ELEC 344
5th Tutorial

Magnetic Circuit Review,
DC Machine & DC Converter

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Overview

1. If you can understand the one quadrant chopper topology perfectly, then you can easily understand the two quadrant chopper which is just a combination of one quadrant chopper.
2. Four quadrant chopper is just a combination of two quadrant chopper.
3. We will review electrical & mechanical characteristics of above choppers and review the DC machine at the same time.
4. Next tutorial, Oct. 28th, will be Q&A session covering all contents that we learned so that you can prepare the Midterm. Questions sent to me by Wednesday, Oct. 26th will be answered.
5. All questions are welcomed!!

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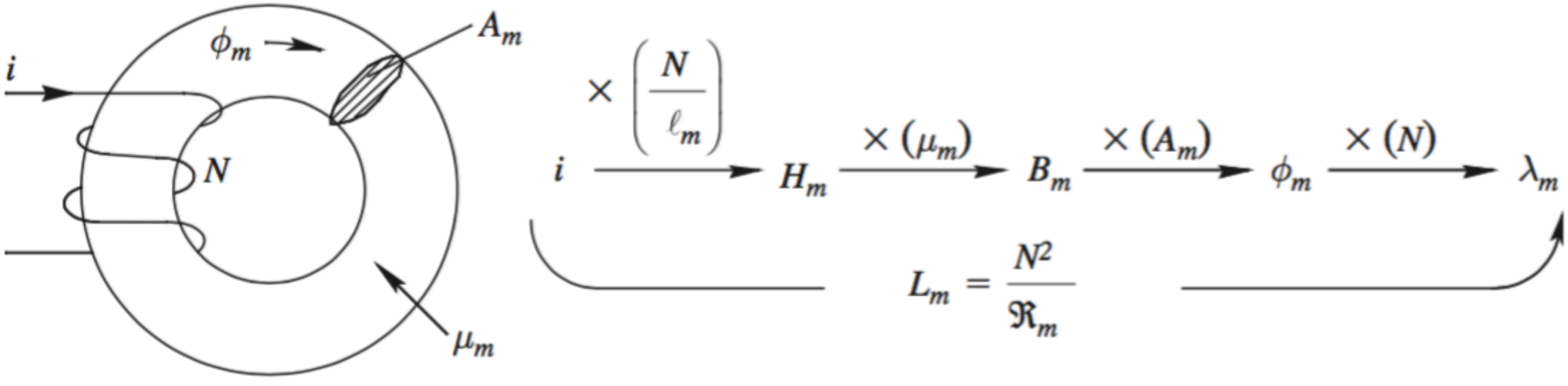
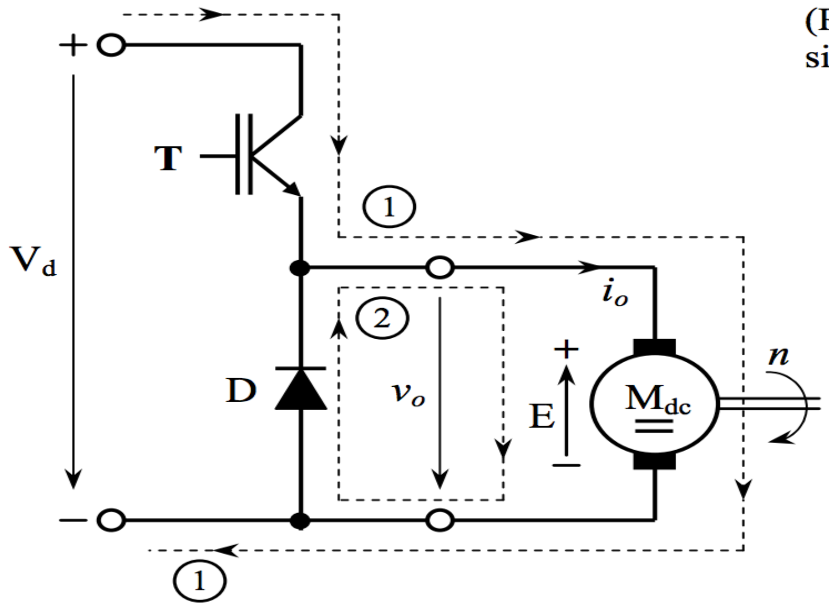
Review of Magnetic circuit (Module 1)



