EECE 571M / 491M: Introduction to Hybrid Systems and Control
Homework #3

Dr. Meeko Oishi

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Problem 1: Stabilizing through switching

Consider the linear system

$$\dot{x} = Ax + Bu, \quad A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{1}$$

with eigenvalues with 0 real part.

1. Consider two possible feedback control laws: $u = -x_1$, $u = 2x_1$. Do either of these controllers result in a subsystem that is asymptotically stable?
2. Consider the switched linear system $\dot{x} = \tilde{A}_i x$, where $\tilde{A}_i = A - BK_i$, corresponding to the above control laws $u = -K_i x$, and $A_3 = \text{diag}([-2, 1])$. Does a quadratic Common Lyapunov Function exist?
3. Consider the Lyapunov function $V(q, x) = x^T x$. Synthesize a stabilizing switching scheme which corresponds to regions for which the Lyapunov function in each mode will decrease.
4. Sketch or plot a phase-plane diagram of the resulting piecewise linear system, with trajectories from arbitrary initial conditions that converge to the origin.

Problem 2: Common Lyapunov functions

Which of the following switched linear systems have a common Lyapunov function? Identify the theorem(s) required to show this or explain why a common Lyapunov function does not exist.

(GS) 1. $A_1 = \begin{bmatrix} -a_1 & 0 \\ b_1 & -c_1 \end{bmatrix}, \; A_2 = \begin{bmatrix} -a_2 & 0 \\ b_2 & -c_2 \end{bmatrix}, a_i, b_i, c_i > 0$

(UG/GS) 2. $A_1 = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_n \end{bmatrix}, \; A_2 = \begin{bmatrix} 1/d_1 & 0 & \cdots & 0 \\ 0 & 1/d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/d_n \end{bmatrix}, \; A_3 = I_{n\times n}, \; d_i < 0$

(UG/GS) 3. $A_1 = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \; A_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

(GS) 4. $A_i = QR_i, i \in \{1, \cdots, m\}$, with Q an orthogonal matrix, R_i an upper-triangular matrix, and A_i Hurwitz.