ELEC 343, Assignment 6: Solution:

Do Study Problem: SP6.2-2 and SP6.2-3; SP6.8-4. and do the Example 6C (this will help you for the Lab-5!) and Example 6D.
Textbook Chapter Problem(s): 15 and 21.

SP6.2-2. (a) The frequency of the rotor currents is

$$\frac{1}{2\pi}(\omega_e - \omega_r) = \frac{1}{2\pi}\Big[(2\pi)(60) - (0.9)(2\pi)(60)\Big] = 6 \text{ Hz}$$

(b) From the rotor both appear to be traveling at $\omega_e - \omega_r$. Thus,
$\omega_e - \omega_r = \omega_e - 0.9\,\omega_e = (0.1)(2\pi)(60) = 37.7$ rad/s, ccw.

(c) From the stator both appear to be traveling at ω_e, ccw; or 377 rad/s, ccw.

SP6.2-3. (a) $\frac{1}{2\pi}(\omega_e - \omega_r) = (1 - 1.1)\,60 = 6$ Hz.
We need not recognize a negative frequency here. Answer same regardless of the number of poles.

(b) For two poles, $\omega_e - \omega_r = -0.1\,\omega_e = 37.7$ rad/s, cw. For six poles, $(\frac{2}{P})\,37.7 = \frac{37.7}{3}$ rad/s cw.

(c) $(\frac{2}{P})\,\omega_e = \frac{377}{3}$ rad/s, ccw.

SP6.8-4. (a) $Z = r_s + j\,\omega_e\,(L_{ls} + L_{ms}) = 20 + j\,(377)(0.025 + 0.3) = 20 + j\,122.5$

$$\tilde{I}_{as} = \frac{\tilde{V}_{as}}{Z} = \frac{115\,\angle 0^\circ}{20 + j\,122.5} = \frac{115\,\angle 0^\circ}{124.1\,\angle 80.75} = 0.927\,\angle{-80.75^\circ}\text{ A}$$

(b) $Z = (r_s + r_r') + j\,\omega_e\,(L_{ls} + L_{lr}') = (20+20) + j\,(377)(0.025+0.025) = 40 + j\,18.85$

$$\tilde{I}_{as} = \frac{\tilde{V}_{as}}{Z} = \frac{115\,\angle 0^\circ}{40 + j\,18.85} = \frac{115\,\angle 0^\circ}{44.3\,\angle 25.2} = 2.6\,\angle{-25.2^\circ}\text{ A}$$

15. From Example 6B with $\omega_e = 377$ rad/s,

$$r_s = 0.531\ \Omega \qquad r_r' = 0.374\ \Omega$$

$$X_{ss} = X_{ls} + X_{ms} = 2.29 + 26.2 = 28.49\ \Omega \qquad X_{rr}' = X_{lr}' + X_{ms} = 2.29 + 26.2 = 28.49\ \Omega$$

Now, from (6.8-28) and (6.8-29),

$$s_m = r_r'\,G$$

$$G = \pm\left[\frac{r_s^2 + X_{ss}^2}{(X_{ms}^2 - X_{ss}X_{rr}')^2 + r_s^2 X_{rr}'^2}\right]^{\frac{1}{2}}$$

(a) $$s_m = 0.374\left\{\frac{(0.531)^2 + (4)(28.49)^2}{(4)\left[(26.2)^2 - (28.49)^2\right]^2 + (0.531)^2\,(4)(28.49)^2}\right\}^{\frac{1}{2}}$$

$$= 0.0845$$

$$\omega_r = (1 - 0.0845)(377)(2) = 690.3 \text{ rad/s}$$

$$\omega_{rm} = \frac{2}{P}\,\omega_r = \left(\frac{2}{4}\right)(690.3) = 345.2 \text{ rad/s}$$

(b) $$s_m = 0.374\left\{\frac{(0.531)^2 + (28.49)^2}{\left[(26.2)^2 - (28.49)^2\right]^2 + (0.531)^2\,(28.49)^2}\right\}^{\frac{1}{2}}$$

$$= 0.0845$$

$$\omega_{rm} = \left(\frac{2}{4}\right)(1 - 0.0845)(377) = 172.6 \text{ rad/s}$$

(c) $$s_m = 0.374\left\{\frac{(0.531)^2 + \left(\frac{1}{4}\right)(28.49)^2}{\left(\frac{1}{4}\right)\left[(26.2)^2 - (28.49)^2\right]^2 + (0.531)^2\,\left(\frac{1}{4}\right)(28.49)^2}\right\}^{\frac{1}{2}}$$

$$= 0.0845$$

$$\omega_{rm} = \left(\frac{2}{4}\right)(1 - 0.0845)\left(\frac{1}{2}\right)(377) = 86.3 \text{ rad/s}$$

(d) $$s_m = 0.374\left\{\frac{(0.531)^2 + \left(\frac{1}{100}\right)(28.49)^2}{\left(\frac{1}{100}\right)\left[(26.2)^2 - (28.49)^2\right]^2 + (0.531)^2\left(\frac{1}{100}\right)(28.49)^2}\right\}^{\frac{1}{2}}$$

$$= 0.0859$$

$$\omega_{rm} = \left(\frac{2}{4}\right)(1 - 0.0859)\left(\frac{1}{10}\right)(377) = 17.2 \text{ rad/s}$$