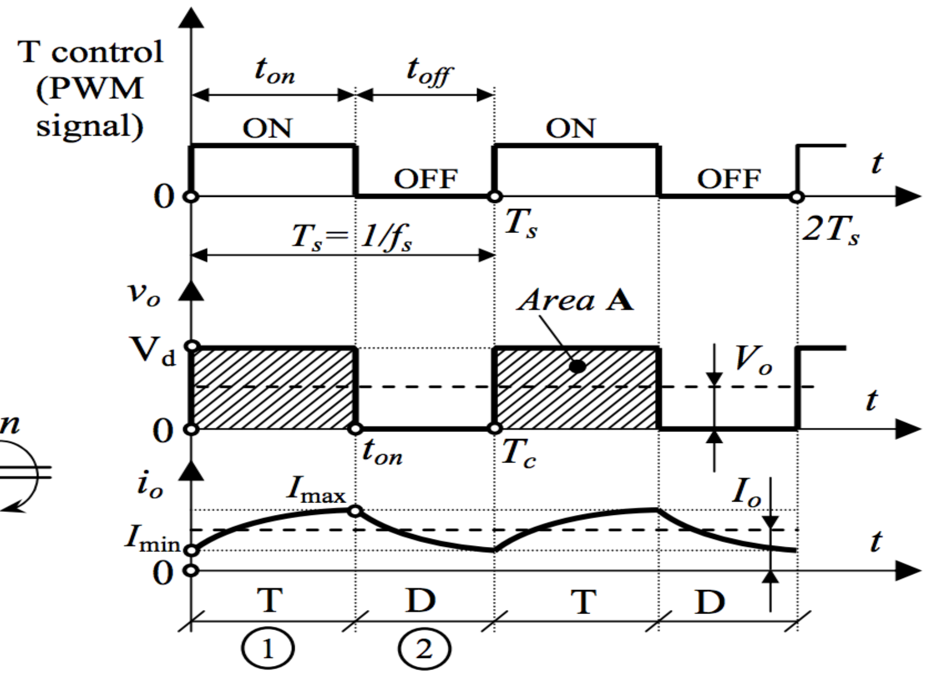
(a) (b)

$$L_m = \frac{\lambda_m}{i} = \frac{\underbrace{\underbrace{\underbrace{\underbrace{\left(\frac{Ni}{\ell_m}\right)}_{H_m}\mu_m}_{B_m} A_m}_{\phi_m} N}_{\lambda_m}}{i} = \frac{N^2}{\left(\frac{\ell_m}{\mu_m A_m}\right)} = \frac{N^2}{\Re_m}$$

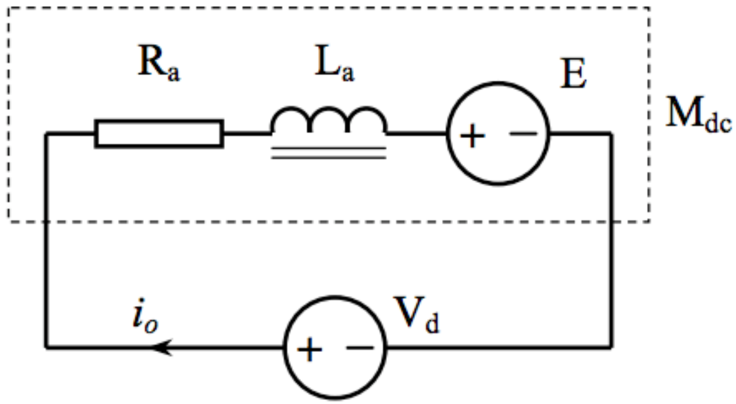
One Quadrant DC-DC Converter (Module 2&3)

