ELEC 391
Electrical Engineering Design Studio

Introduction to Spectrum Analyzers

Introduction to project management. Problem definition. Design principles and practices. Implementation techniques including circuit design, software design, solid modeling, PCBs, assembling, and packaging. Testing and evaluation. Effective presentations. Pre-requisite: Two of EECE 352, EECE 356, EECE 359, EECE 360, EECE 364, EECE 373. [2-6-0]

- During this lecture, the instructor will bring up many points and details not given on these slides. Accordingly, it is expected that the student will annotate these notes during the lecture.
- The lecture only introduces the subject matter. Students must review these notes after class and complete the reading assignments and problems if they are to master the material.

Learning Objectives

- Upon completion of this lecture and review of supplementary materials, students will be able to:
  - Understand and perform calculations using decibel measures such as dBm, dBW and dBV.
  - Describe various approaches to representing periodic signals as Fourier series.
  - Compare the strengths and limitations of alternative methods for resolving the frequency content of signals.
  - Describe the function and operation of swept-frequency spectrum analyzers.
  - Explain how the pole-zero approach simplifies filter design.
  - Describe the operation of FM modulators and demodulators.

Outline

1. Decibels
2. Principles of Fourier Analysis
3. Spectrum Analysis
4. Filter Analysis and Design
5. Frequency Modulation

1. Decibels

- Decibels are a logarithmic measure of the ratio of two powers

$$P(\text{dB}) = 10\log\frac{P_1}{P_2} = 20\log\frac{V_1}{V_2}$$

← measured across the same impedance

- You should know common ratios and their decibel equivalent from memory

-30 dB	0.001	30 dB	1000
-20 dB	0.01	20 dB	100
-10 dB	0.1	10 dB	10
-6 dB	0.25	6 dB	4
-3 dB	0.5	3 dB	2
		0 dB	1

Decibels - 2

- It's often convenient to express the relationship between a given signal and a known reference quantity, *e.g.,*

