EECE 568: Control Systems
Problem Set #4

Dr. Oishi
`http://courses.ece.ubc.ca/568`

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Problems without asterisks emphasize basic material, problems marked with asterisks (*) are more difficult, time-consuming, or open-ended. All basic problems **plus** any two of the (*) problems should be handed in for full credit. If you hand in more than two (*) problems, all will be evaluated, but only the top two will be counted towards your mark.

1. Two-car train (20/20* points)

Consider a two-car train with two inputs and one output, and the two masses connected via a spring with spring constant k. The input force for each mass is generated via a motor on the drive wheels of each car. You may assume for simplicity that car masses are $m_1 = m_2 = 1$ kg, and $k = 1$ N/m. For each question, clearly state the input or output matrix used.

1. Is the system controllable using only one motor? Using both motors?

2. Is the system observable if only the position of the first car is measurable? If only the velocity of the first car is measurable?

Now consider the case in which the motor on the first car only is used. We wish to stabilize the system, that is, to return the train to rest at the origin.

*3. Find the gain matrix K in the control law $u = -Kx$ to place the poles of the closed-loop system at $\lambda = -1 \pm j$ and $\lambda = -100 \pm j100$. Use Matlab's `place` to compute K.

*4. With initial condition $x_0 = [1, 2, -1, -3]$, simulate all 4 state trajectories of the closed-loop system in Matlab with `initial` for 10 seconds. Compare this to a simulation of the zero-input response of the open-loop system from the same initial condition. Discuss your results. What is happening to the physical system in both cases?

2. Harmonic oscillator (20/20* points)

Consider the following uncontrollable and observable system $\dot{x} = Ax + Bu, y = Cx + Du$:

$$A = \begin{bmatrix} 0 & \omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$

1. Put the system into standard form.

2. Is it possible for the closed-loop system to have poles at $\lambda = -1 \pm j, -2$? Why or why not?

*3. Calculate the transfer function for the open-loop system *without* inverting a 3×3 matrix (e.g., use information obtained from the Kalman Decomposition Theorem).

*4. Consider the case in which the closed-loop system will have at least one pole at $\lambda = -2$ with feedback controller $u = -Kx + r$. Discuss qualitatively the zero-state response of the closed-loop system when the reference input is a step input.

3. System design for controllability / observability (0/20* points)

Consider the system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \\ b_{51} & b_{52} \end{bmatrix} u \\ y &= \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} \end{bmatrix} x \end{aligned} \tag{2}$$

*1. Is it possible to find a set of b_{ij} such that the system is controllable? If so, find appropriate restrictions on b_{ij}.

*2. Is it possible to find a set of c_{ij} such that the system is observable? If so, find appropriate restrictions on c_{ij}.

4. Aircraft longitudinal dynamics (20/0* points)

Consider the longitudinal dynamics of an aircraft, with $x = [\alpha, \theta, q]$, consisting of the aircraft angle of attack α, pitch θ, and pitch rate q, and input due to the angle of deflection of the elevators.

$$A = \begin{bmatrix} -1 & 1 & 0 \\ -5 & -.5 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 \\ -9 \\ 0 \end{bmatrix} \tag{3}$$

1. Is the open-loop system asymptotically stable, stable in the sense of Lyapunov, or unstable?
2. Could the closed-loop system be asymptotically stable? Why or why not?

5. Decoupled system (0/30* points)

Consider the system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} x \end{aligned} \tag{4}$$

*1. Show that the system is uncontrollable and unobservable.

*2. Use the Kalman decomposition theorem to put the system into standard form.

*3. Find a reduced system in state-space form which is equivalent to the original system, in the sense that the original and reduced systems both have the same transfer function. (You do not need to compute the transfer functions.)