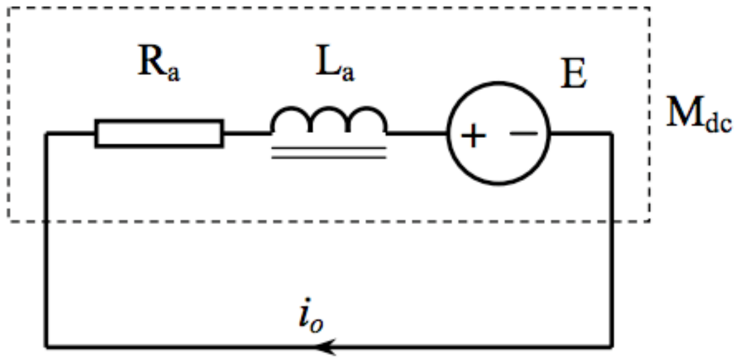


(a)



(b)





One Quadrant DC-DC Converter

1. Electrical Analysis (Voltage & Current)

- Voltage & Current equation

$$V_d - E = R_a \cdot i_o + L_a \cdot \frac{di_o}{dt}, \qquad i_o(t) = I_{\min} \cdot e^{-\frac{t}{\tau}} + \frac{V_d - E}{R_a} \cdot \left(1 - e^{-\frac{t}{\tau}}\right), \qquad 0 \le t \le t_{on}$$

$$-E = R_a \cdot i_o + L_a \cdot \frac{di_o}{dt}, \qquad i_o(t) = I_{\max} \cdot e^{-\frac{t}{\tau}} - \frac{E}{R_a} \cdot \left(1 - e^{-\frac{t}{\tau}}\right), \qquad t_{on} < t < T_c$$

where: $\tau = L_a / R_a$ is the time constant of the loop

- Average value of Voltage at the terminal of DC machine

$$V_o = \text{average value of } v_o(t) = \frac{1}{T_s}\int_0^{T_s} v_o(t)\cdot dt = \frac{1}{T_s}\int_0^{t_{on}} V_d \cdot dt + \frac{1}{T_s}\int_{t_{on}}^{T_s} 0 \cdot dt$$

$$= \frac{Area\ \mathbf{A}}{T_s} = \frac{1}{T_s}\cdot V_d \cdot [t]_0^{t_{on}} = V_d \cdot \frac{t_{on}}{T_s} = V_d \cdot d$$

where: $\frac{t_{on}}{T_s} \overset{not}{=} d$ is the ***duty ratio*** of the T switch. Because $0 \le t_{on} \le T_s \Rightarrow 0 \le d \le 1$.

$$0 \le d \le 1 \Rightarrow 0 \le V_o \le V_d$$

In the first quadrant of the electrical plane the output power is always positive ($P_o = U_o \cdot I_o > 0$) which means that the flow direction of the power through the converter is from the input to the output, from the V_d source to the DC motor.

One Quadrant DC-DC Converter

2. Mechanical Analysis (Speed & Torque)

- Speed & Torque equation

$$V_o = E + R_a \cdot I_o = k_e \cdot \Phi_{ex} \cdot n + R_a \cdot I_o \quad \Rightarrow$$

$$\Rightarrow \quad n = \frac{V_o}{k_e \cdot \Phi_{ex}} - \frac{R_a \cdot I_o}{k_e \cdot \Phi_{ex}} \quad \text{(in steady state)}$$

Knowing that the current value absorbed by the motor is proportional to the mechanical load (load torque - T_{load}) in steady state: $T_{load} = T_{em} = k_m \cdot \Phi_{ex} \cdot I_o$ we obtain:

$$n = \frac{V_o}{k_e \cdot \Phi_{ex}} - \frac{R_a \cdot T_{load}}{k_e \cdot k_m \cdot \Phi_{ex}^2}$$

