EECE 568: Control Systems
Problem Set #3

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Problems without asterisks emphasize basic material, problems marked with asterisks (*) are more difficult, time-consuming, or open-ended. All basic problems plus any two of the (*) problems should be handed in for full credit. If you hand in more than two (*) problems, all will be evaluated, but only the top two will be counted towards your mark.

1. LTI Stability (20/20* points)

Consider the system matrix

$$A = \begin{bmatrix} -.9 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \tag{1}$$

and non-zero input matrix $B \in \mathbb{R}^{3\times1}$ and non-zero output matrix $C \in \mathbb{R}^{1\times3}$. For the following questions, feel free to use Matlab to assist in any numerical calculations.

1. Is the system $\dot{x} = Ax$ stable? Asymptotically stable? Unstable?

2. Is the system $x[k+1] = Ax[k]$ stable? Asymptotically stable? Unstable?

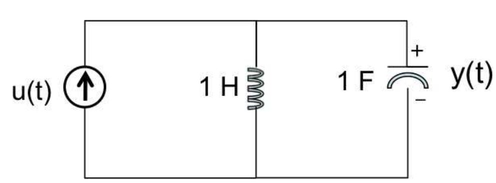
*3. Is the system $\dot{x} = Ax + Bu$, $y = Cx$, BIBO stable? BIBO unstable? Or is further information required to determine BIBO stability?

*4. Is the system $x[k+1] = Ax[k] + Bu[k]$, $y[k] = Cx[k]$, BIBO stable? BIBO unstable? Or is further information required to determine BIBO stability?

2. BIBO Stability (10/10* points)

1. Is the LTI system with impulse response $H(t) = \left[te^{-t}, \frac{1}{1+t}\right]$, $t \geq 0$, BIBO stable?

Now consider the following LC circuit with input current u and output y, the voltage across the capacitor. You may assume $L = C = 1$ for simplicity.



*2. Find the transfer function between input u and output y, then show that the system is not BIBO stable by finding a bounded input that produces an unbounded output.

3. LTV System Stability (0/20* points)

Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ -e^{-3t} & 0 \end{bmatrix} x \tag{2}$$

*1. Find the solution $\Phi(t, t_0)$.

*2. Is the system stable? Asymptotically stable?

4. Lyapunov Techniques (10/20* points)

1. Use the Lyapunov theorem for stability of continuous-time LTI systems to show that all eigenvalues of $A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & -1 \end{bmatrix}$ have negative real part. That is, show that the Lyapunov equation holds for any real, symmetric, negative definite Q and corresponding real, symmetric, positive definite P. *Hint: The leading principal minors may be useful in evaluating definiteness.*

*2. Show that all eigenvalues of $A \in \mathbb{R}^{n\times n}$ have real parts less than $-\mu < 0$ if and only if, for any given positive definite symmetric matrix $Q \in \mathbb{R}^{n\times n}$, the equation $A^T P + PA + 2\mu P = -Q$ has a unique symmetric solution $P \in \mathbb{R}^{n\times n}$ that is positive definite. (You can reference (as opposed to duplicate) portions of the standard stability proof covered in class, as needed).

*3. Consider a system $x[k+1] = Ax[x], x \in \mathbb{R}^n$. For each question, fill in the blank with one of the following: *may be*, *must be*, *must not be*.

   (a) If the system is stable in the sense of Lyapunov, there _______ a real, symmetric $P > 0$ such that $A^T PA + A = Q$ for any real, symmetric $Q < 0$.

   (b) If the system is stable in the sense of Lyapunov, there _______ a real, symmetric $P > 0$ such that $A^T PA + A = Q$ for any real, symmetric $Q \leq 0$.

   (c) If the system is asymptotically stable, there _______ a real, symmetric $P < 0$ such that $A^T PA + A = Q$ for any real, symmetric $Q < 0$.

   (d) If there exists real, symmetric $P > 0, Q < 0$ pair such that $A^T PA + A = Q$, the system _______ asymptotically stable.