EECE 568: Control Systems
Problem Set #2

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1. Linear transformations (30 points)

Consider the LTI system $\dot{x} = Ax + Bu, y = Cx$, with

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 \end{bmatrix} \tag{1}$$

1. Find the modal-form equivalent equation.
2. Compute the transfer function from input u to output y.
3. Compute the step response of the LTI system (assuming initial condition $x(0) = [0, 0, 0]$) using two different methods.

2. Controllability and Range Spaces (10 points)

Consider the equation

$$x[n] = A^n x[0] + A^{n-1} b u[0] + A^{n-2} b u[1] + \cdots + A b u[n-2] + b u[n-1] \tag{2}$$

with $A \in \mathbb{R}^{n \times n}$, $b \in R^{n \times 1}$. Notice that this can be rewritten as

$$x[n] - A^n x[0] = \begin{bmatrix} b & Ab & \cdots & A^{n-1}b \end{bmatrix} \begin{bmatrix} u[n-1] \\ u[n-2] \\ \vdots \\ u[0] \end{bmatrix} \tag{3}$$

Under what conditions on A and b will there exist an input sequence $u[0], u[1], \cdots, u[n]$ to meet equation (2) for any $x[n]$ and $x[0]$?

3. State transition matrix (20 points)

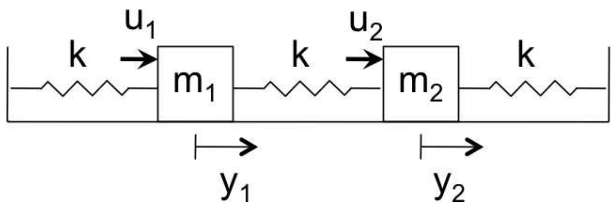
Let $\Phi(t, t_0)$ be the state transition matrix $\dot{x}(t) = A(t)x(t)$, with

$$\Phi(t, t_0) = \begin{bmatrix} \Phi_{11}(t, t_0) & \Phi_{12}(t, t_0) \\ \Phi_{21}(t, t_0) & \Phi_{22}(t, t_0) \end{bmatrix}, \text{ and } A(t) = \begin{bmatrix} A_{11}(t) & A_{12}(t) \\ \mathbf{0} & A_{22}(t) \end{bmatrix} \tag{4}$$

Show that $\Phi_{21}(t, t_0) = \mathbf{0}$ for all t and t_0, and that $\frac{\partial}{\partial t}\Phi_{ii}(t, t_0) = A_{ii}\Phi_{ii}(t, t_0)$ for $i = 1, 2$.

4. Modes of a Spring-Mass System (40 points)

Consider a frictionless two-mass system as shown below, with forces u_1 and u_2 applied to masses m_1 and m_2, respectively.



1. Show that the state-space representation of the system, with state $x = [y_1, y_2, \dot{y}_1, \dot{y}_2]$, input $u = [u_1, u_2]$, and output $y = [y_1, y_2]$ is given by:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} \mathbf{0} & I \\ -M^{-1}K & \mathbf{0} \end{bmatrix} x + \begin{bmatrix} \mathbf{0} \\ M^{-1} \end{bmatrix} u \\ y &= \begin{bmatrix} I & \mathbf{0} \end{bmatrix} \end{aligned} \tag{5}$$

   with $M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $m_1 = m_2 = 1$ kg and $K = \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}$, $k = 1$ N/m.

2. Simulate the zero-input response from initial condition $x_0 = [1, 0, 0, 0]$. (Using `lsim` or `initial` is fine).

   (a) What is the representation of x_0 in modal coordinates? (The Matlab command `eig` may be useful.)

   (b) Explain the observed output in terms of the modal dynamics.

3. Now consider the initial condition $x_0 = [1, 1, 0, 0]$. Simulate the zero-input response from this initial condition and explain the observed output. Why is the response a simple oscillation?

4. Find another initial condition which triggers a different "mode" of the system dynamics.

BONUS: (+2 points) How many modes does this system have? Why?