$$P(\text{expressed in dBm}) = 10\log\left(\frac{P}{1\,\text{mW}}\right)$$

$$P(\text{expressed in dBW}) = 10\log\left(\frac{P}{1\,\text{W}}\right)$$

$$P(\text{expressed in dBV}) = 20\log\left(\frac{V}{1\,\text{V}}\right)$$

Decibels - 3

- Given a sinusoidal signal with peak amplitude V_p across an impedance Z_o,

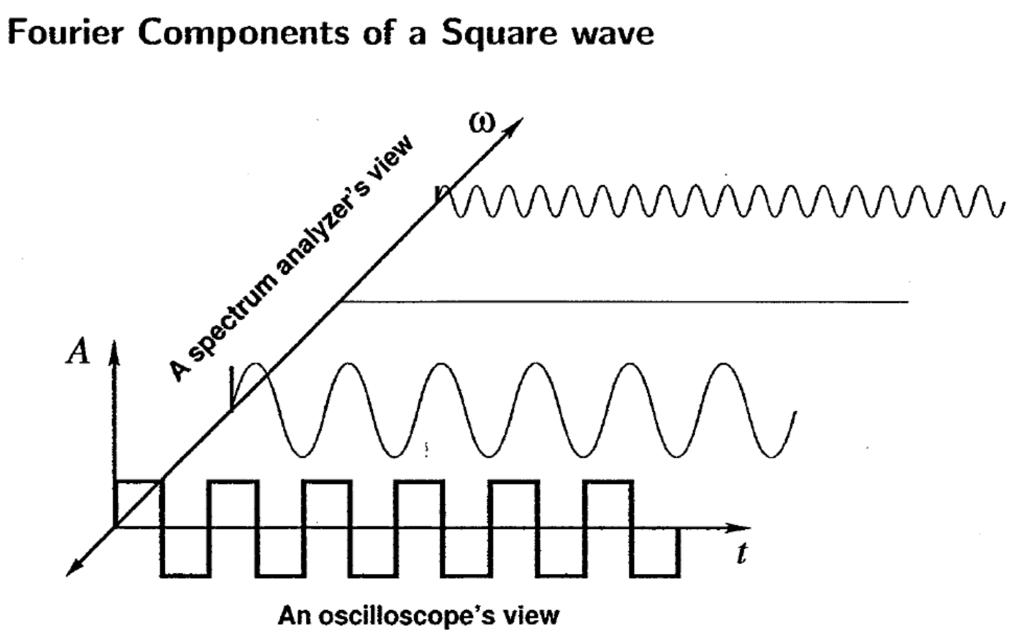
$$P = \frac{V_p^2}{2Z_0}$$

$$P(\text{dBW}) = 10\log\left(\frac{P}{1\,\text{W}}\right)$$

$$P(\text{dBm}) = P(\text{dBW}) + 30$$

$$\text{mW} = W \times 1000$$

2. Principles of Fourier Analysis



The Synthesis and Analysis Equations

- the complex exponential Fourier series representation of a periodic signal $x(t)$ with fundamental period T_0 is given by

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0} \qquad \text{(the synthesis equation)}$$

- the *complex Fourier coefficients* c_k are given by

$$c_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t}\, dt \qquad \text{(the analysis equation)}$$

  where $\int_{T_0}$ denotes integration over a single fundamental period (the precise limits are arbitrary)

- the Dirichlet conditions are sufficient but not necessary conditions for a Fourier series representation of a given periodic signal to exist

Amplitude, Power and Phase Spectra

- Let the complex Fourier coefficients c_k in

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

  be expressed as $c_k = |c_k| e^{j\phi_k}$

- the amplitude spectrum of the signal is a plot of $|c_k|$ versus ω
- the power spectrum of the signal is a plot of $|c_k|^2/Z_L$ versus ω (Proof?)
- the phase spectrum of the signal is a plot of ϕ_k versus ω
- because k assumes only discrete values, these are referred to as *line spectra*

Example

$x(t)$ is a rectangular wave with period 0.1 sec, duty cycle of 0.2, with $V_{\max}$ of 1.0 V and $V_{\min}$ of 0.0 V as measured across a 50 Ω load. Find its power spectrum.

- first, sketch the signal; note that offsets in time are irrelevant because time shifts affect only the *phase* of the frequency spectrum
- note that the Dirichlet conditions are satisfied (as in the case of almost all real signals) so apply the analysis equation
- here,

$$c_k = \frac{1}{0.1} \int_{-0.01}^{0.01} e^{-jk20\pi t}\, dt$$

- the power spectrum is given by $|c_k|^2/50$

Why is there no factor of ½ in this expression?

Harmonic Form of the Fourier Series

- another form of the Fourier series representation of a real periodic signal $x(t)$:

$$x(t) = \frac{C_0}{2} + \sum_{k=1}^{\infty} C_k \cos(k\omega_0 t - \theta_k) \qquad \omega_0 = \frac{2\pi}{T_0}$$

- C_0 is the *dc component*
- $C_k \cos(k\omega_0 t - \theta_k)$ is the *kth harmonic component*
- C_k and θ_k are the *harmonic amplitudes* and *phase angles*, respectively

3. Spectrum Analysis

Why Measure the Spectrum of a Signal?

- to characterize noise and interference
- to measure distortion (both intermodulation and harmonic)
- to characterize amplitude, phase, frequency or pulse modulated signals
- to estimate spectrum usage and occupancy

How to Measure the Frequency Content of a Signal?

- Fourier theory works well when applied to mathematical abstractions.
- What about physical signals?
- Options:
  - analog filter bank
  - analog-to-digital conversion followed by a discrete Fourier transform
  - tunable bandpass filter
  - swept frequency spectrum analyzer
- Each approach has particular strengths and weaknesses!

Analog Filter Bank or *Fourier Analyzer*

- The first widely used technique to estimate frequency spectra, analog filter banks of various types have been used for over a century.
- They can be implemented by mechanical, electromechanical, and electronic means.
- *Strengths*: simplicity, low cost
- *Weaknesses*: poor resolution, inflexible configuration, poor performance at low frequencies.
- *Example*: spectrum displays of the sort often found in audio systems.

