EECE 571M
Nonlinear Systems and Control
Problem Set #2

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1. Consider the damped inverted pendulum

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \tfrac{g}{L}\sin x_1 - \tfrac{b}{mL^2}x_2 \end{aligned} \tag{1}$$

with x_1 the angle from the vertical (where $x_1 = 0$ indicates the mass is inverted), x_2 the rotational speed of the pendulum, and with positive constants g (due to gravity), L the length of the massless arm, b the damping coefficient of the joint, m the mass of the pendulum.

   (a) Find the equilibrium points and nullclines of the system.

   (b) Determine the stability and type of the equilibrium points.

   (c) Show that there cannot be any periodic orbits.

2. Consider the problem of jet surge, an oscillatory phenomena in aircraft jet engines that can cause inefficient operation or even engine stall. Mass flow x enters the plenum (a chamber in the jet engine) to undergo an increase in pressure y, modulated by adjusting the throttle at the exit of the plenum. The mass flow depends on the compressor characteristic $c(x)$, the compressor speed b, and the plenum pressure rise y; the plenum pressure is dependent on the throttle characteristic $d(x)$ and the current mass flow x. Hence we only consider $x, y > 0$.

$$\dot{x} = b(c(x) - y), \quad \dot{y} = \frac{1}{b}(x - d^{-1}(y)), \tag{2}$$

with

$$\begin{aligned} c(x) &= -x^3 + 6x^2 - 9x + 6 \\ d(x) &= x^2 \cdot \mathrm{sign}(x) \end{aligned} \tag{3}$$

and

$$\mathrm{sign}(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases} \tag{4}$$

   (a) This model is capable of reproducing *rotating stall*, a malfunction that can occur in actual jet engines when a stable limit cycle exists. Say that measurements taken from this model show a limit cycle that oscillates between $x \in [1, 3]$. Applying the Poincare

Theorem, how many equilibrium points would you expect (without solving the cubic equation explicitly) to occur within $x \in [1, 3]$?

(b) Now sketch or plot the nullclines of the system for $b = 0.1$, $b = 0.3$, and $b = 1$. Note that the nullclines will have only one intersection. *Hint: Find the local minima and maxima of $c(x)$ by computing its first and second derivatives.*

(c) Show that the equilibrium point satisfies $c(x^*) = d(x^*)$. The command `roots` may be useful if you want a numerical solution, although you are also welcome to simply identify the solution as (x^*, y^*) without numerical values.

(d) Linearize the system about its equilibrium point (x^*, y^*) and characterize its stability. Assume that b is an unknown, positive parameter.

(e) Using Matlab and/or pplane, demonstrate the effect of increasing the compressor speed b by creating phase plots for the above 3 values of b. Briefly explain your results (3-4 sentences at most).

(f) Calculate $D_x f_\mu$. Show that bifurcations occur as b increases.

(g) At what values of b does stability change? What type of bifurcations occur at these values? Ia this consistent with your analysis from (d)?