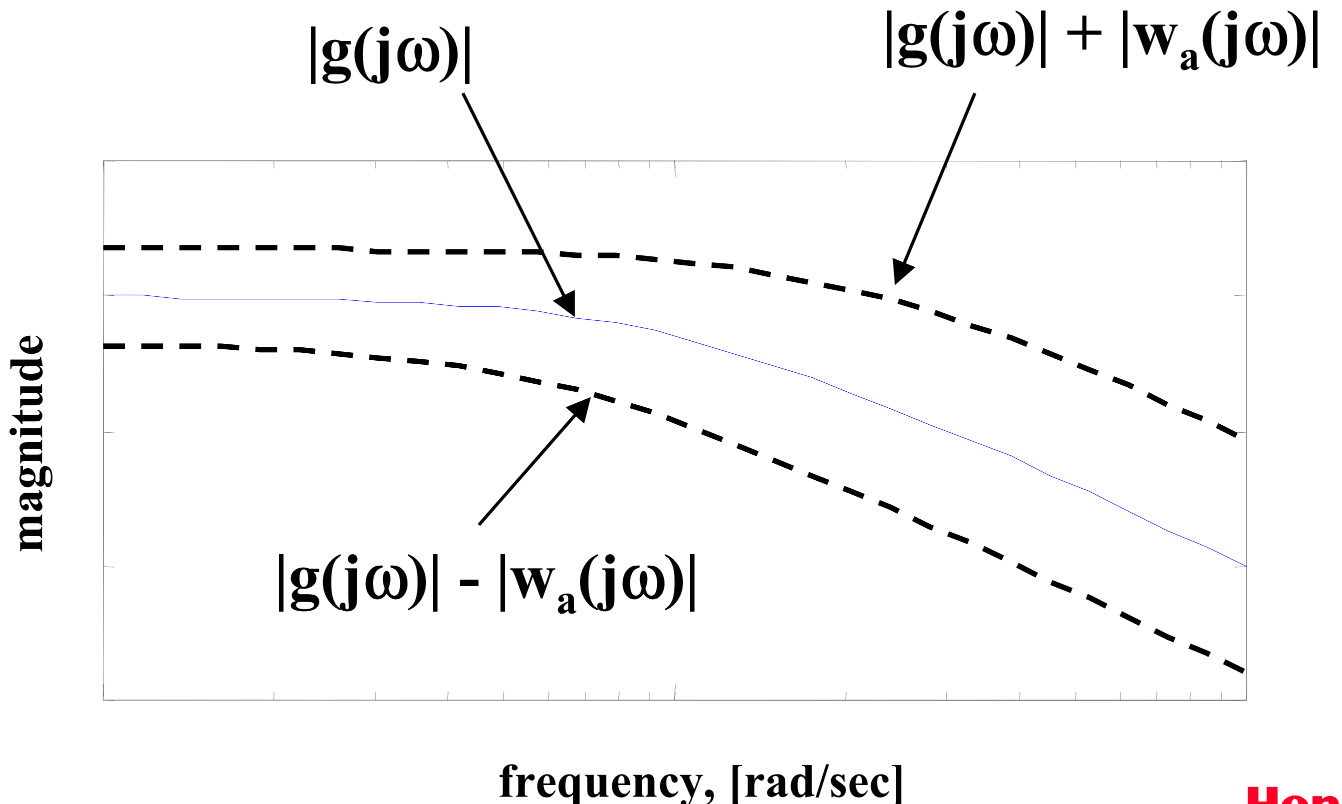
$\Delta_a(s) = 0$

$\Delta_a(s) = e^{-\theta s}$

... and infinitely many others!

