REFERENCES

- E. H. Abed, P. K. Houpt, and W. M. Hosny, "Bifurcation analysis of surge and rotating stall in axial flow compressors," *ASME J. Turbomachinery*, vol. 115, pp. 817–824, 1993.
- [2] O. O. Badmus, S. Chowdhury, K. M. Eveker, C. N. Nett, and C. J. Rivera, "Simplified approach for control of rotating stall part 1: Theoretic development," *J. Propulsion Power*, vol. 11, no. 6, pp. 1195–1209, 1995.
- [3] O. O. Badmus, K. M. Eveker, and C. N. Nett, "Control-oriented high-frequency turbomachinery modeling: General one-dimensional model development," ASME J. Turbomachinery, vol. 117, pp. 320–335, 1995.
- [4] M. M. Bright, H. K. Qammar, H. J. Weigl, and J. D. Paduano, "Stall precursor identification in high-speed compressor stages using chaotic time series analysis method," *ASME J. Turbomachinery*, vol. 119, no. 3, pp. 491–499, 1997.
- [5] S.-K. Chang, P.-L. Hsu, and K.-L. Lin, "A parametric transfer matrix approach to fault-identification filter design and threshold selection," *Int. J. Syst. Sci.*, pp. 741–754, 1995.
- [6] X. Ding and P. M. Frank, "Fault detection via factorization approach," Syst. Control Lett., vol. 14, pp. 431–436, 1990.
- [7] P. M. Frank and B. Köppen-Seliger, "Fuzzy logic and neural network applications to fault diagnosis," *Int. J. Approximate Reasoning*, vol. 16, pp. 67–88, 1997.
- [8] B.-C. Kuo, Automatic Control Systems, 5th ed. Englewood Cliffs: Prentice Hall, 1987.
- [9] P. B. Lawless, K. H. Kim, and S. Fleeter, "Characterization of abrupt rotating stall initiation in an axial-flow compressor," *J. Propulsion Power*, pp. 709–715, 1994.
- [10] D.-C. Liaw and E. H. Abed, "Active control of compressor stall inception: A bifurcation-theoretic approach," *Automatica*, vol. 32, pp. 109–115, 1996.
- [11] D.-C. Liaw and J.-T. Huang, "Fuzzy control for stall recovery in compressor dynamics," *J. Control Syst. Technol.*, vol. 6, no. 4, pp. 231–241, 1998.
- [12] —, "Global stabilization of axial compressors using nonlinear cancellation and backstepping design," *Int. J. Syst. Sci.*, pp. 1345–1361, 1998.
- [13] F. K. Moore and E. M. Greitzer, "A theory of post-stall transient in axial compression systems. Part I—Development of equations," ASME J. Engine Gas Turbines Power, vol. 108, pp. 68–76, 1986.
- [14] G. Vachtsevanos, H. Kang, J. Cheng, and I. Kim, "Detection and identification of axial flow compressor instabilities," *AIAA J. Guid., Control, Dyna.*, vol. 15, no. 5, pp. 1216–1223, 1992.

A New Controller Architecture for High Performance, Robust, and Fault-Tolerant Control

Kemin Zhou and Zhang Ren

Abstract—In this note, we propose a new feedback controller architecture. The distinguished feature of our new controller architecture is that it shows structurally how the controller design for performance and robustness may be done separately which has the potential to overcome the conflict between performance and robustness in the traditional feedback framework. The controller architecture includes two parts: one part for performance and the other part for robustness. The controller architecture works in such a way that the feedback control system will be solely controlled by the performance sont of the robustification controller will only be active when there are model uncertainties or external disturbances.

Index Terms—Fault-tolerant control, H_{∞} control, internal model control, robust control.

I. INTRODUCTION

A fundamental reason for using feedback control is to achieve desired performance in the presence of external disturbances and model uncertainties. It is well known that there is an intrinsic conflict between performance and robustness in the standard feedback framework, see [3], [9], [11], [20], [21] for some detailed analyzes and discussions. In other words, one must make a tradeoff between achievable performance and robustness against external disturbances and model uncertainties. For example, a high-performance controller designed for a nominal model may have very little robustness against the model uncertainties and external disturbances. For this reason, worst-case robust control design techniques such as H_{∞} control, L_1 control, μ synthesis, etc, have gained popularity in the last twenty years or so, see, for example, [1], [2], [6], [8], [13], [17], [20], [21] and references therein. Unfortunately, it is well recognized in the robust control community that a robust controller design is usually achieved at the expense of performance. This is not hard to understand since most robust control design techniques are based on the worst possible scenario which may never occur in a particular control system.

