ELEC 343, Home Work Assignment # 1:
Follow through Example 1A, and do Study Problem, SP1.2-2, SP1.2-4
Follow through Example 1B, and do Study Problems SP1.3-1, SP1.3-3
Follow through Example 1D, and do Study Problems SP1.5-2 and SP1.5-3.
Textbook Chapter Problem(s): 3, 8, 9, 15, and 16.
Solutions:

SP1.2-2

From SP1.2-1, we have phasors of voltage and current:

$\tilde{V} = 1\angle 0^\circ$, $\tilde{I} = 1\angle 180^\circ$

Then we have $\omega = 2 \cdot \pi \cdot 60 \approx 377$ r/s

$V = \sqrt{2} \cdot \cos(\omega t) = \sqrt{2} \cdot \cos(377 \cdot t)$

$I = \sqrt{2} \cdot \cos(\omega t + \pi) = \sqrt{2} \cdot \cos(377 \cdot t + \pi)$

$P = V \cdot I = \sqrt{2} \cdot \cos(377 \cdot t) \cdot \sqrt{2} \cdot \cos(377 \cdot t + \pi)$

$= 2\left(\frac{1}{2} \cdot \cos(754 \cdot t + \pi) + \frac{1}{2}\cos(-\pi)\right) =$

$= -1 + \cos(754 \cdot t + \pi)$

SP 1.2-4 For simplicity, and without lack of generality, neglect resistance and assume the voltage is along the real axis. In this case the relationship between voltage and current is given by:

$$V\angle 0 = j(X_L - X_C)\tilde{I} + \tilde{E}$$

One can observe that the magnitude and phase of $\tilde{E}$ controls the magnitude and direction of $\tilde{I}$. For example, if $\tilde{E} = 2\angle 0^\circ$ and $X_L > X_c$, $\tilde{I}$ will lead $\tilde{V}$. In contrast, if $\tilde{E} = 0.5\angle 0^\circ$ and $X_L > X_c$, $\tilde{I}$ will lag $\tilde{V}$.

SP1.3-1. $\mathrm{Ni} = \mathcal{R}_{ab}(\Phi_1 + \Phi_2) + \mathcal{R}_{bcda}\Phi_1$

$$\Phi_1 = \frac{1}{\mathcal{R}_{bcda}}\left[\mathrm{Ni} - \mathcal{R}_{ab}(\Phi_1 + \Phi_2)\right]$$

$$\Phi_1 = \frac{1}{358{,}099}\left[-(109{,}419)(2.547 \times 10^{-3})\right] = 2.014 \times 10^{-3}\ \mathrm{Wb}$$

SP1.3-3. With windings as shown in Fig. 1B-1 and with the center leg removed, the total mmf is

$$\mathrm{mmf}_t = \mathrm{mmf}_1 + \mathrm{mmf}_2 = \mathrm{N}_1\mathrm{I}_1 + \mathrm{N}_2\mathrm{I}_2$$
$$= (150)(9) + (90)(-15) = 0$$

SP1.5-2. During steady-state conditions, the time rate-of-change of i_1 is zero; therefore, a voltage is not induced in the 2-winding. Hence, for the 2-winding open or short circuited $I_1 = \frac{V}{r_1} = \frac{12}{6} = 2$ A and $I_2 = 0$.

SP1.5-3. $Z = r_1 + j\,\omega_e\,(L_{l1} + L_{m1}) = 6 + j\,(100)(13.5 + 263.9) \times 10^{-3}$

$$= 6 + j\,27.74 = 28.38\,\underline{/77.8^\circ}$$

$$\tilde{I}_1 = \frac{\tilde{V}_1}{Z} = \frac{10\underline{/0^\circ}}{28.38\underline{/77.8^\circ}} = 0.352\,\underline{/-77.8^\circ}\text{ A}$$