Additive Model Uncertainty

- **The bounds of the possible models can be drawn in the frequency domain.**



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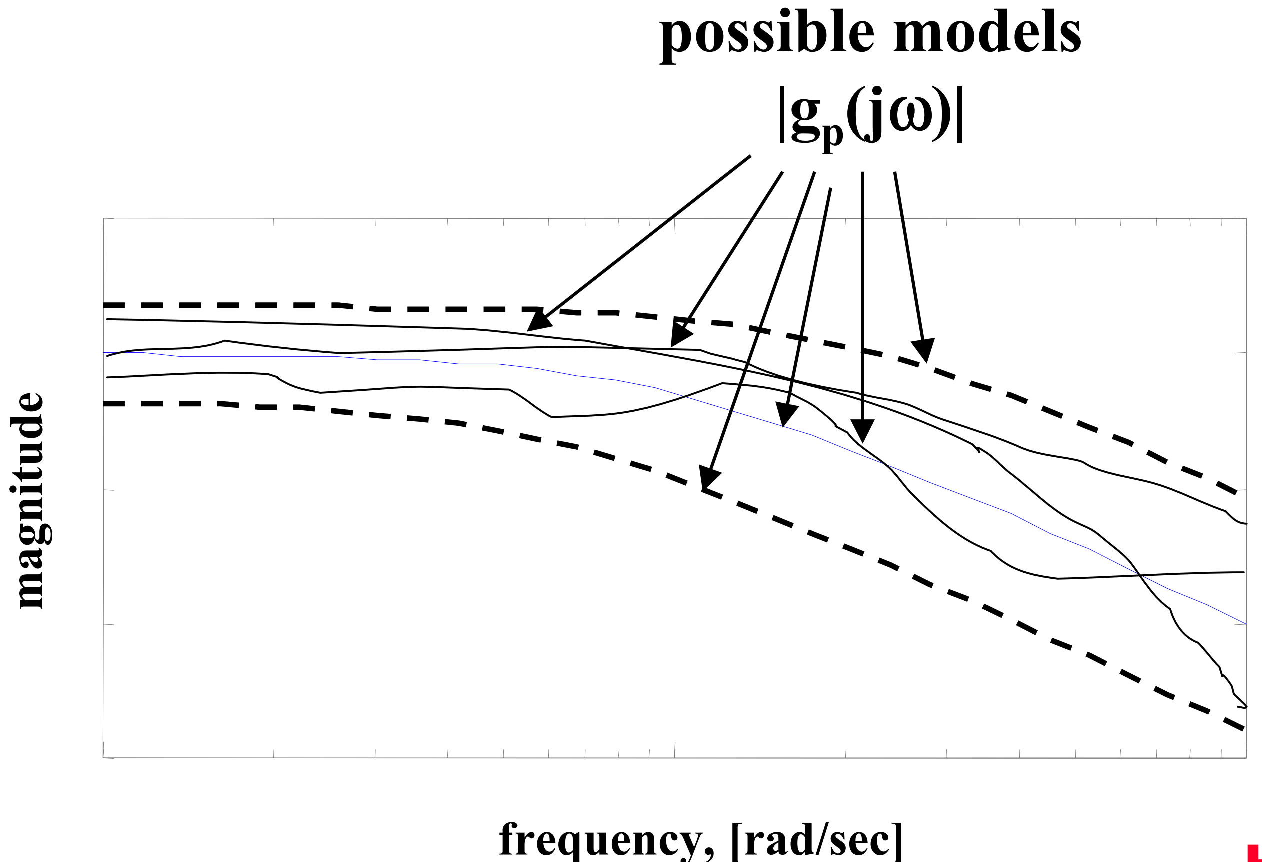
Additive Model Uncertainty

- **This structure permits a family of potential models in a bounded neighbourhood of the nominal model.**



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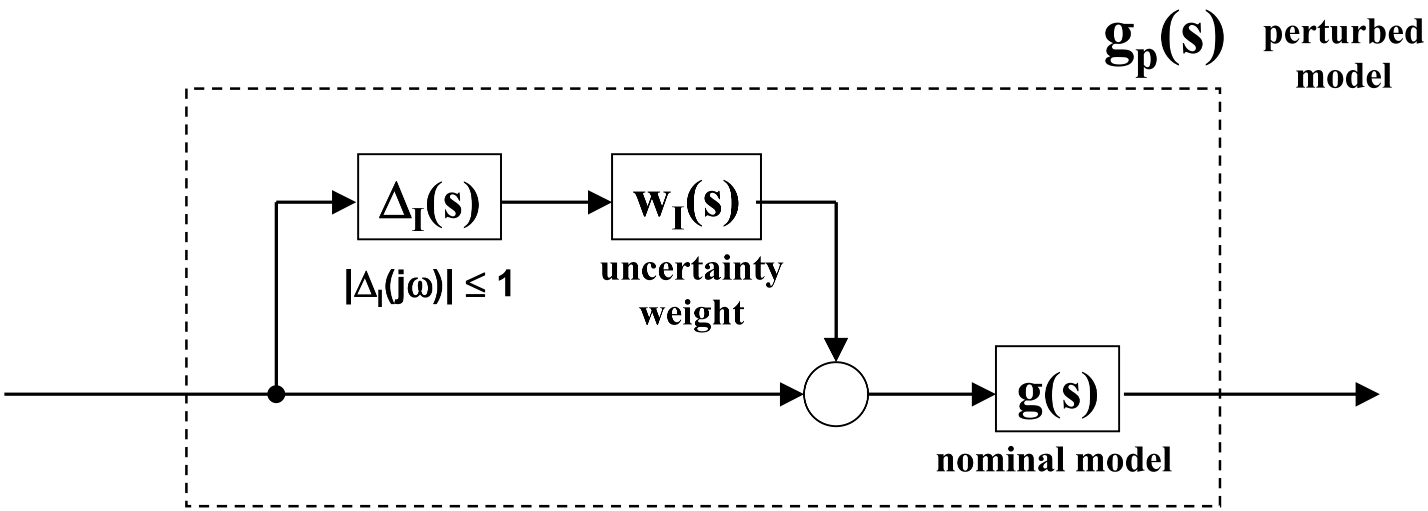
Multiplicative Model Uncertainty



- configuration is different, but
- each transfer function block has a similar interpretation as with the additive case.