In this note, we shall propose a new controller architecture that has the potential to overcome the conflict between performance and robustness in the traditional feedback framework. This controller architecture uses the well-known Youla controller parameterization in a nontraditional way. The distinguished feature of our new controller architecture is that it shows structurally how the controller design for performance and robustness may be done separately. First of all, a high performance controller, say K_0 , can be designed using any method, and then a robustification controller, say Q, can be designed to guarantee robust stability and robust performance using any standard robust control techniques. The feedback control system will be solely controlled by the high performance controller K_0 when there is no model uncertainties and external disturbances while the robustification controller

Manuscript received December 22, 1999; revised September 25, 2000 and February 1, 2001. Recommended by Associate Editor R. Tempo. This work was supported in part by ARO under Grant DAAH04-96-1-0193, the Air Force Office for Scientific Research under Grant F49620-99-1-0179, and LEQSF under Grant DOD/LEQSF(1996–99)-04.

K. Zhou is with the Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA 70803 USA (email: kemin@ ee.lsu.edu).

Z. Ren is with College of Marine Engineering, Northwestern Polytechnical University, Xian 710072, China (email: renzhag@nwpu.edu.cn).

Publisher Item Identifier S 0018-9286(01)09533-2.



Fig. 1. Standard feedback configuration.

Q will only be active when there are model uncertainties or external disturbances. This controller architecture also offers a way to build the fault-tolerant control strategy based on the normal working controllers.

This note is organized as follows. Section II introduces the Youla controller parameterization. We propose the new controller architecture in Section III. In Section IV, we show how our controller architecture can be used to design high performance and fault-tolerant controllers. Section V discusses how to design the robustification controllers in this new controller framework. Section VI makes the connection between this new controller architecture and the two degree of freedom controller structure [17], [18]. In particular, we shall show that this controller architecture can in some sense be regarded as a special implementation of all two degree of freedom controller parameterization. Section VII considers an dual version of this new controller architecture and makes some connections with the well known internal model control (IMC) structure. Some concluding remarks are given in Section VIII.

The following notation will be used throughout this note. Let $M \in \mathbb{C}^{n \times m}$. Then $\overline{\sigma}(M)$ denotes the largest singular value of M. H_{∞} denotes the Banach space of bounded analytic functions with the ∞ norm defined as $\|F\|_{\infty} = \sup_{\omega} \overline{\sigma}(F(j\omega))$ for any $F \in H_{\infty}$. A state space realization of a rational proper transfer function G(s) is denoted by $G(s) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = C(sI - A)^{-1}B + D$. Let P be a block matrix $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$. Then the linear fractional transformation of

trix $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$. Then the linear fractional transformation of P over F is defined as $\mathcal{F}_{\ell}(P,F) = P_{11} + P_{12}F(I - P_{22}F)^{-1}P_{21}$ where F is assumed to have appropriate dimensions and $(I - P_{22}F)^{-1}$ is well defined.

II. PRELIMINARY

Consider a standard feedback configuration shown in Fig. 1 where P is a linear time invariant plant and K is a linear time invariant controller. We shall assume without loss of generality that the feedback system is well-posed, i.e., $\det(I - P(\infty)K(\infty)) \neq 0$.

The following lemma is a simple variation of the well-known Youla controller parameterization [17], [20], [21] and will play the key role to our development in this note.

Lemma 1: Suppose that K_0 stabilizes internally the feedback system shown in Fig. 1. Let K_0 and P have the following right and left coprime factorizations:

$$K_0 = UV^{-1} = \tilde{V}^{-1}\tilde{U}, \quad P = NM^{-1} = \tilde{M}^{-1}\tilde{N}.$$

Then every controller K that internally stabilizes the feedback system shown in Fig. 1 can be written in the following form:

$$K = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})$$

for some $Q \in H_{\infty}$ such that $\det(\tilde{V}(\infty) - Q(\infty)\tilde{N}(\infty)) \neq 0$, or, equivalently

$$K = (U + MQ)(V - NQ)^{-1}$$

for some $Q \in H_{\infty}$ such that $\det(V(\infty) - N(\infty)Q(\infty)) \neq 0$.



Fig. 2. Youla controller parameterization.