- Average value of Voltage at the terminal of DC machine

One Quadrant DC-DC Converter

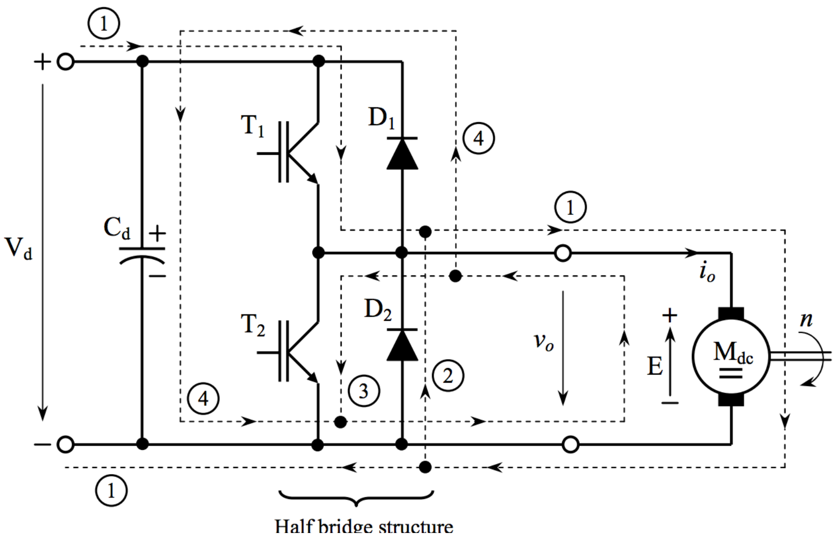
3. Operation Mode (Motor)

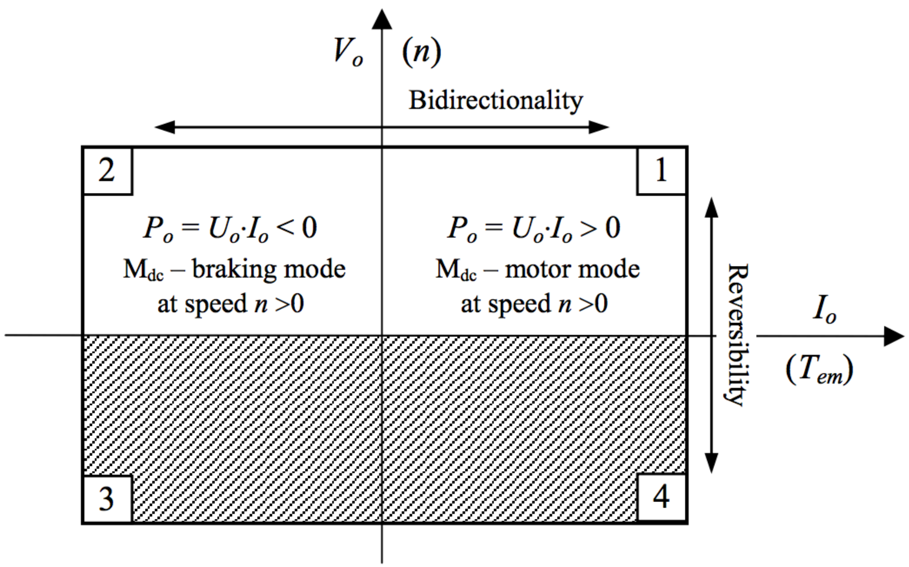
It is known that the rotational speed n of a DC motor is proportional to the supply DC voltage and the electromagnetic torque T_{em} is proportional to the motor current if the excitation magnetic flux Φ_{ex} is constant. Thus, if we associate to the electrical variables V_o, I_o, the mechanical variables n, respectively T_{em}, we can say that the DC machine operates only in the first quadrant of the mechanical plane n - T_{em}. From the load's point of view this means that the DC machine M_{dc} takes the power transferred through the chopper and converts this electrical energy into mechanical energy. Therefore, the machine operates in ***motor mode*** at a positive speed (n >0). There is not the possibility for the DC machine of operating in the ***braking mode***, in the second quadrant of the mechanical plane, since the DC/DC converter cannot operate in the second quadrant of the electrical plane.