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Multiplicative Model Uncertainty

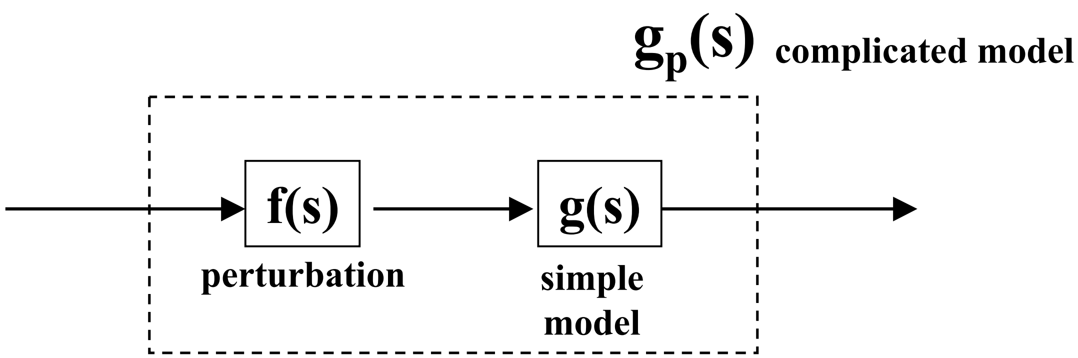
- In SISO systems, we can find simple equivalence between additive and multiplicative model uncertainty.

- Multiplicative uncertainty seems to be used more often than additive.

- Robust stability and performance calculations are simpler with multiplicative than additive uncertainty.

Deliberate Model Uncertainty

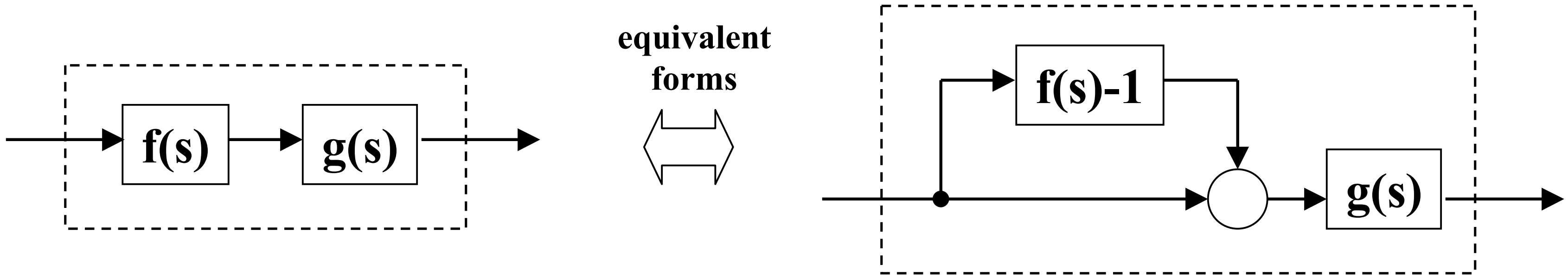
- **Sometimes a control engineer will deliberately simplify a process model before designing a controller.**



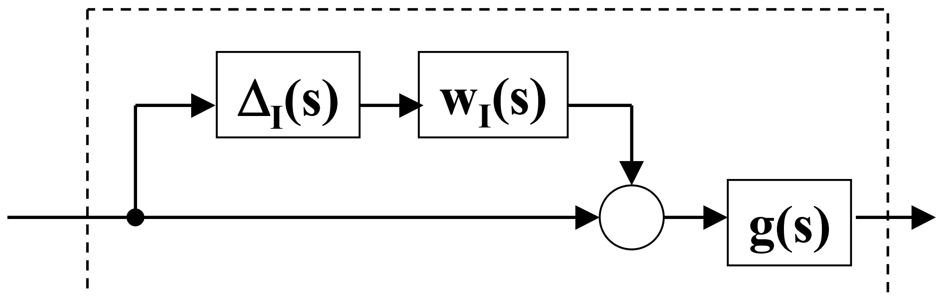
- **where f(s) is a *known* transfer function,**

$$g_p(s) = g(s)\, f(s)$$

Deliberate Model Uncertainty



- This may be "covered" by a complex perturbation,



- if we create $w_I(s)$ such that

$$| w_I(j\omega) | \geq | f(j\omega) - 1 |$$

Deliberate Model Uncertainty

- This trick is used when the engineer believes that it will be easier to design a controller for a simple g(s) plus some perturbation (robust control).
- Used when "true" model $g_p(s)$ is complicated.
- Examples include time delays, high-order models, etc.
- Technique should be used carefully, since it "hides" model information from the controller design.

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Robust Stability

Definition: Robust Stability

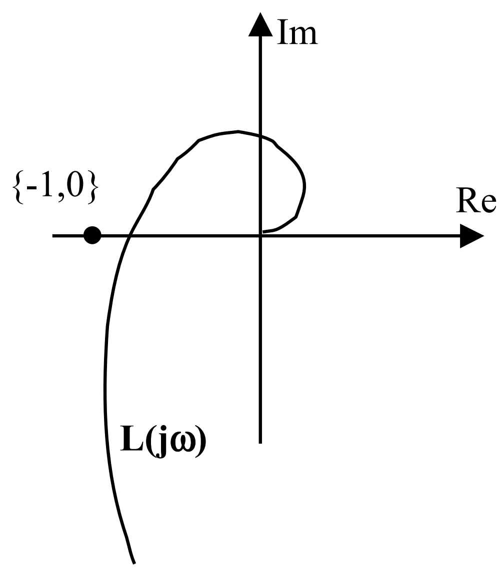
- A closed-loop system with controller K(s) is said to be robustly stable if it is stable for every possible plant in the uncertainty set,

$$G_p(s) \in \Pi$$

Nominal Stability: graphical interpretation

- **Remember that (nominal) closed-loop stability requires:**

$$L(j\omega) = K(j\omega)G(j\omega) \quad \text{does not encircle } \{-1,0\} \text{ in complex plane.}$$