It is noted that in the standard Youla controller parameterization, U, \tilde{V} , U and V are chosen so that $\tilde{U}N + \tilde{V}M = I$ and $\tilde{N}U + \tilde{M}V = I$, in particular, K_0 is chosen to be an observer based stabilizing controller. Unfortunately, this choice of K_0 is not always desirable in the subsequent development. The controller parameterizations in the above lemma do not impose such constraints. In fact, we shall always choose K_0 as our nominal controller that satisfies our nominal design objectives. In particular, K_0 can be a simple PID controller.

Note that the feedback system with a controller

$$K = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})$$

can be implemented either as shown in Fig. 1 after obtaining a total transfer function K or as shown in Fig. 2 with five blocks. For a fixed Q, it is clear that there is no advantage in using the implementation in Fig. 2 and, in fact, this implementation is usually not desirable since it needs much higher order controller implementation. It does have some advantages when Q is made to be adaptive, see [14]. We shall not discuss this issue further.

III. A NEW CONTROLLER ARCHITECTURE

It is well understood that the model P is in general not perfectly known. What one actually knows is a nominal model P_0 . Now assume that K_0 is a stabilizing controller for the nominal plant P_0 and assume P_0 and K_0 have the following coprime factorizations:

$$K_0 = UV^{-1} = \tilde{V}^{-1}\tilde{U}$$
 $P_0 = NM^{-1} = \tilde{M}^{-1}\tilde{N}$

Then by Lemma 1, every stabilizing controller for P_0 can be written in the following form:

$$K = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})$$

for some $Q\in H_\infty$ such that $\det(\tilde V(\infty)-Q(\infty)\tilde N(\infty))\neq 0,$ or, equivalently

$$K = (U + MQ)(V - NQ)^{-1}$$

for some $Q \in H_{\infty}$ such that $\det(V(\infty) - N(\infty)Q(\infty)) \neq 0$. We shall now propose a new way of implementing the controller

$$K = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})$$

as shown in Fig. 3. Note that the feedback diagram in Fig. 3 is not equivalent to the diagram in Fig. 2 since the reference signal r enters into the system from a different location. Nevertheless, the internal stability of the system is not changed since the transfer function from y to u is not changed. Thus this controller implementation also stabilizes internally the feedback system with plant P_0 for any $Q \in H_{\infty}$ such that $\det(\tilde{V}(\infty) - Q(\infty)\tilde{N}(\infty)) \neq 0$.



Fig. 3. Generalized internal model control structure.

Due to the similarity with the well-known IMC, see [13] for details, we shall call our controller framework as *generalized internal model control (GIMC)*. We shall see later on their connections and the possible advantages of our new GIMC over the traditional IMC.

The distinguished feature of this controller implementation is that the inner loop feedback signal f is always zero, i.e., f = 0, if the plant model is perfect, i.e., if $P = P_0$. The inner loop is only active when there is a model uncertainty or other sources of uncertainties such as disturbances and sensor noises. Thus Q can be designed to robustify the feedback systems. Thus our new controller design architecture has a clear separation between performance and robustness.

Controller Design: A high performance robust system can be designed in two steps: (a) Design $K_0 = \tilde{V}^{-1}\tilde{U}$ to satisfy the system performance specifications with a nominal plant model P_0 ; (b) Design Q to satisfy the system robustness requirements. Note that the controller Q will not affect the system nominal performance.

It should be emphasized that K_0 is not just any stabilizing controller as in most of controller parameterizations used in the literature, it is designed to satisfy certain performance specifications. For example, K_0 may be a simple PI controller

$$K_0(s) = \frac{K_p(s+a)}{s}$$

that satisfies our design specifications, in which case we can take $\tilde{U} = 1$ and $\tilde{V}^{-1} = K_0 = (K_p(s+a))/s$.

Suppose that P_0 has the following state-space realization:

$$P_0 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

and assume that (A, B) is stabilizable and (C, A) is detectable. Let F and L be such that A + BF and A + LC are stable. Then the left coprime factorization $P_0 = \tilde{M}^{-1}\tilde{N}$ can be chosen as

$$\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} A + LC & B + LD & L \\ \hline C & D & I \end{bmatrix}$$

Denote the state vector of $\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix}$ by \hat{x} and note that

$$f = \tilde{N}u - \tilde{M}y.$$

Then we have

$$\dot{\hat{x}} = (A + LC)\hat{x} + (B + LD)u - Ly$$
$$f = (C\hat{x} + Du) - y$$

i.e., f is the estimated output error.