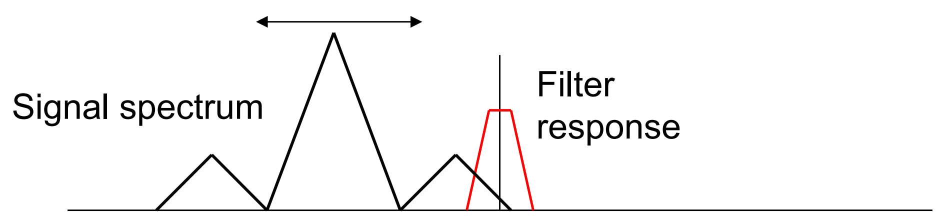
Digital Fourier Analyzer or *Vector Signal Analyzer*

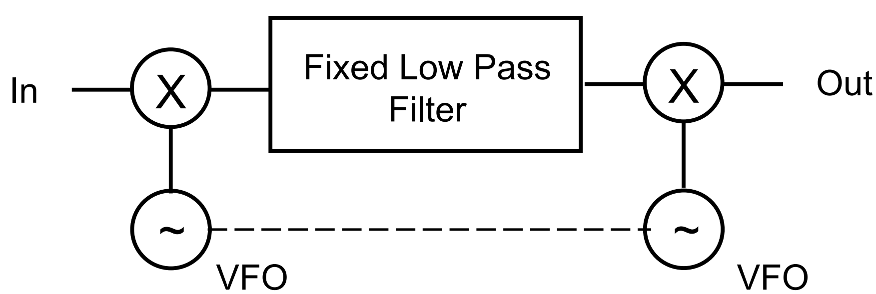
- Digital techniques became practical almost forty years ago with the introduction of the minicomputer and the fast Fourier transform.
- First major application: vibration analysis and biomedical signal processing.
- Initially limited to relatively low frequencies due to hardware limitations, but modern VSA's work up to 10 GHz and beyond.
- *Strengths*: flexibility, stability, recovers both amplitude and phase spectra, recovers all frequencies simultaneously, suitable for measuring transient signals.
- *Weaknesses*: Relatively poor dynamic range (limitation of ADC), and low frequency span (limitations of sampling rate), and high cost (particularly at high frequencies).

Tunable or Frequency Agile Bandpass Filter

- One possibility is to apply the signal of interest to a tunable bandpass filter then sweep the filter through the frequency range of interest while observing the output
- This effectively overcomes the most serious limitation of analog filter banks but implicitly assumes that the spectrum is either time invariant or at least changing much more slowly than the sweep rate.
- The biggest problem: How to build such a filter?
  - Tunable SAW or crystal filters are out of the question. Tunable LC or RC filters are also difficult.
  - One possibility is to use an LO and mixer at the input and another at the output to translate the signal spectrum past a fixed lowpass filter.

Tunable or Frequency Agile Bandpass Filter - 2





Although one appears to be shifting the corner frequency of the filter, one is actually translating the spectrum of the signal!