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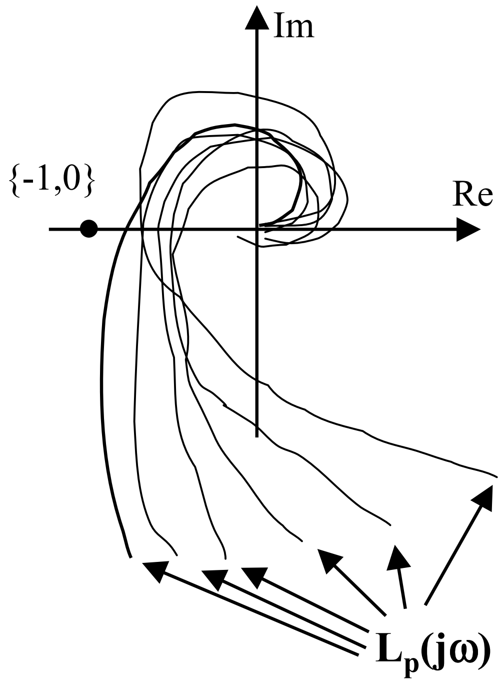
Robust Stability: graphical interpretation

Robust stability requires that

$L_p(j\omega) = K(j\omega)G_p(j\omega)$ **does not encircle {-1,0} in complex plane, for any $G_p(s) \in \Pi$**



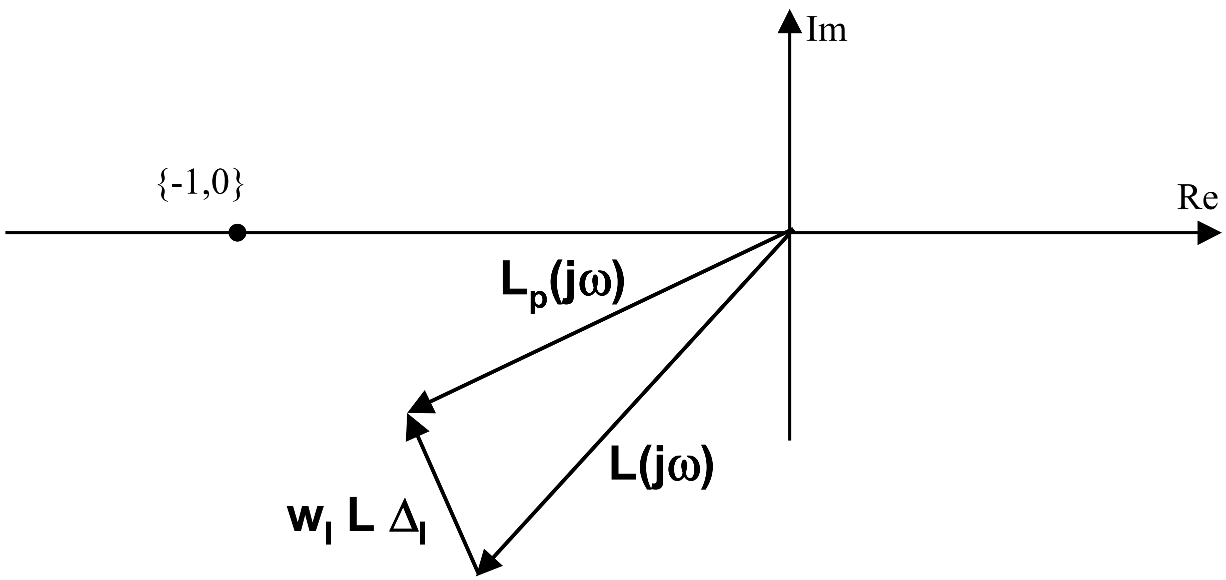
How can we guarantee this?

Derivation of Robust Stability

Assume multiplicative uncertainty for the moment:

$$L_p(s) = G_p(s)\ K(s)$$
$$= GK\ (1 + w_I \Delta_I) = L + w_I\ L\ \Delta_I$$

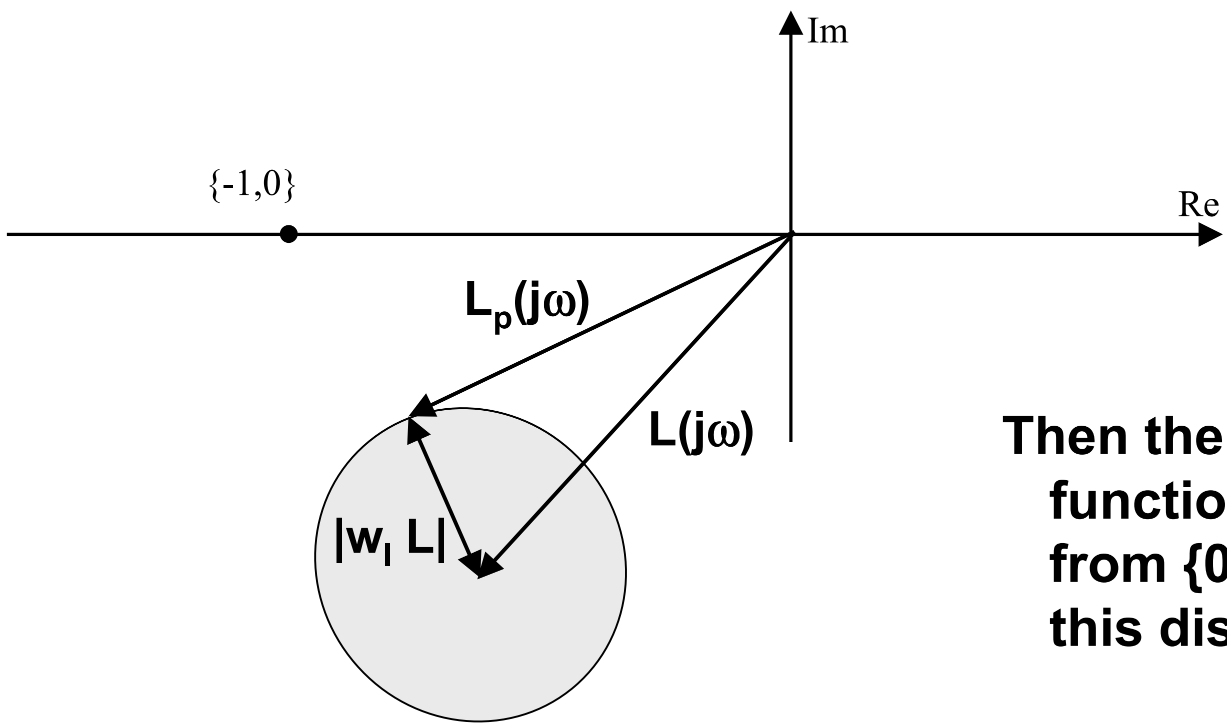
which is a vector equation in the complex plane:



Derivation of Robust Stability

Now, since $\Delta_I(j\omega)$ can have any phase and $|\Delta_I(j\omega)| \leq 1$, then the vector

$w_I\, L\, \Delta_I$ defines a disk of radius $|w_I\, L|$



Then the loop transfer function $L_p(j\omega)$ is a vector from {0,0} to any point in this disk.

Derivation of Robust Stability

Since RS is satisfied if $L_p(j\omega)$ does not encircle the {-1,0} point, then we need to ensure that the disk never touches {-1,0}.

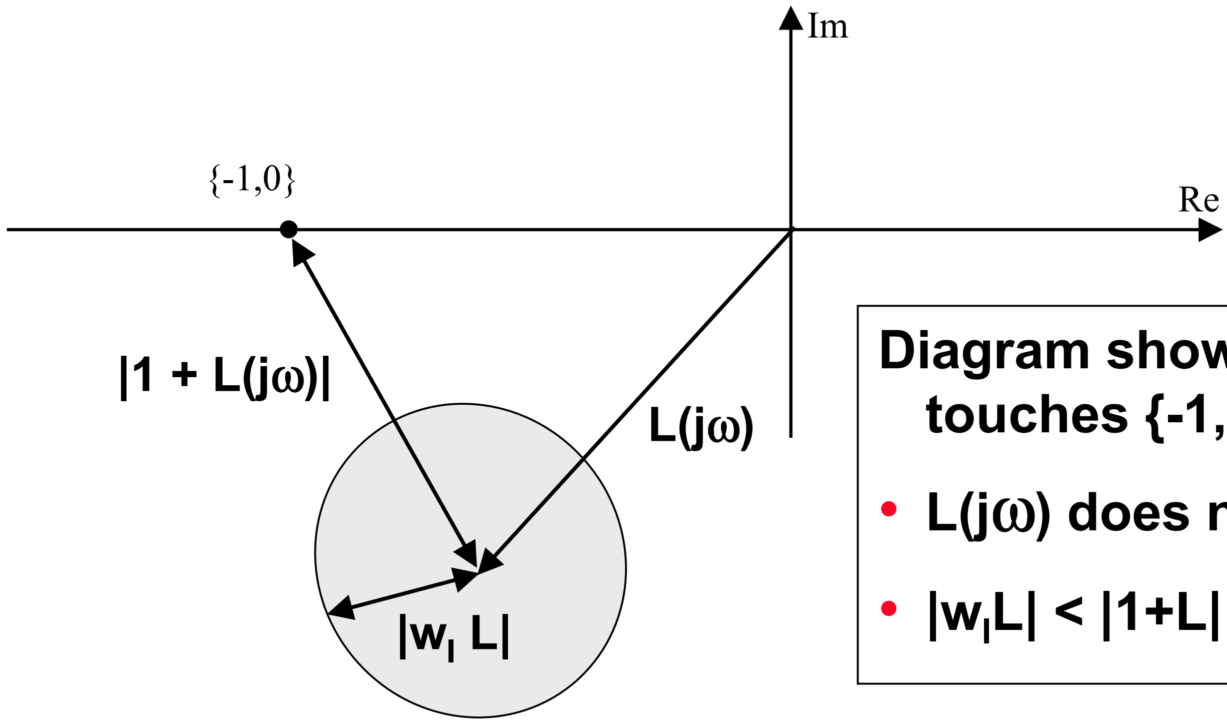


Diagram shows that the disk never touches {-1,0} if

- $L(j\omega)$ does not encircle {-1,0}, and
- $|w_I L| < |1+L|$, for all ω

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Derivation of Robust Stability