We should also point out that we do not have to implement our GIMC controllers using five blocks of transfer functions. For example, assume K_0 has the following stabilizable and detectable realization:

$$K_0 = \begin{bmatrix} A_k & B_k \\ \hline C_k & D_k \end{bmatrix}$$



Fig. 4. Alternative implementation of GIMC.



Fig. 5. GIMC with stable plant.

then there exists an L_k such that $A_k + L_k C_k$ is stable and $K_0 = \tilde{V}^{-1} \tilde{U}$ with

$$\begin{bmatrix} \tilde{V} & \tilde{U} \end{bmatrix} = \begin{bmatrix} A_k + L_k C_k & L_k & B_k + L_k D_k \\ \hline C_k & I & D_k \end{bmatrix}.$$

Then the GIMC structure can be redrawn as shown in Fig. 4 where

$$\begin{bmatrix} I - \tilde{V} & \tilde{U} \end{bmatrix} = \begin{bmatrix} \underline{A_k + L_k C_k} & -L_k & B_k + L_k D_k \\ \hline C_k & 0 & D_k \end{bmatrix}$$
$$\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} \underline{A + LC} & \underline{B + LD} & \underline{L} \\ \hline C & D & I \end{bmatrix}.$$

It is noted that our GIMC structure may result in a high order controller. Thus it might be necessary to do a controller order reduction. We suggest the following controller reduction scheme:

$$\min_{\hat{Q}} \left\| Q \begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} - \hat{Q} \right\|_{\infty}$$

where \hat{Q} is restricted to a lower order transfer function. This problem can be approximately solved using Hankel norm model reduction method or balance truncation method, see [10], [21], and the references therein.

When the plant itself is stable, we can take

$$\tilde{N} = P_0 \quad \tilde{M} = I$$

Then the feedback system shown in Fig. 3 becomes Fig. 5. It is clear from this diagram that f is the error between the output of the nominal model and the output of the true system.

IV. APPLICATION TO FAULT TOLERANT CONTROL

Surprisingly, the estimated output error f defined in Fig. 3 is in fact the *residual signal* used in fault diagnosis literature [4], [5], [7]. (q = Qf is also considered in the fault diagnosis literature as a



Fig. 6. A fault-tolerant control scheme.

residual signal but the motivation for the choice of Q is quite different from here.) In the fault diagnosis literature, f is used to detect the possible faults in actuators and/or sensors. Unfortunately, only a very few published papers have dealt with how to use the residual signal to design fault-tolerant controllers, see [4], [15], and the references therein. The existing approaches to the design of fault-tolerant controllers are mostly based on robust control techniques. More precisely, a single controller is usually designed using robust control methods by assuming the possible actuators and/or sensors failures as model uncertainties. For example, a possible actuator fault in the first channel of an m actuator system with $B = [B_1, B_2, \ldots, B_m]$ can be represented by introducing an uncertainty in the corresponding input matrix

$$\dot{x} = Ax + B_1(1+\delta)u_1 + B_2u_2 + \dots + B_mu_m$$

 $\delta \in [-1,0]$

where $\delta = -1$ implies a total failure of the actuator and $\delta = 0$ implies no actuator failure. Then a robust controller is designed for this uncertain system and the resulting controller is implemented using the standard feedback structure shown in Fig. 1. This is clearly the worst case design and it is not surprising to see that such fault-tolerant feedback system may perform very poorly compared with a nonfault-tolerant control system when there is no actuator and/or sensor failure. On the other hand, our GIMC structure potentially gives all possible fault-tolerant controllers. Our fault-tolerant controllers can be designed such that they provide adequate performance when there are no faults in the systems and as much tolerance as possible by any other fault-tolerant or robust controllers. Such controllers can be designed in two steps.

- a) Design $K_0 = \tilde{V}^{-1}\tilde{U}$ to satisfy the system performance by assuming no faults (and model uncertainties).
- b) Design Q to tolerate possible actuators and/or sensors failures (and model uncertainties). This Q can be designed using standard robust control techniques, fuzzy control methods, adaptive control techniques, etc.