3. The reluctance of the iron is

$$\mathscr{R}_m = \frac{l}{\mu_i\,A} = \frac{(4)(0.25)}{(4000)(4\pi \times 10^{-7})(0.05)^2} = 79{,}577\ \mathrm{H}^{-1}$$

$$L_{12} = \frac{N_1\,N_2}{\mathscr{R}_m} = \frac{(50)(100)}{79{,}577} = 0.0628\ \mathrm{H}$$

$$L_{m1} = \frac{N_1^2}{\mathscr{R}_m} = \frac{50^2}{79{,}577} = 0.0314\ \mathrm{H}$$

$$L_{m2} = \frac{N_2^2}{\mathscr{R}_m} = \frac{100^2}{79{,}577} = 0.1257\ \mathrm{H}$$

8. $\tilde{V}_1 = \frac{10}{2\sqrt{2}}\,\underline{/0^\circ} = 3.54\,\underline{/0^\circ}$ V

$$Z = r_1 + r_2' + j\,\omega_e\,(L_{l1} + L_{l2}') = 10 + 10 + j\,(2\pi)(30)(30 + 30) \times 10^{-3}$$

$$= 20 + j\,11.31\ \Omega$$

$$\tilde{I}_1 = \frac{\tilde{V}_1}{Z} = \frac{3.54\,\underline{/0^\circ}}{20 + j\,11.31} = 0.154\,\underline{/-29.5^\circ}\text{ A}$$

9. Since $\omega_e = 400$, $X_{m1} = 400\ \Omega$. Neglecting the magnetizing current $i_1 = -i_2'$.

(a) $\tilde{V}_1 = \frac{2}{\sqrt{2}}\,\underline{/0^\circ} = \sqrt{2}\,\underline{/0^\circ}$

$$\tilde{I}_1 = \frac{\tilde{V}_1}{(r_1 + r_2 + R_L) + j\,\omega_e\,(L_{l1} + L_{l2})} = \frac{\sqrt{2}\,\underline{/0^\circ}}{4 + j\,(400)(0.02)} = 0.158\,\underline{/-63.4^\circ}\text{ A}$$

(b) $I_1 = \sqrt{2}\,0.158\cos(400t - 63.4^\circ)$

15. $L_m(x) = \dfrac{k}{k_o + x}$

$$k = \frac{N^2 \mu_0 A_i}{2} = \frac{(500)^2(4\pi \times 10^{-7})(4 \times 10^{-4})}{2} = 2\pi \times 10^{-5}$$

$$k_0 = \frac{l_i}{2\mu_{ri}} = \frac{20 \times 10^{-2}}{(2)(1000)} = 10^{-4}$$

$$L_m(x) = \frac{2\pi \times 10^{-5}}{10^{-4} + x} \text{ H}$$

The approximation for $x > 0$ is

$$L_m(x) = \frac{2\pi \times 10^{-5}}{x} \text{ H}$$

Now to find minimum value of x

$$\frac{2\pi \times 10^{-5}}{x} = 1.1 \frac{2\pi \times 10^{-5}}{10^{-4} + x}$$

Solving for x yields $x = 1$ mm. Thus, the approximate expression is 10% in error at $x = 1$ mm and less than this for $x > 1$ mm.

16. $L_m(x) = \dfrac{k}{k_0 + x}$

$$\frac{\partial L_m(x)}{\partial x} = \frac{-k}{(k_0 + x)^2}$$

$$v = ri + (L_l + \frac{k}{k_0 + x})\frac{di}{dt} - i\frac{k}{(k_0 + x)^2}\frac{dx}{dt}$$

$$v = r\sqrt{2}\, I_s \cos\omega_e t - (L_l + \frac{k}{k_0 + t})\,\omega_e \sqrt{2}\, I_s \sin\omega_e t - \sqrt{2}\, I_s \cos\omega_e t \left[\frac{k}{(k_0 + t)^2}\right]$$

Gathering terms

$$v = \left[r - \frac{k}{(k_0 + t)^2}\right]\sqrt{2}\, I_s \cos\omega_e t - (L_l + \frac{k}{k_0 + t})\,\omega_e \sqrt{2}\, I_s \sin\omega_e t$$

Taking the limit as $t \to \infty$

$$v = r\sqrt{2}\, I_s \cos\omega_e t - L_l\, \omega_e \sqrt{2}\, I_s \sin\omega_e t$$

which is the voltage equation for a linear r-L circuit. In phasor form,

$$\tilde{V} = (r + j\,\omega_e L_l)\tilde{I}$$