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# Introduction to Cryptography

EECE 412



## Session Outline

- Historical background
  - Caesar and Vigenère ciphers
  - One-time pad
  - One-way functions
  - Asymmetric cryptosystems
- The Random Oracle model
  - Random functions: Hash functions
  - Random generators: stream ciphers
  - Random Permutations: block ciphers
  - Public key encryption and trapdoor one-way permutations
  - Digital signatures



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## Historical Background

To read:
5.1-5.2 Anderson's book
8.1-8.2 Bishop's book



## Letter Indices in English Alphabet

| A | B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

| N | O | P | Q | R | S | T | U | V | W | X | Y | z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |



## Caesar Cipher

- Plaintext is `HELLO WORLD`
- Change each letter to the third letter following it (X goes to A, Y to B, Z to C)
  - Key is 3, usually written as letter 'D'
  - **C = P + K mod 26**
- Ciphertext: `KHOOR ZRUOG`

```
Plain      HELLOWORLD
Key        DDDDDDDDDD
Cipher     KHOORZRUOG
```



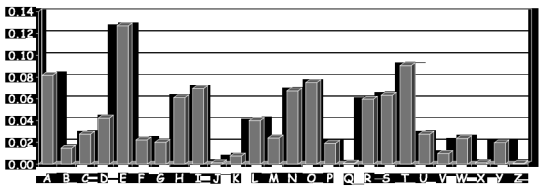
## Monoalphabetic Cipher

Invented by Arabs in 8th or 9th centuries

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | .. | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | T | W | S | G | M | P | A | Z | C | L | V | O | D | .. | B |

```
Plain   HELLOWORLD
Key
Cipher  AGVVYEYZVS
```

## Frequency Analysis






## Polyalphabetic **Vigenère Cipher**

proposed by Blaise de Vigenere from the court of Henry III of France in the sixteenth century

Like Cæsar cipher, but use a phrase

- Example
  - Message: TO BE OR NOT TO BE THAT IS THE QUESTION
  - Key: RELATIONS
  - Encipher using Cæsar cipher for each letter:

```
Plain   TO BE OR NOT TO BE TH AT  IS  THE  QUESTION
Key     RE LA T I  ONS  RE LA  T I ON  SR ELA T I ONSREL
Cipher  KS ME HZ BBL  KS ME MPOG AJ  XSE J CSFLZSY
```



## Cryptanalysis of Vigenère Cipher

Factoring of distances

- KSMEHZBBLKSMEMPOGAJXSEJCSFLZSY
- 012345678012345678012345678012



## One-Time Pad

A Vigenère cipher with a random key at least as long as the message

- Provably unbreakable
- Why?

| Plain text | D O I T | D O N T |
|---|---|---|
| Key | A J I Y | A J D Y |
| Cipher text | D X Q R | D X Q R |

- Warning: keys *must* be random, or you can attack the cipher by trying to regenerate the key



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## Little Bit of History

90 years ago,
January 19, 1917 …



## Codebook

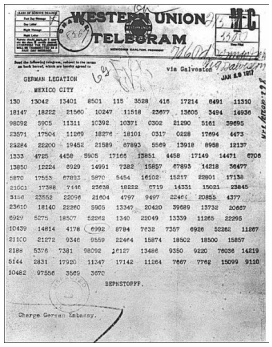
- Literally, a book filled with "codewords"
- Zimmerman Telegram encrypted via codebook

| | |
|---|---|
| Februar | 13605 |
| fest | 13732 |
| finanzielle | 13850 |
| folgender | 13918 |
| Frieden | 17142 |
| Friedenschluss | 17149 |
| : | : |

- Modern block ciphers are codebooks!

# Zimmerman Telegram

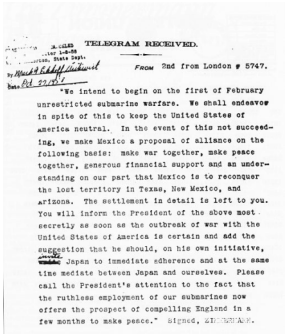
- One of most famous codebook ciphers ever
- Ciphertext shown here…






# Zimmerman Telegram Decrypted

- British had recovered partial codebook
- Able to fill in missing parts
- Led to US entry in WWI


TELEGRAM RECEIVED.

From 2nd from London # 5747.

"We intend to begin on the first of February unrestricted submarine warfare. We shall endeavor in spite of this to keep the United States of America neutral. In the event of this not succeeding, we make Mexico a proposal of alliance on the following basis: make war together, make peace together, generous financial support and an understanding on our part that Mexico is to reconquer the lost territory in Texas, New Mexico, and Arizona. The settlement in detail is left to you. You will inform the President of the above most secretly as soon as the outbreak of war with the United States of America is certain and add the suggestion that he should, on his own initiative, Japan to immediate adherence and at the same time mediate between Japan and ourselves. Please call the President's attention to the fact that the ruthless employment of our submarines now offers the prospect of compelling England in a few months to make peace." Signed, ZIMMERMAN.