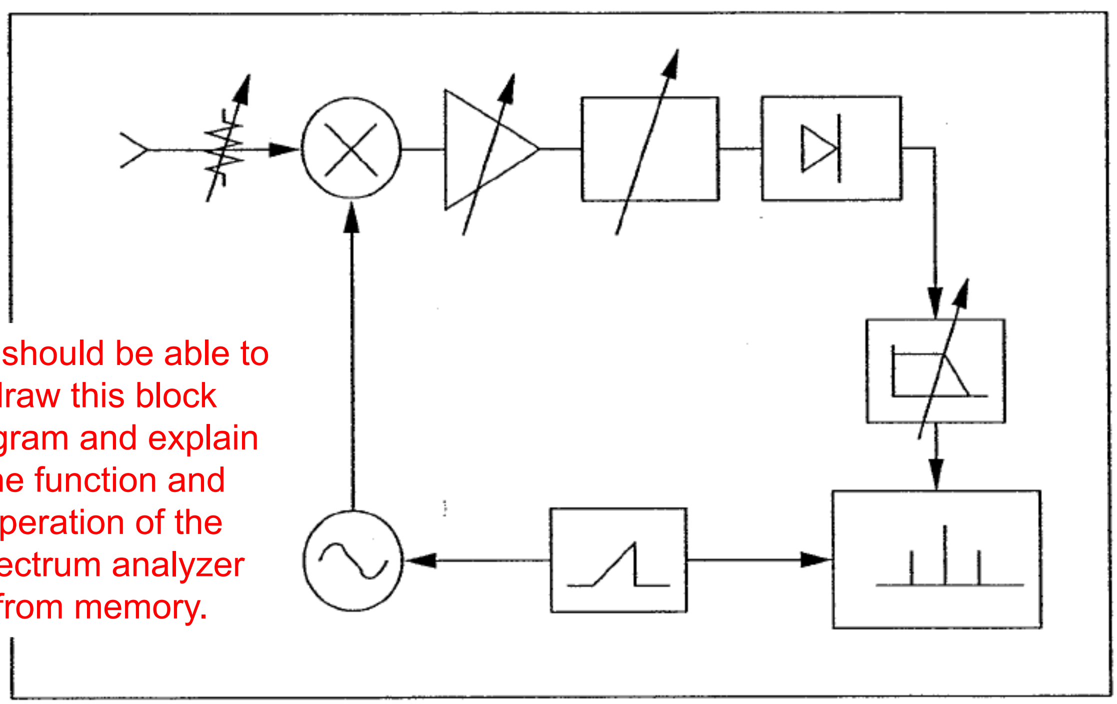
• for the details, see Oppenheim & Willsky, pp. 325-6

Swept-tuned Spectrum Analyzer

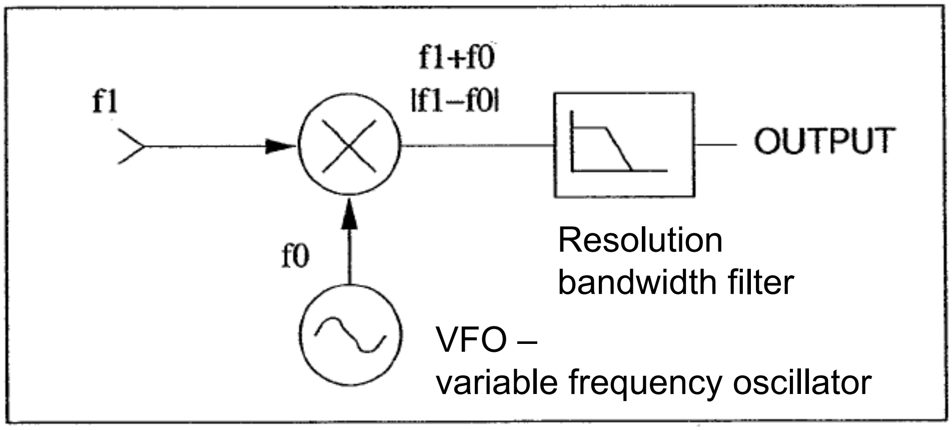
- Swept-tuned spectrum analyzers were introduced during the Second World War as a method for measuring the characteristics of pulsed radar signals.
- They have since become the standard method for measuring the frequency content of RF and microwave signals.
- Used in conjunction with a tracking generator, they can be used to characterize the frequency response of two-port devices.
- *Strengths*: Good dynamic range, broad frequency span, relatively low cost.
- *Weaknesses*: Recovers only the power spectrum, scans the frequency spectrum sequentially, are less suitable for measuring time variant spectra or transient signals.

Inside the Swept-Tuned Spectrum Analyzer

You should be able to draw this block diagram and explain the function and operation of the spectrum analyzer from memory.



The Fundamental Principle of Swept-Tuned Spectrum Analysis



As the f_0 increases, the spectrum of the input signal is translated past the centre frequency of the resolution bandwidth filter. For practical reasons, we normally must employ a bandpass filter rather than the low pass filter shown.