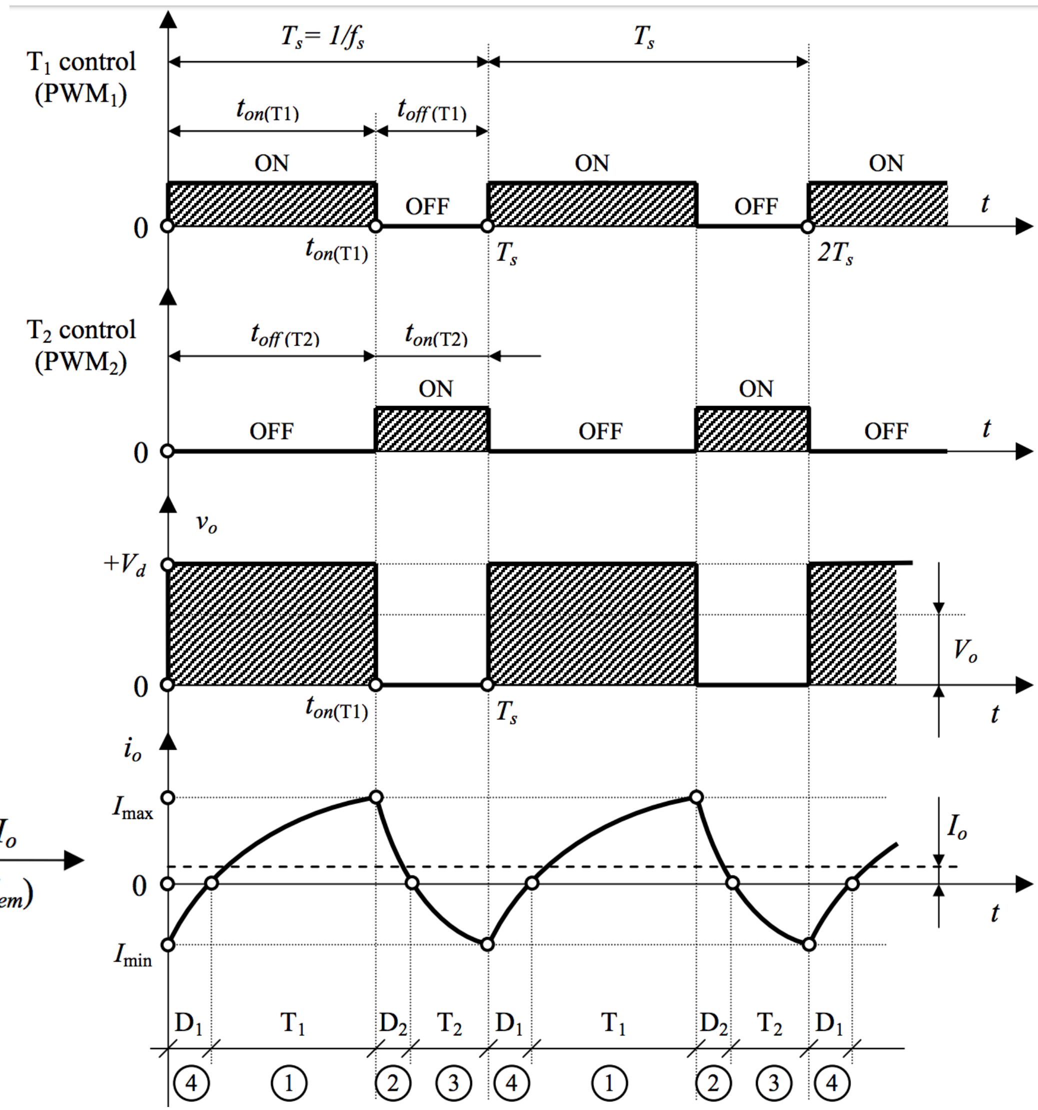
If we increase the average voltage from the chopper output, the DC motor accelerates entering in an ***electromechanical transitory state*** towards a new ***electromechanical steady state,*** at a higher rotational speed. The new steady-state speed value is fixed by the new value of the V_o voltage and by the mechanical load on the motor shaft:

Two Quadrant DC-DC Converter (*Half-bridge*)

<u>Note that the control signal for the two power transistors (switches) are of a PWM type and ***COMPLEMENTARY***!!!</u>







Two Quadrant DC-DC Converter

1. Electrical Analysis (Voltage & Current)

- Duty Ratio $(t_{on(T1)} + t_{on(T2)}) = T_c \Leftrightarrow \dfrac{t_{on(T1)}}{T_c} + \dfrac{t_{on(T2)}}{T_c} = 1 \Rightarrow$

$$d_{(T1)} + d_{(T2)} = 1$$

- Voltage & Current Equation

a. Cycle 1 & 4: Same as one quadrant chopper during $\boldsymbol{t_{on}}$ interval

b. Cycle 2 & 3: Same as one quadrant chopper during $\boldsymbol{t_{off}}$ interval

- Average value of Voltage at the terminal of DC machine

$$V_o = V_d \cdot d \geq 0$$

- Average value of Current at the terminal of DC machine is approximately at halfway between I_{max} and I_{min}.

$$I_0 \cong \frac{I_{max} + I_{min}}{2}$$

Depending on how are located the two extremes I_{max} and I_{min} ***the average output current can be positive or negative.***

Two Quadrant DC-DC Converter

2. Mechanical Analysis (Speed & Torque)

- The same as Quadrant DC-DC Converter

3. Operation Mode (Motor & Braking)

- Motor mode ($I_0 > 0$)

Producing the positive electromagnetic torque in the same direction as the rotation speed.

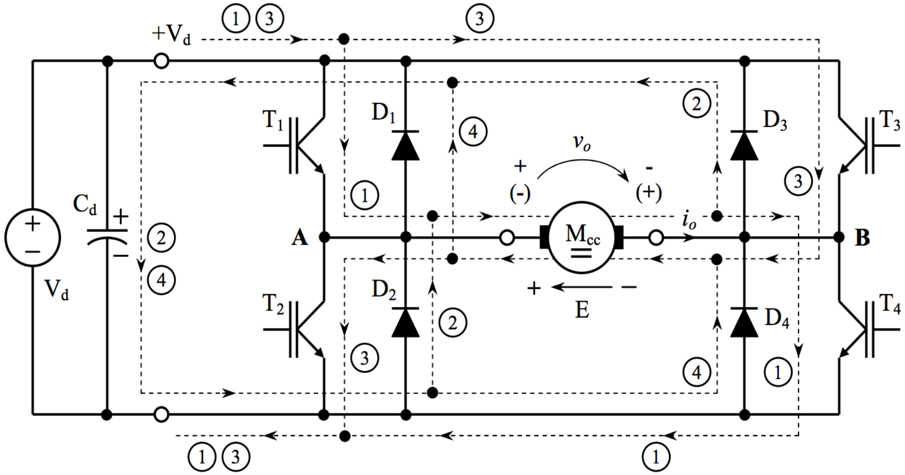
- Braking mode ($I_0 < 0$)

