EECE 568: Control Systems
Problem Set #1

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1. System Properties (10 points)

Consider a linear system with input u, output y, and state x. Three experiments are performed on the system using the inputs $u_1(t), u_2(t), u_3(t)$ for $t \geq 0$. In each case, the initial state $x(0)$ at $t = 0$ is the same. The corresponding outputs are denoted y_1, y_2, y_3.
Consider the following statements:

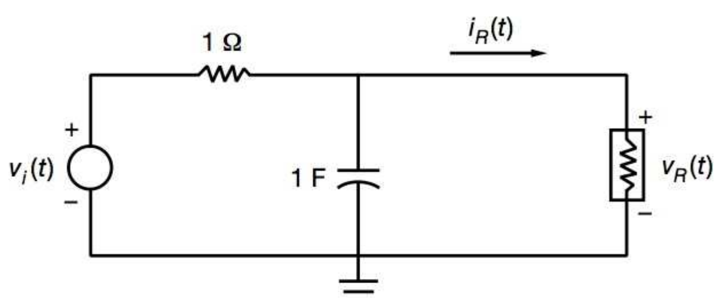
A. If $u_3 = u_1 + u_2$, then $y_3 = y_1 + y_2$.
B. If $u_3 = 0.5(u_1 + u_2)$, then $y_3 = 0.5(y_1 + y_2)$.
C. If $u_3 = u_1 - u_2$, then $y_3 = y_1 - y_2$.

1. Which, if any, of the above three statements are correct for any $u_1, u_2, x(0)$ if $x(0) \neq 0$?

2. Which, if any, of the above three statements are correct for any $u_1, u_2, x(0)$ if $x(0) = 0$?

2. Linearization (30 points)

In the circuit shown below, $v_i(t)$ is a voltage source and the nonlinear resistor obeys the relation $i_R = 1.5v_R^3$, with $v_i(t)$ the circuit input and $v_R(t)$ the circuit output.



1. Derive the differential equation for this circuit, and put the system into state-space form.

2. Linearize the system dynamics for the case when the circuit operates in steady-state about the point $v_i = 14$.

3. Find the analytic solution for the zero-state response of the linearized system when the input is $v_i(t) = 15, t \geq 0$ (and 0 otherwise). Plot the approximate solution to the circuit equation, $\delta x(t) + x_0(t)$, as a function of time. (FYI, use Matlab's `lsim` to calculate $\delta x(t)$ numerically.)

BONUS: (+5 points) Plot the zero-state response of the nonlinear system identified in question 1 and compare to your result from question 3.

3. State-space format (20 points)

Consider a system described by

$$\ddot{y} + 2\dot{y} - 3y = \dot{u} - u \tag{1}$$

1. Put the above system into state-space form with input $\dot{u}$ and output y. Clearly define the state vector and its dimension. What are $A(t)$, $B(t)$, $C(t)$, $D(t)$?
2. What is the transfer function for the system?

4. Integration Methods (40 points)

Numerical methods are often required to solve ordinary differential equations, e.g., when analytic solutions are not possible. One of the simplest methods is Euler's method. For the one-dimensional dynamical system

$$\dot{x} = f(t, x), \quad x(t_0) = x_0, \quad t_0 = 0, \quad t_0 \le t \le t_f \tag{2}$$

the numerical solution via Euler's method (also known as the forward rectangular rule) is

$$x[i+1] = x[i] + hf(t[i], x[i]), \quad i = 0, 1, 2, \ldots, \tag{3}$$

where $h = t[i+1] - t[i]$ is the constant integration step, $t[0] = t_0$, and $x[0] = x_0$. One interpretation of this method is that the area below the solution curve $x(t)$ is approximated by a sequence of sums of rectangular areas.

Alternatively, the Runge-Kutta family of integration methods are among the most widely used techniques. The main idea is to form a weighted average, by looking ahead (in time) by one-half or by a whole time-step h to determine the values of the derivative at several points. The solution to (2) via the Runge-Kutta method is approximated by

$$x[i+1] = x[i] + k \tag{4}$$

with

$$\begin{aligned} k &= \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \\ k_1 &= hf(t[i], x[i]), \\ k_2 &= hf(t[i] + \tfrac{1}{2}h, x[i] + \tfrac{1}{2}k_1), \\ k_3 &= hf(t[i] + \tfrac{1}{2}h, x[i] + \tfrac{1}{2}k_2), \\ k_4 &= hf(t[i] + h, x[i] + k_3) \end{aligned} \tag{5}$$

and $t[i+1] = t[i] + h$, $x[0] = x_0$.
Now consider the dynamical system

$$\dot{x} = 3x, \quad x(t_0) = 5, \quad t_0 = 0, \quad t_0 \le t \le 10 \tag{6}$$

1. Use Euler's method to obtain a solution to (6) with $h = 0.2$. Plot $x[k]$ vs. $t[k]$.
2. Use the Runge-Kutta method to obtain a solution to (6) with $h = 0.2$. On the same plot as in question 1, again plot $x[k]$ vs. $t[k]$.
3. Solve (6) analytically for $x(t)$, $t_0 \le t \le 10$. Again, on the same plot, plot $x(t)$ vs t.
4. Discuss your results. Which method works better for this system? Why?