Note that it is shown in [12] that all nonlinear and time varying stabilizing controllers for a linear time invariant and strictly proper plant P_0 , i.e., $P_0(\infty) = 0$, can also be parameterized as

$$K = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})$$

as long as Q is allowed to be any nonlinear and time varying stable system. Thus the system stability is guaranteed as long as Q is chosen to be a stable nonlinear and time varying system. Hence, we can choose a fixed Q or a nonlinear and time varying Q. One can also design a Qfor each failure mode, then switch among the Q's when a certain failure mode is detected from the residual signal f. This is shown schematically in Fig. 6.

It is also easy to see that Q can also be used as a redundant reliable controller in a reliable control system. We believe a reliable controller

designed using this framework may potentially performance much better than a one by the conventional design methods based on the worst-case robust control techniques such as the Riccati equation method proposed in [16].

It is not hard to see that our generalized internal model control can also be made to be adaptive or used for gain scheduling control. One way to design an adaptive robust control law is to devise a mechanism to adjust the free stable controller Q online. Switching among several predefined controllers may also be used. We believe that this adaptive robust control scheme has the potential to perform better than the conventional adaptive robust control scheme if the nominal performance controller $K_0 = \tilde{V}^{-1}\tilde{U}$ is suitably designed.

V. ROBUSTIFICATION

In this section, we shall consider how to design the controller Q for robustness. We shall start with a system where the plant is described by a family of coprime factorizations.

Suppose that the true plant is described by

$$P = (N + \Delta_n)(M + \Delta_m)^{-1}$$

with the model uncertainties satisfying

$$\begin{bmatrix} \Delta_n \\ \Delta_m \end{bmatrix} \in H_{\infty} \quad \left\| \begin{bmatrix} \Delta_n \\ \Delta_m \end{bmatrix} \right\|_{\infty} < \frac{1}{\gamma}.$$

Then it is fairly easy to show that the controller

$$K = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})$$

will robustly stabilize the uncertain feedback system if and only if

$$\left\| \left(\tilde{U}N + \tilde{V}M \right)^{-1} \left(\begin{bmatrix} \tilde{U} & \tilde{V} \end{bmatrix} + Q \begin{bmatrix} \tilde{M} & -\tilde{N} \end{bmatrix} \right) \right\|_{\infty} < \gamma.$$

In the case when K_0 is an observer-based controller, i.e.,

$$K_0 = \begin{bmatrix} A + BF + LC + LDF & L \\ F & 0 \end{bmatrix}$$

where F and L are state feedback gain and observer gain such that A + BF and A + LC are stable, for instance K_0 may be a LQG controller, then we can take

$$\begin{bmatrix} \tilde{V} & \tilde{U} \end{bmatrix} = \begin{bmatrix} \frac{A+LC}{F} & -(B+LD) & L \\ \hline F & I & 0 \end{bmatrix}$$
$$\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} \frac{A+LC}{C} & B+LD & L \\ \hline C & D & I \end{bmatrix}$$
$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} \frac{A+BF}{F} & L \\ \hline -(C+DF) & I \end{bmatrix}$$
$$\begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} \frac{A+BF}{F} & B \\ \hline F & I \\ C+DF & D \end{bmatrix}.$$

It is easy to verify that

$$\tilde{U}N + \tilde{V}M = I$$



Fig. 7. GIMC with general uncertain plant.



Fig. 8. General linear fractional transformation form.

and

$$G = \begin{bmatrix} \tilde{U} & \tilde{V} & \vdots & I \\ \dots & \dots & \dots \\ \tilde{M} & -\tilde{N} & \vdots & 0 \end{bmatrix}$$
$$= \begin{bmatrix} A + LC & L & -(B + LD) & \vdots & 0 \\ \hline F & 0 & I & \vdots & I \\ \dots & \dots & \dots & \dots \\ C & I & -D & \vdots & 0 \end{bmatrix}.$$

Thus an H_{∞} controller Q satisfying

$$\left\| \begin{bmatrix} \tilde{U} & \tilde{V} \end{bmatrix} + Q \begin{bmatrix} \tilde{M} & -\tilde{N} \end{bmatrix} \right\|_{\infty} = \left\| \mathcal{F}_{\ell}(G, Q) \right\|_{\infty} < \gamma$$

can be found by solving one algebraic Riccati equation, see [21, Th. 17.1] for details.

In general, we can assume without loss of generality that the uncertain system can be described by a linear fractional transformation as shown in Fig. 7 where Δ includes all model uncertainties and are generally in block diagonal form, d includes all disturbances and sensor noises, and z includes all signals to be controlled such as the weighted control signal and weighted output signal. Then the problem can be put in a general linear fractional transformation form as shown in Fig. 8 and Q can be designed using standard robust control techniques, see [1], [20], and [21] for details.