- **The condition,**

$$|w_I L| < |1+L|$$

is equivalent to,

$$\left|\frac{L(j\omega)}{1+L(j\omega)}\right| < \left|\frac{1}{w_I(j\omega)}\right|$$

and finally in terms of the "complementary sensitivity"

RS for multiplicative uncertainty	$\Longleftrightarrow$	$\lvert T(j\omega)\rvert < \left\lvert\dfrac{1}{w_I(j\omega)}\right\rvert$

Robust Stability: Comments

- Note the the left-hand-side is stated completely in terms of the nominal transfer functions.
- The right-hand-side is stated in terms of the magnitude of the model uncertainty.
- This robust stability condition is ***not*** conservative. It is both sufficient and necessary to guarantee stability.
- RS conditions for other model uncertainty structures are derived using similar calculations.

Robust Stability: Comments

- Similar RS conditions exist for other model uncertainty structures.

- For example, the RS condition for additive model uncertainty is derived using similar arguments:

RS for additive uncertainty $\iff$ $$\left|\frac{K(j\omega)}{1+L(j\omega)}\right| < \left|\frac{1}{w_A(j\omega)}\right|$$

Robust Stability: parting thought

- Like the majority of robustness tests, these magnitude-based conditions assume that the system is nominally stable.

RULE #1: Always check nominal stability (NS), before applying a robust stability (RS) condition.

Robust Performance

Definition: Robust Performance

- A closed-loop system with controller K(s) is said to possess robust performance if the closed-loop satisfies performance specifications (whatever they may be) for every possible plant in the uncertainty set,

$$G_p(s) \in \Pi$$

  – Remark: In the vast majority (maybe all) of cases, performance specifications will include closed-loop stability.

Robust Performance

- Let us consider the <u>special case</u> of
  - multiplicative uncertainty
  - a performance specification of

$$|w_p(j\omega)\ S(j\omega)| < 1$$

- where $S = (1+L)^{-1} = (1+GK)^{-1}$, and typically we will have the performance weight

$|w_p(j\omega)| >> 1,$ for low frequencies ω

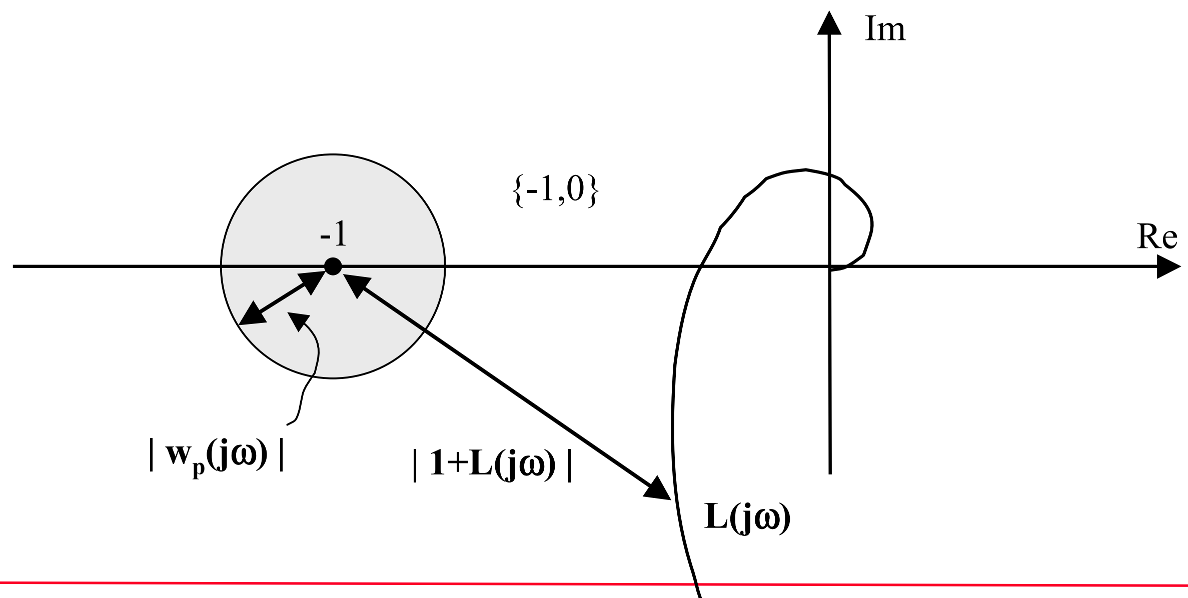
$|w_p(j\omega)| < 1,$ for high frequencies ω

