Stationary Magnetically Coupled Circuits: Two-Winding Transformer
Consider a two-winding step-up transformer with following parameters: turn-ratio \( a = 0.4505 \), \( r_1 = 0.11 \Omega \), \( x_{1l} = 0.3 \Omega \), \( r_2 = 0.4 \Omega \), \( x_{1/2} = 1.2 \Omega \), \( x_m = 98.25 \Omega \), and \( \omega_p = 2 \cdot \pi \cdot 60 \text{ rad/sec} \) (60 Hz base frequency). Assume that your transformer is supplied from a nominal source of 10.28V rms (as on p. 24-26 of your notes!) and secondary side is connected to resistive load \( r_{load} \). Use a Matlab script file to store and load the circuit parameters!

1) Develop a Simulink model (first without saturation). Here, for compactness, you can calculate and use A, B, C, D matrices. Compute the system eigenvalues with \( r_{load} = 0 \), \( r_{load} = 50 \), \( r_{load} = 500 \), and \( r_{load} = 5000 \). Determine the stiffness ratio and estimate possible step-size that would be required for explicit and implicit ODE solvers for different loading conditions.

2) Assume zero initial conditions. Set \( r_{load} = 720 \Omega \) and simulate the energizing transients supplying the primary winding from a sinusoidal source of 10.28V rms. Experiment with the fixed and with variable step solvers available in Simulink. Observe the time step(s) and the simulation speed obtained by several different selected solvers. Select a solver and the appropriate step size limits and tolerances that you think work well for this system. Consider two cases:
   a) when the input voltage is a sine wave 10.28V rms, and
   b) when the input voltage is a cosine wave 10.28V rms.

Output to the Workspace and plot the variables \( i_1, V_1, i_2, V_2, \Psi_m , P_{in} \) (use the Matlab SUBPLOT function to plot all variables on one page); where \( P_{in} \) is the average input power per cycle.

Determine (based on simulation results and the eigenvalues!) when the steady-state is reached in the case a) and in b)?

3) **Incorporate magnetic saturation**: The characteristic \( \Psi_m(i_m) \) will be posted on the class webpage. Implement the direct approach with algebraic-loop, flux correction with algebraic-loop, and flux correction without no-algebraic-loop approaches discussed in class. Incorporate the power dissipation due to core losses in your model assuming that \( r_c = 720 \Omega \) resistor is connected to the output terminals. Repeat the studies of step 2a) and 2b). Use the Simulink Profiler to determine the number of calls to the functions that implement saturation. What is the difference between the models with and without the algebraic loop? Here, also plot \( \Psi_m(i_1) \). Comment on the effect of saturation in both cases. How do your results compare to the measurements included in your notes on p. 24-26? What does your model predict well and not-so-well in steady state and transient? Briefly state and explain why.

In addition to magnetic saturation, try to implement the model with “variable \( r_c \)” that changes depending on the saturation of magnetic core such that your model better predicts the overall transient in your notes on p. 24-26. Show and explain your best results compared to step 3).

5) You can use the LINMOD function and/or the Linear Analysis tool. Obtain the transformer input-to-output transfer-function \( H^{unsat}(s) = V_{out}/V_{in} \) and the input impedance \( Z^{unsat}(s) = V_{in}/I_{in} \) at zero state operating point and at an operating point that corresponds to the peak of the magnetizing flux linkage per second \( \Psi_m \) of 16V.
   
   (a) Obtain \( H^{sat}(s) \) and compare it to \( H^{unsat}(s) \) on the same plot;
   
   (b) Obtain \( Z^{unsat}(s) \) and compare it to \( Z^{sat}(s) \) on the same plot.

Calculate the effective inductance at 60Hz from \( Z(s) \) in step (b) and compare it to the transformer parameters (nominal and saturated). What can you say about the system’s eigenvalues and the frequency response? How can you explain the system’s behavior in the very low and very high frequency range? Explain the difference between the unsaturated and saturated cases in steps (a) and (b). What could be the ways of obtaining a similar transfer-function from a hardware prototype?

6) Assume 3 identical transformers connected as Yg/Y. The primary side Y is grounded through a resistor: a) \( r_g = 0 \Omega \); and

   b) \( r_g = 1000 \Omega \). The secondary sides are connected to resistive loads \( r_{load} = 5000 \Omega \). Include your block diagram and explain inputs/outputs of the modules. How the presence of grounding resistor changes model? Assume a symmetrical three-phase source. Repeat step 2) to study the inrush current and grounding. Explain your results.

7) Verify your Simulink model against the SymPowerSystems transformer model and repeat the step 3). Comment of results.

**Reporting**: The assignment report should demonstrate the individual work and models, include printouts of the model and subsystems, parameter script file, simulation results, as well as very brief discussions, comments/conclusions wherever appropriate, and overall summary of what you have learned in this assignment. The discussions and comments should not be long (avoid writing long reports), but they should demonstrate your understanding of the problem.