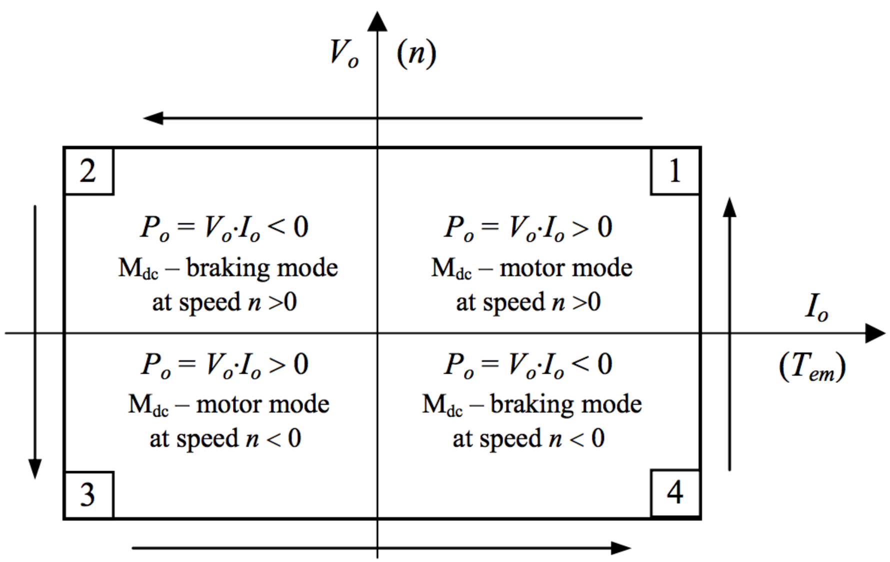
The duty ratio of two transistors are suddenly modified so that the DC output voltage is less than back EMF. Then, the average current of the machine becomes negative as below;

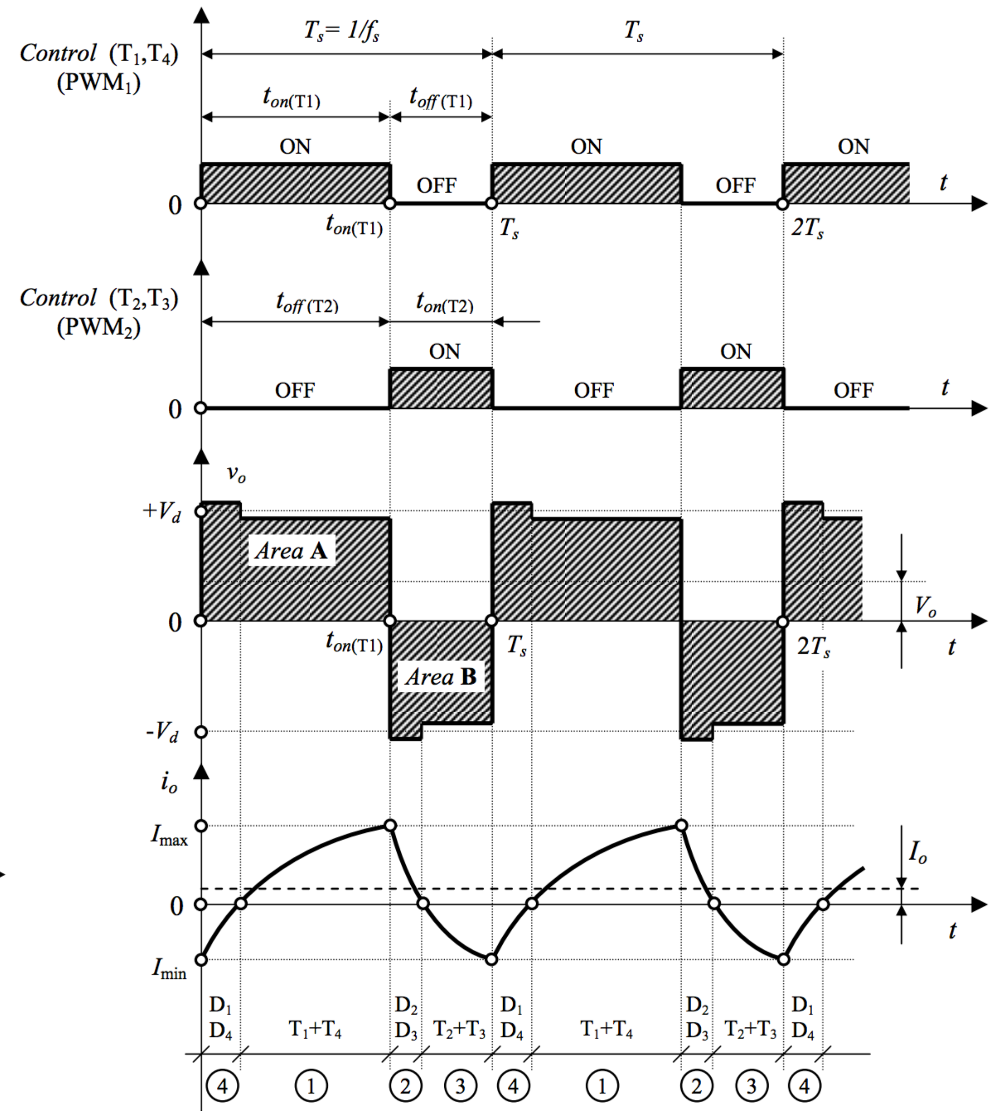
$$\left.\begin{aligned} V_o &= E + R_a \cdot I_o \\ V_o &< E = k_e \cdot \Phi_{ex} \cdot n \end{aligned}\right\} \Rightarrow I_o = \frac{U_o - E}{R_a} < 0$$