Robust Performance

- First, rewrite the nominal performance specification,

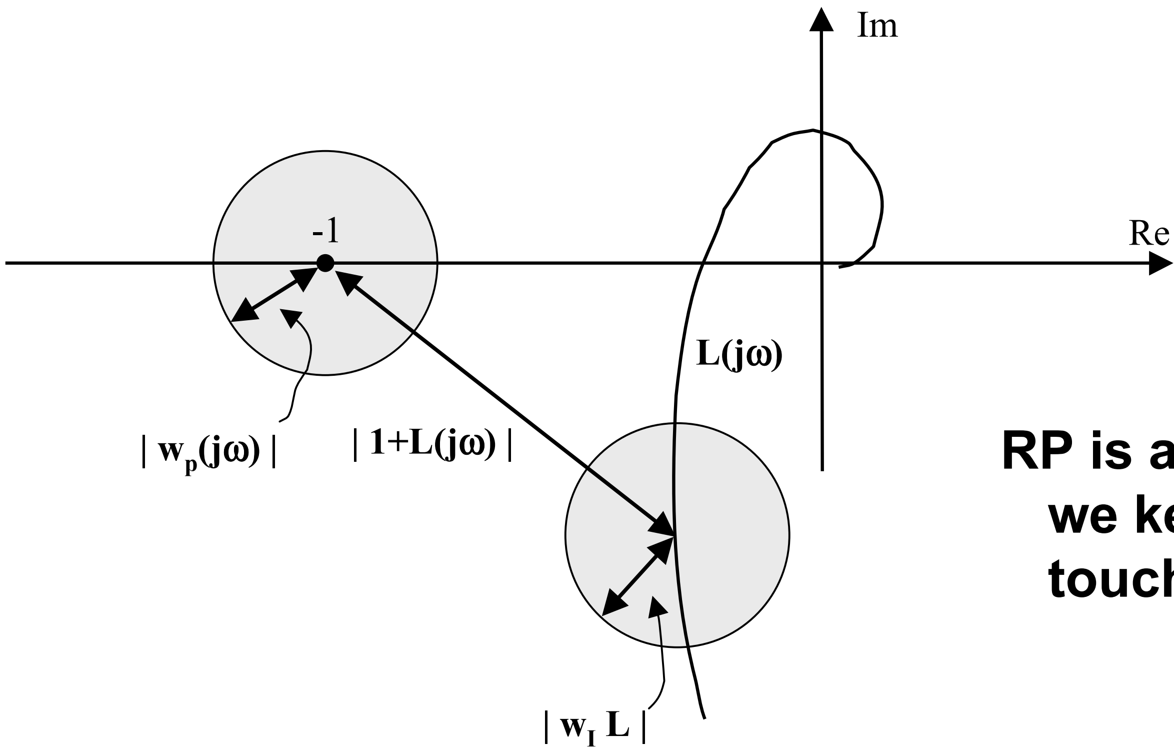
$$\left|\frac{w_p(j\omega)}{1+L(j\omega)}\right| < 1 \quad \Longleftrightarrow \quad |w_p(j\omega)| < |1+L(j\omega)|$$

- which can be represented in a diagram as,



Robust Performance

- Then to apply the performance specification to all loop gains defined by $L_p(s)$, we can re-use our RS diagram.

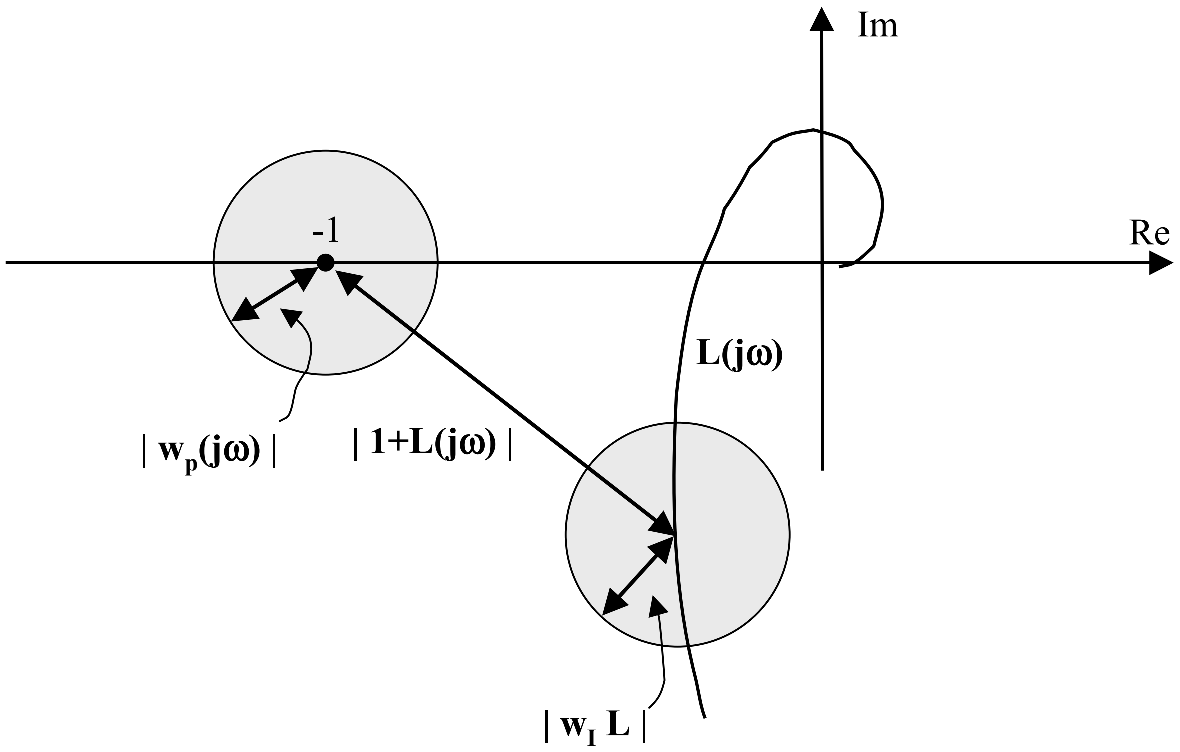


RP is achieved as long as we keep these disks from touching.