Key Differences between Fourier Theory and Spectrum Analyzer Measurements

	Fourier Theory	Spectrum Analyzer
Spectrum	Two-Sided	One-Sided
Amplitude Spectrum	Yes	Yes
Phase Spectrum	Yes	No
Noise	No	Yes
Spectral Lines	Delta functions	Finite Width

Configuring a Spectrum Analyzer

1. Ensure signal is << 30 dBm; connect to spectrum analyzer input, possibly through a 20 dB attenuator
2. Set center frequency and frequency span *or* start and stop frequencies
3. Set reference level and dB/division
4. Set resolution bandwidth (video bandwidth and sweep time will be set automatically unless manual mode is chosen)
5. Adjust settings as required to obtain best view of the frequency content of the signal
6. Activate markers and use readout to precisely measure signal amplitudes

4. Filter Analysis and Design

- Filters modify the relative amplitudes or phases of the frequency components of a signal.
- Accordingly, they are most easily analyzed using frequency-domain techniques.
- However, the impulse response of a filter can also have important consequences, as demonstrated by the loss of accuracy that results if a spectrum analyzer is swept too quickly.
- It is convenient to divide filters into two classes: frequency-shaping filters and frequency-selective filters.
- Filter analysis (determining the properties of a given filter) and filter synthesis (design of a filter with specified characteristics) are complementary tasks.

Frequency-Shaping Filters

- An important class of filters is used to modify the shape of a signal's spectrum.
- *Example*: Emphasis or de-emphasis of bass (low frequency) and treble (high frequency) signals in audio systems.
- Such equalizers are used to compensate for the acoustic response of the speakers and the enclosed volume in which they are located, or to match the personal tastes of the listener.
- *Example*: Similar devices are used to mitigate effects (fading, intersymbol interference) over wireless channels
- Here, the problem is to estimate the channel characteristics then synthesize a filter that will invert the channel response in order to yield something close to the identity system with some time delay.

- *Example*: Emphasis or de-emphasis of rapid variations in a signal.
- Differentiators emphasize the high frequency components (temporal or spatial) in a signal.
- Integrators emphasize the low frequency components (temporal or spatial)

Frequency Selective Filters

- Another important class of filters is used to select some bands of frequencies and reject others.
- *Example*: Channel selection or interference rejection in communications systems.
- Standard terms:

  - low pass filter
  - high pass filter
  - band pass filter
  - band stop or band reject filter
  - pass band
  - stop band
  - "x" dB bandwidth
  - shape factor

Gain and Phase from the Filter Transfer Function

- A voltage transfer function $H(s) = V_o/V_i$ is usually expressed as a function of the complex frequency variable $s = \sigma + j\omega$.
- Standard circuit analysis rules apply, but the impedance of resistors capacitors, and inductors are given by R, $1/sC$ and sL.
- The result normally takes the form of a ratio of polynomials; this implies that a voltage transfer function can be completely specified by the location of the poles and zeros of the transfer function and a constant gain factor.
- If the coefficients of the transfer function are real, the poles and zeros will either be real or come in conjugate pairs.
- Substituting $j\omega$ for s yields the voltage transfer function in terms of angular frequency; this is easily evaluated using MATLAB or similar.