The negative current causes a negative electromagnetic torque in opposition to the rotational movement, with the significance of a braking torque which will decrease the motor speed. Therefore, the DC machine operates in ***braking mode*** as a ***DC generator***. This technique is essential in the controlled electrical drive systems since we obtain short response times for speed adjustment.

Four Quadrant DC-DC Converter (Full-bridge)







Four Quadrant DC-DC Converter

1. Electrical Analysis (Voltage & Current)

- Duty Ratio

$$(t_{on(T1)} + t_{on(T2)}) = T_c \Leftrightarrow \frac{t_{on(T1)}}{T_c} + \frac{t_{on(T2)}}{T_c} = 1 \Rightarrow$$

$$d_{(T1)} + d_{(T2)} = 1$$

where: $d_{(T_1)} = t_{on(T_1)} / T_s$ is ***duty ratio*** *of the transistors pair* (T_1, T_4).

- Voltage & Current Equation

a. Cycle 1 & 4: Same as one quadrant chopper during t_{on} interval

b. Cycle 2 & 3: $v_0(t) = -V_d$

- Average value of Voltage at the terminal of DC machine

$$V_o \overset{not}{=} \text{average value of } v_o(t) = \frac{1}{T_s}\int_0^{T_c} v_o(t)\cdot dt \approx \frac{1}{T_s}\int_0^{t_{on(T1)}} (+V_d)\cdot dt + \frac{1}{T_s}\int_{t_{on}(T1)}^{T_c} (-V_d)\cdot dt =$$

$$= \frac{Area\ \mathbf{A} + Area\ \mathbf{B}}{T_s} = \frac{1}{T_s}\cdot\left[V_d\cdot[t]_0^{t_{on(T1)}} - V_d\cdot[t]_{t_{on(T1)}}^{T_s}\right] = V_d\left(2\cdot d_{(T_1)} - 1\right)$$

- Average value of Current at the terminal of DC machine is approximately at halfway between I_{max} and I_{min}.

Four Quadrant DC-DC Converter

2. Mechanical Analysis (Speed & Torque)

- The same as two Quadrant DC-DC Converter

3. Operation Mode (Motor & Braking)

- Reversible property: $0 \leq d_{(T_1)} \leq 1 \Rightarrow -V_d \leq V_e \leq +V_d$

DC motor can be turned in both directions with variable speed adjustment.

- Bidirectionality:

DC motor can be accelerated, rotated and braked in both rotating directions.