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## Asymmetric Cryptosystems



## Public Key Cryptography

- Two keys
  - Sender uses recipient's **public key** to encrypt
  - Receiver uses his **private key** to decrypt
- Based on **trap door, one way function**
  - Easy to compute in one direction
  - Hard to compute in other direction
  - "Trap door" used to create keys
  - Example: Given p and q, product N=pq is easy to compute, but given N, it is hard to find p and q



## Public Key Cryptography

- Encryption
  - Suppose we encrypt M with Bob's public key
  - Only Bob's private key can decrypt to find M
- Digital Signature
  - **Sign** by "encrypting" with private key
  - Anyone can **verify** signature by "decrypting" with public key
  - But only private key holder could have signed
  - Like a handwritten signature (and then some)



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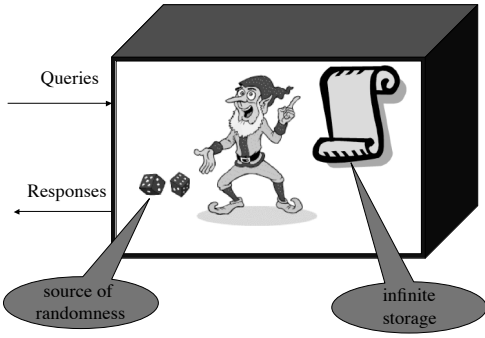
## Random Oracle Model

5.3 (Anderson's book)

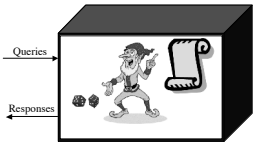
## What is Random Oracle Model?




19

## Random Function as Random Oracle

- In: string of any length



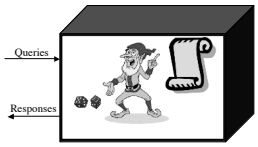

- Out: random string of fixed length
- Applications:
  - One-way functions
  - Hash functions
    - Message digests
    - Time stamping

Properties
- "One-wayness"
- No input inference from output h(M|K)
- Few collisions

20

## Random Generator (Stream Cipher) as Random Oracle

- In:
  - short string (key)
  - length of the output




- Out: long random stream of bits (keystream)
- Applications:
  - Communications encryption
  - Storage encryption

Properties
- Should not reuse
  - Use *seed*

21

## Example: A5 stream cipher for GSM

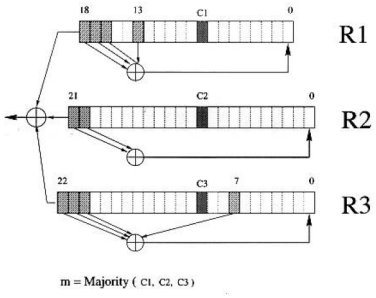
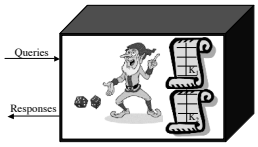



Figure 1: The A5/1 stream cipher.

22 From: Alex Biryukov, Adi Shamir, David Wagner "Real Time Cryptanalysis of A5/1 on a PC"

## Random Permutation (Block Cipher) as Random Oracle

- In
  - fixed size short string (plaintext) M,
    - DES -- 64 bits
  - Key K




- Out
  - same fixed size short string (ciphertext) C

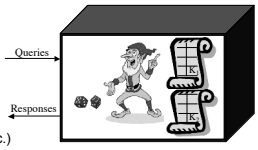
Notation
- $C = \{ M \}_K$
- $M = \{ C \}_K$

Properties
- Invertible

23

## Attacks on Block Ciphers

- Attack types
  - Known plaintext attack
  - Chosen plaintext attack
  - Chosen ciphertext attack
  - Chosen plaintext/ciphertext attack
  - Related key attack (K +1, K + 2, etc.)

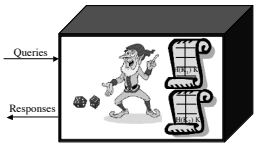



- Attack objectives
  - forgery attacks-- deduce the answer to the query which the attacker has not made yet
  - key recover attacks -- recover the key
- Why attack types are important?
  - DES
    - $2^{47}$ chosen plain texts
    - $2^{43}$ known plain texts

24

## Public Key Encryption and Trap-door One-Way Permutation as Random Oracle

- Public Key Encryption Scheme:
  - Key pair (KR, $KR^{-1}$) generation function from random string R
    - $KR \rightarrow KR^{-1}$ is infeasible
  - $C = \{M\}_{KR}$
  - $M = \{C\}_{KR^{-1}}$

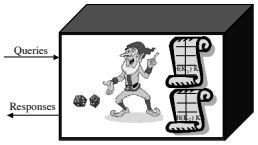



- In:
  - fixed size short string (plaintext) M,
  - Key KR
- Out: fixed size short string (ciphertext) C



## Digital Signature as Random Oracle

- Public Key Signature Scheme:
  - Key pair ($\sigma R$, VR) generation function
    - $VR \rightarrow \sigma R$ is infeasible
  - $S = Sig_{\sigma R}(M)$
  - $\{True, False\} = Ver_{VR}(S)$




| | Signing | Verifying |
|---|---|---|
| Input | Any string M + $\sigma R$ | S + VR |
| Output | S = hash(M) \| cipher block | "True" or "False" |



## Summary

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