Robust Performance

- **The disks do not touch if,**

$$|\,1+L(j\omega)\,| > |\,w_p(j\omega)\,| + |\,w_I(j\omega)\,L(j\omega)\,|$$



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Robust Performance

- The RP condition

$$| 1+L(j\omega) | > | w_p(j\omega) | + | w_I(j\omega)\ L(j\omega) |$$

can be rewritten as,

$$\left|\frac{w_p(j\omega)}{1+L(j\omega)}\right| + \left|\frac{w_I(j\omega)L(j\omega)}{1+L(j\omega)}\right| < 1$$

- and is typically presented in the form:

$$\left|w_p(j\omega)S(j\omega)\right| + \left|w_I(j\omega)T(j\omega)\right| < 1$$

Robust Performance: Comments

$$\left|w_p(j\omega)S(j\omega)\right| + \left|w_I(j\omega)T(j\omega)\right| < 1$$

- **This is a RP condition for a *specific* performance criterion and a *specific* model uncertainty structure.**
- **RP conditions for other uncertainty structures may be derived.**
- **Note that satisfying this RP condition automatically includes the RS condition for multiplicative uncertainty $|w_I(j\omega)\ T(j\omega)| < 1$.**

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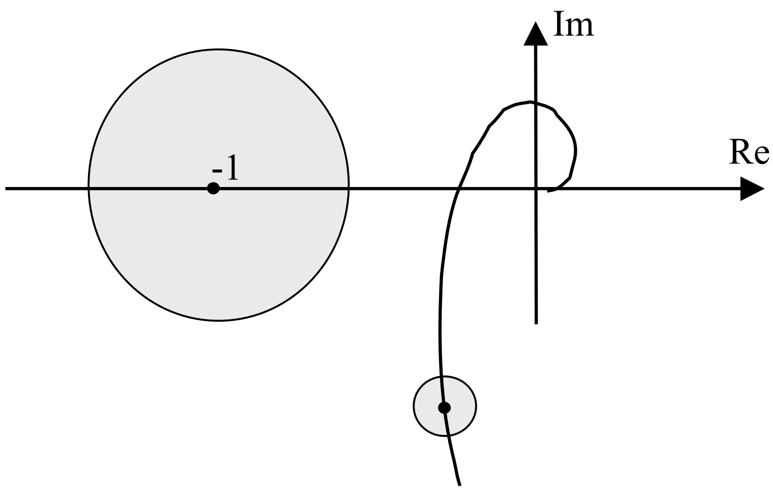
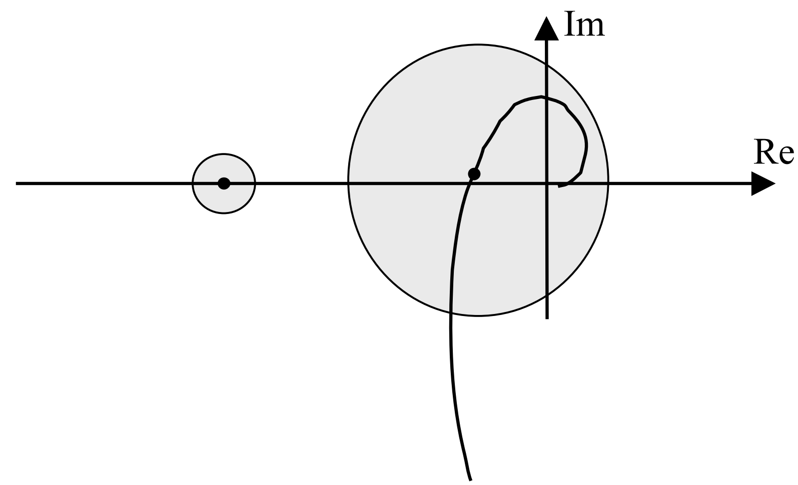
Robust Performance: Practically Speaking

- In practical robust performance, the size of these disks will depend on the frequency ω.

Low frequencies, ω
$|w_p(j\omega)|$ large
$|w_I(j\omega)|$ small



High frequencies, ω
$|w_p(j\omega)|$ small
$|w_I(j\omega)|$ large



Robust Performance: Parting Thought

$$\left|w_p(j\omega)S(j\omega)\right| + \left|w_I(j\omega)T(j\omega)\right| < 1$$

- **The derivation of this result assumed nominal stability (NS).**

RULE #2: Always check nominal stability (NS), before applying a robust performance (RP) condition.

Conclusions

- Control systems contain uncertainty in signals and in models.
- Perturbations due to model uncertainty can destabilize a closed-loop. Bounded signal perturbations cannot.
- Robust control concentrates on addressing model uncertainty.

Conclusions

- Complex model perturbations are commonly used to represent model uncertainty.
- Assuming nominal stability (NS):
  - a robust stability (RS) condition was derived in terms of the magnitude of these perturbations.
  - a robust performance (RP) condition was derived in terms of the model uncertainty and the performance specification.
- In practice, the assumption of NS is valid since practical design techniques produce a NS closed-loop.