21. (a) $\tilde{I}_{as} = \dfrac{\tilde{V}_{as}}{Z}$

Neglecting the current flowing in X_M

$$\begin{aligned} Z &= (r_s + r_r') + j\,(X_{ls} + X_{lr}') \\ &= (0.3 + 0.15) + j\,(377)(1.5 + 0.7) \times 10^{-3} \\ &= 0.45 + j\,0.829 = 0.944 \,\underline{/61.5^\circ}\ \Omega \end{aligned}$$

$$\tilde{I}_{as} = \frac{110\,\underline{/0^\circ}}{0.944\,\underline{/61.5^\circ}} = 116.6\,\underline{/-61.5^\circ}\ \text{A}$$

From (6.8-26),

$$T_e = \frac{(\frac{3}{2})(2)(\frac{P}{2})\frac{X_M^2}{\omega_e} r_r' s\, |\tilde{V}_{as}|^2}{\left[r_s r_r' + s\,(X_M^2 - X_{ss} X_{rr}')\right]^2 + (r_r' X_{ss} + s\, r_s X_{rr}')^2}$$

For a 60 Hz supply,

$$\begin{aligned} X_M &= \omega_e\,(\frac{3}{2})L_{ms} \\ &= (377)(\frac{3}{2})(35 \times 10^{-3}) = 19.79\ \Omega \end{aligned}$$

$$\begin{aligned} X_{ss} &= \omega_e\,(L_{ls} + \frac{3}{2}L_{ms}) = (377)[1.5 + (\frac{3}{2})(35)] \times 10^{-3} \\ &= 20.36\ \Omega \end{aligned}$$

$$\begin{aligned} X_{rr}' &= \omega_e\,(L_{lr}' + \frac{3}{2}L_{ms}) = (377)[0.7 + (\frac{3}{2})(35)] \times 10^{-3} \\ &= 20.06\ \Omega \end{aligned}$$

With $s = 1$,

$$\begin{aligned} T_e &= \frac{(3)(\frac{4}{2})\frac{(19.79)^2}{377}(0.15)(110)^2}{[(0.3)(0.15) + (19.79)^2 - (20.36)(20.06)]^2 + [(0.15)(20.36) + (0.3)(20.06)]^2} \\ &= 31.9\ \text{N·m} \end{aligned}$$

(b) $\tilde{I}_{as} = \dfrac{\tilde{V}_{as}}{Z}$

$$\begin{aligned} Z &= r_s + j\,X_{ss} = 0.3 + j\,20.36 \\ &= 20.36\,\underline{/89.2^\circ} \end{aligned}$$

$$\tilde{I}_{as} = \frac{110\,\underline{/0^\circ}}{20.36\,\underline{/89.2^\circ}} = 5.40\,\underline{/-89.2^\circ}\ \text{A}$$

Additional Problem:

Consider a 25hp, 3-phase, Y-connected, 220V line-to-line, 60Hz Induction Motor NEMA* Type B. The motor operates at full nominal load and the following is known: input electrical power $P_{e,in} = 20.8\text{kW}$; output mechanical power $P_{m,out} = 25\text{HP}$; stator phase current $I_{as} = 64\text{A(rms value)}$; motor shaft speed $n = 830rpm$. Establish and/or calculate the following:

a) Number of magnetic poles f_r

b) Actual electrical frequency of the rotor currents f_r (assuming stator frequency $f_s = f_e$ is 60Hz)

c) Input power factor pf as seen by the source

d) Mechanical (load) torque T_m

e) Motor efficiency $\eta[\%]$ under the given load

Number os poles P

$$n_{syn} = \frac{120}{p} \cdot f_e \; ; \quad p = \frac{120 f_e}{n_{syn}} = \left\lfloor \frac{120 f_e}{n} \right\rfloor$$

$$p = \left\lfloor \frac{120 \cdot 60}{830} \right\rfloor = \lfloor 8.67 \rfloor = 8 \; ; \; n_{syn} = 900 rpm$$

Seip $S = \frac{n_{syn} - n}{n_{syn}} = \frac{900 - 830}{900} = 0.0778 = 7.78\%$

Frequency os rotor currents f_r

$$f_r = S \cdot f_e = 0.0778 \cdot 60 = 4.67 Hz$$

Power socsor $pf = \cos\varphi = \frac{P_{in}}{\sqrt{3} \cdot V_L \cdot I_L} = \frac{20.8 kW}{\sqrt{3} \cdot 220 \cdot 64} = 0.853$

Torque $T_m = \frac{P_{out}}{\omega_{rm}} = \frac{30}{\pi} \cdot \frac{25 \cdot 746}{830} = 214 N \cdot m$

Essiciency $\eta = \frac{P_{out}}{P_{in}} = \frac{25 \cdot 746}{20.8 kW} = 0.8966 = 89.66\%$