EECE 571M / 491M: Introduction to Hybrid Systems and Control
Homework #2
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Problem 1: Lyapunov's indirect method

Consider the dynamical system

$$\begin{aligned} \dot{x}_1 &= -x_1 + a x_2 - b x_1 x_2 + x_2^2 \\ \dot{x}_2 &= -(a+b)x_1 + b x_2 - x_1 x_2 \end{aligned} \tag{1}$$

with $a > 0$ and $b \neq 0$.

1. Find all equilibrium points of the system.
2. Determine the type and stability (if known) of each equilibrium point for all values of $a > 0$ and $b \neq 0$.
3. For each of the following cases, construct the phase-plane diagram and discuss the qualitative behavior of the system.
   (a) $a = 1, b = 1$
   (b) $a = 1, b = -0.5$
   (c) $a = 1, b = -2$

Problem 2: Longitudinal aircraft dynamics

Consider the longitudinal dynamics of a large civil jet aircraft, simplified by assuming that the aircraft is traveling at a steady-state cruising altitude and speed. Deviations from the steady-state condition are modeled by the LTI system

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} \tag{2}$$

as a function of aircraft alpha of attack α, pitch rate q, and pitch θ.

1. Analyze the eigenvalues of the system matrix to determine whether the system is asymptotically stable, stable, or unstable?
2. Consider a quadratic Lyapunov function $V(x) = x^T P x$ with $P > 0$. Will the function $\dot{V}(x) = -x^T Q x$ have Q positive definite or positive semi-definite?
3. Use the Matlab LMI Toolbox to compute a feasible matrix P and the resulting matrix Q.

Problem 3: Lyapunov functions for time-varying linear systems

While we focused solely on LTI systems in class, Lyapunov functions can also be used on linear time-varying systems such as those shown below. For each of the following linear systems $\dot{x} = A(t)x$, use a quadratic Lyapunov function to show that the origin is asymptotically stable. In all cases, $\alpha(t)$ is continuous and bounded for all $t \geq 0$.

1. $A(t) = \begin{bmatrix} -1 & \alpha(t) \\ \alpha(t) & -2 \end{bmatrix}$, $|\alpha(t)| \leq 1$

2. $A(t) = \begin{bmatrix} 0 & 1 \\ -1 & -\alpha(t) \end{bmatrix}$, $\alpha(t) \geq 2$

3. $A(t) = \begin{bmatrix} -1 & 0 \\ \alpha(t) & -2 \end{bmatrix}$

[GS] Problem 4: Region of attraction

Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2(1 - x_1^2) \\ \dot{x}_2 &= -(x_1 + x_2)(1 - x_1^2) \end{aligned} \tag{3}$$

1. Identify all equilibria of the system.

2. Use a quadratic Lyapunov function $V(x) = x^T P x$ to show that the system is locally asymptotically stable about the origin. What are P and Q, such that $\dot{V}(x) = -x^T Q x$?

3. Sketch or plot the phase-plane diagram.

4. Estimate the region of attraction using the computed Lyapunov function.