We should point out that, as long as r and e are not involved directly in the design of Q (i.e., it does not shown in Fig. 8), our controller implementation should in principle perform no worse than the standard robust controller implementation does with regard to the robustness and the performance of the controlled signal z since the transfer function from y to u, the standard robust controller, is always the same and is



Fig. 9. Two degree of freedom controller.



Fig. 10. A GIMC implementation of 2DOF controller.

independent of the nominal controller K_0 . In the worst case (i.e., when the uncertainties are in the worst case), our controller implementation will be equivalent to the existing robust control design. Of course, if there is no uncertainty, our controller will perform as well as a nominal controller does. In fact, our framework provides a great flexibility in controller design, for example, one could still use all the robust and H_{∞} design techniques here. All one has to do is to start with a good performance controller and then everything can proceed as in the standard robust control design procedure to find the robust controller Q. The only difference is that we are not interested in plugging Q into the controller parameterization to find the total controller rather we will implement the performance controller and the robust controller Q separately.

VI. CONNECTIONS WITH TWO DEGREE OF FREEDOM CONTROLLERS

It turns out that our GIMC structure is closely related to the two degree of freedom control strategy proposed in the literature, see [17], [18], and the references therein. Consider a two degree of freedom feedback system shown in Fig. 9.

It is shown in [17] that all two degree of freedom controllers can be parameterized as

$$\begin{bmatrix} K_1 & K_2 \end{bmatrix} = (\tilde{V} - Q\tilde{N})^{-1} \begin{bmatrix} R & \tilde{U} + Q\tilde{M} \end{bmatrix}$$

where $Q \in H_{\infty}$ and $R \in H_{\infty}$ are any systems such that $\det(\tilde{V}(\infty) - Q(\infty)\tilde{N}(\infty)) \neq 0$.

Now take R = UR for any $R \in H_{\infty}$. Then the two degree of freedom controller can be alternatively implemented as shown in Fig. 10, which is in fact a general form of our GIMC. Of course, the conventional two degrees of freedom (2DOF) controllers are not implemented in this fashion. Nevertheless, we believe this is probably a more suitable alternative implementation if the computational demand due to the high order controllers can be managed.

VII. DUAL STRUCTURE

A dual GIMC structure can be obtained by using the right coprime factorization approach as shown in Fig. 11. This dual structure was actually first proposed in one of the author's book [19] (page 78).

However, we believe this GIMC structure is less favorable comparing with the GIMC structure using left coprime factorization. One reason is that Q is always active even with a perfect model. Since the



Fig. 11. GIMC using right coprime factorization.



Fig. 12. Standard IMC structure.

output of M adds additional signal to the actuator, it may saturate the actuator easily even though the net effect of this Q controller in the ideal case is cancelled in the feedback loop. Another reason is that it is not clear how this structure can be used for fault-tolerant control. Nevertheless, this approach is closely related to the well-known IMC structure. Indeed, if P_0 is stable, then we can pick M = I and $N = P_0$. Since $K_0 = 0$ is a stabilizing controller, one can also choose U = 0 and V = I. Then feedback system shown in Fig. 11 is exactly the well-known IMC system as shown in Fig. 12. The design of such Q is discussed in detail in [13].

VIII. CONCLUSION

In this note, we have proposed a controller architecture that we hope to have some impact on modern control system design. We also hope that this controller architecture may offer an alternative way to look at robust control, fault-tolerant control, adaptive robust control, etc.

REFERENCES

- G. Balas, J. C. Doyle, K. Glover, A. Packard, and R. Smith, μ-Analysis and Synthesis Toolbox. Natick, MA: MUSYN, Inc., and The Math-Works, Inc., 1991.
- [2] T. Başar and P. Bernhard, "H_∞—optimal control and related minimax design problems: A dynamic game approach," in Systems and Control: Foundations and Applications. Boston, MA: Birkhäuser, 1991.
- [3] J. Chen, "Sensitivity integral relations and design trade-offs in linear multivariable feedback systems," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 1700–1716, Oct. 1995.
- [4] J. Chen and R. J. Patton, Robust Model-Based Fault Diagnosis for Dynamic Systems. Norwell, MA: Kluwer, 1999.
- [5] E. Y. Chow and A. S. Willsky, "Analytic redundancy and the design of robust detection systems," *IEEE Trans. Automat. Contr.*, vol. AC-29, pp. 603–614, July 1984.
- [6] M. Dahleh and I. J. Diaz-Bobillo, Control of Uncertain Systems: A Linear Programming Approach. Upper Saddle River, NJ: Prentice-Hall, 1995.
- [7] X. Ding and P. M. Frank, "Fault detection via factorization approach," Syst. Control Lett., vol. 14, no. 5, pp. 431–436, 1990.
- [8] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-space solutions to standard \mathcal{H}_{∞} and H_{∞} control problems," *IEEE Trans. Automat. Control*, vol. AC-34, no. 8, pp. 831–847, 1989.

