EECE 571M / 491M: Introduction to Hybrid Systems and Control
Homework #1
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Problem 1: Spring mass damper with dead zone

Spring-mass-damper systems can have a "dead zone" – an area in the spring force curve for which the spring provides no restorative force. A simple way to approximate this behavior is through the following:

$$\begin{aligned} m\ddot{x} &= -b\dot{x} - F_{\text{spring}}(x) + f_{\text{external}}, \\ F_{\text{spring}}(x) &= \begin{cases} k(x-\epsilon) & x \geq \epsilon \\ k(x+\epsilon) & x \leq -\epsilon \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{1}$$

Assume for now that the external force acting on the system is $f_{\text{external}} = 0$.

1. Model the system as a hybrid automata. Identify and label modes, continuous dynamics, domains, guards, reset maps, and events.

2. For each of the three values of m, b, k, with $\epsilon = 0.10$, plot or sketch the phase-plane diagram for the system in coordinates $(x, \dot{x})$. Compare the response of the system to a standard (e.g., no dead-zone) spring-mass system with the same constant values. The phase-plane toolbox `pplane7` may be useful.

   (a) $m = 1, b = 0, k = 2$.

   (b) $m = 1, b = 1, k = 2$.

   (c) $m = 1, b = 3, k = 2$.

3. Sketch the phase-plane of the underdamped spring-mass-damper system in modal coordinates that correspond to the non-dead zone dynamics.

4. Find the conditions m, b, k must fulfill in order for a standard spring-mass-damper system (with zero external force) to have

   (a) Exactly one eigenvalue at 0.

   (b) Both eigenvalues at 0.

   Describe the physical system which corresponds to both cases.

Problem 2: Four-speed car

Consider a four-speed car with an automatic transmission. The continuous state is $x = [y, v]$, where y is the position of the car and v is its speed. Each of the four gears has associated with it an efficiency function that relates the speed of the car to the engine power available within a given gear. The dynamics of the car are

$$\dot{x} = \begin{bmatrix} v \\ \alpha_i(x) u_{\text{gas}} \end{bmatrix} \tag{2}$$

with u_{gas} representing the amount that the driver depresses the gas pedal, and efficiency function α_i described by Figure 1. The car should only use a gear when its efficiency in that gear is above

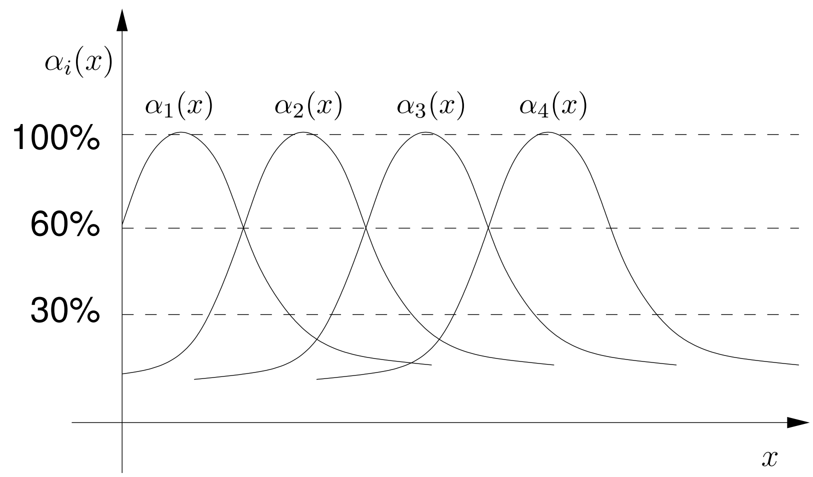

Figure 1: Car efficiency in each gear.

60%. In addition, the driver always has the option to brake, which results in the dynamics

$$\dot{x} = \begin{bmatrix} v \\ u_{\text{brake}} \end{bmatrix} \tag{3}$$

with u_{brake} the amount that the brake pedal is depressed. When braking, the car will remain in the same gear until it reaches the speed which corresponds to a 30% efficiency. Automatic downshifting is then required.

1. Model this problem as hybrid automaton with both continuous and discrete inputs due to the driver's actions. Identify any additional specifications required to make the system both non-blocking and deterministic.

Problem 3

Consider the discontinuous differential equation

$$\begin{aligned} \dot{x}_1 &= \operatorname{sgn}(x_1) + 2\operatorname{sgn}(x_2) \\ \dot{x}_2 &= -2\operatorname{sgn}(x_1) + \operatorname{sgn}(x_2) \end{aligned} \tag{4}$$

where $x(0) \neq (0,0)$, and

$$\operatorname{sgn}(z) = \begin{cases} +1 & \text{if } z > 0 \\ -1 & \text{if } z < 0 \\ \text{undefined} & \text{otherwise} \end{cases} \tag{5}$$

This system defines a hybrid automaton with four discrete modes having domains corresponding to the four quadrants.

1. Specify a hybrid automaton H to model the system.
2. Is H Zeno? Why or why not?

GS: Simulate this system from various initial conditions in Matlab, using `ode23` or `ode45`. These functions can explicitly handle boundary crossings or other hybrid phenomena when the `events` option is on.