- [9] J. S. Freudenberg and D. P. Looze, "Frequency domain properties of scalar and multivariable feedback systems," in *Lecture Notes in Control* and Information Science. Berlin, Germany: Springer-Verlag, 1988, vol. 104.
- [10] K. Glover, "All optimal Hankel-norm approximations of linear multivariable systems and their L_∞-error bounds," Int. J. Control, vol. 39, pp. 1115–1193, 1984.
- [11] I. M. Horowitz, Synthesis of Feedback Systems. London, U.K.: Academic, 1963.
- [12] P. P. Khargonekar and K. R. Poolla, "Uniformly optimal control of linear time-invariant plants: Nonlinear time-varying controllers," *Syst. Control Lett.*, vol. 6, no. 5, pp. 303–308, 1986.
- [13] M. Morari and E. Zafiriou, *Robust Process Control.* Upper Saddle River, NJ: Prentice-Hall, 1989.
- [14] T. Tay, I. Mareels, and J. B. Moore, *High Performance Control*. New York: Springer-Verlag, 1997.
- [15] N. E. Wu and T. J. Chen, "Feedback design in control reconfigurable systems," *Int. J. Robust Nonlinear Control*, vol. 6, pp. 561–570, 1996.
- [16] R. J. Veillete, J. V. Medanic, and W. R. Perkins, "Design of reliable control systems," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 290–304, Mar. 1992.
- [17] M. Vidyasagar, Control System Synthesis. Cambridge, MA: MIT Press, 1985.
- [18] W. A. Wolovich, Automatic Control Systems: Basic Analysis and Design. Oxford, U.K.: Oxford Univ. Press, 1993.
- [19] K. Zhou, Solutions Manual: Essentials of Robust Control. Upper Saddle River, NJ: Prentice-Hall, 1998.
- [20] K. Zhou and J. C. Doyle, *Essentials of Robust Control*. Upper Saddle River, NJ: Prentice-Hall, 1998.
- [21] K. Zhou, J. C. Doyle, K. Glover, and H. Prentice-Hall, Robust and Optimal Control Upper Saddle River, NJ, 1996.

On the Stable \mathcal{H}_{∞} Controller Parameterization Under Sufficient Condition

Youngjin Choi and Wan Kyun Chung

Abstract—Suboptimal \mathcal{H}_{∞} controller is said to internally stabilize the closed loop transfer matrix. However, the stability of controller is not assured in conventional \mathcal{H}_{∞} control theory. Unstable controller can damage the whole system when the sensor fails or actuator saturates. This note presents a stable *n*-dimensional \mathcal{H}_{∞} controller and its parameterization for the LTI system based on the sufficient condition for existence of stable \mathcal{H}_{∞} controller. The stability of \mathcal{H}_{∞} controller and closed-loop transfer matrix are guaranteed if the positive–semidefinite solutions for the suggested three Riccati equations exist.

Index Terms—Parameterization, stable \mathcal{H}_{∞} control, strong stabilization.

I. INTRODUCTION

The stability of controller has been neglected in its design procedure. However, if sensor failure or actuator saturation happens, then stable controllers can relatively protect the entire control system comparing to unstable controllers. Also, the unstable controller brings undesired right half plane zeros in the closed loop and it degrades the tracking performance and affects the sensitivity to disturbances. The problem

Manuscript received October 20, 2000; revised March 1, 2001. Recommended by Associate Editor P. Apkarian.

The authors are with the Robotics and Bio-Mechatronics Laboratory, Pohang University of Science and Technology (POSTECH), Pohang, Korea (e-mail: yjchoi@postech.edu; wkchung@postech.edu).

Publisher Item Identifier S 0018-9286(01)09534-4.