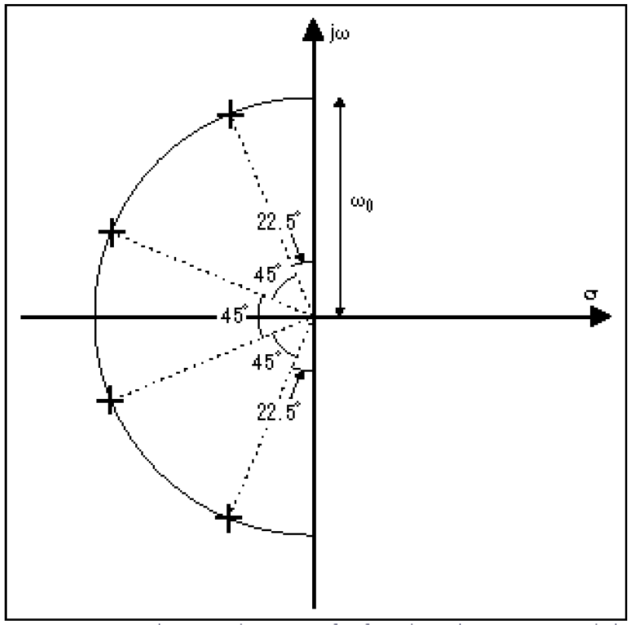
Principle of Filter Design

- The pole-zero approach provides a useful layer of abstraction when designing filters: (1) first decide upon the locations of the poles and zeros required to realize a given frequency response, (2) then determine how to realize that transfer function using circuit elements
- Thus, the fundamental problem of analog filter design is to determine the optimal number and location of the poles and zeros in the voltage transfer function $H(s)$
- Given specific goals, we could optimize the performance of a filter with a given number of poles (and possibly zeros) by either manual or automatic methods.
- In fact, depending upon the chosen criterion (maximal flatness, flat phase response, *etc*.), there are well-known optimal solutions

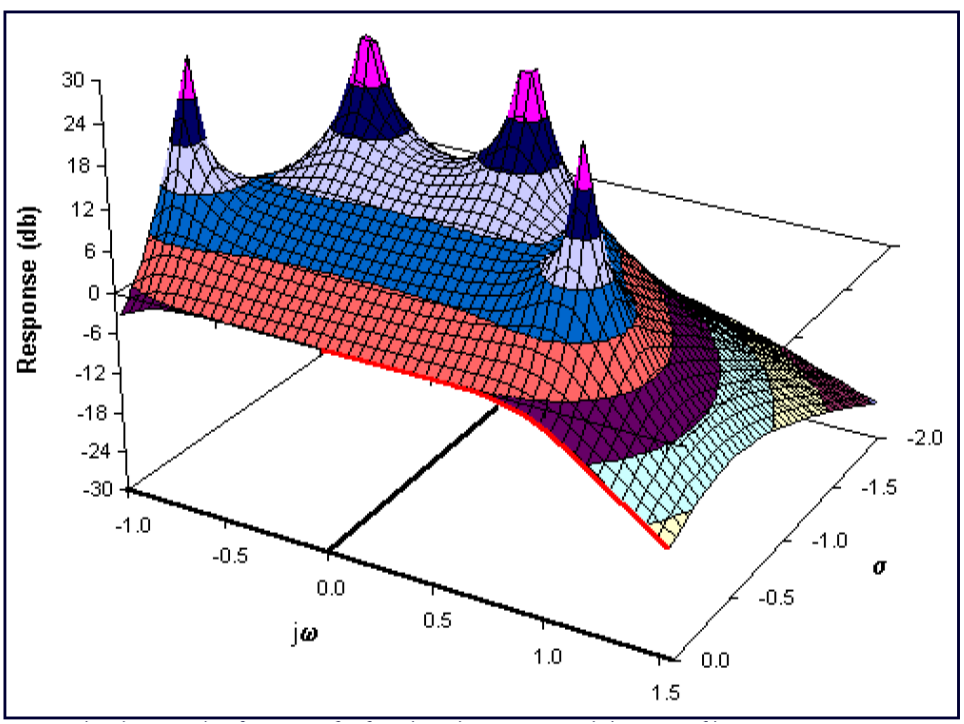
Examples of Optimal Filters

- **Butterworth** filters are all-pole filters that seek to achieve maximal flatness in the passband
  - its poles fall on a half-circle within the left half-plane
- **Chebychev** filters are all-pole filters that seek to steepen the descent from the passband into the stopband by bringing some of the poles closer to the $j\omega$ axis
  - its poles fall on a half-ellipse within the left half plane
- **Bessel** filters are all-pole filters that seek to achieve a flatter phase response at the expense of a shallower descent from the passband into the stopband
  - its poles fall on the locus of something that apparently resembles a parabola within the left half plane

A pole-zero diagram of a fourth-order Butterworth low-pass filter



The transfer function of a fourth-order Butterworth low-pass filter



Response of a Butterworth Filter

- Because the filter transfer function contains only poles, we can write the response as

$$H(s) = \frac{1}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + 1}$$

- Alternatively,

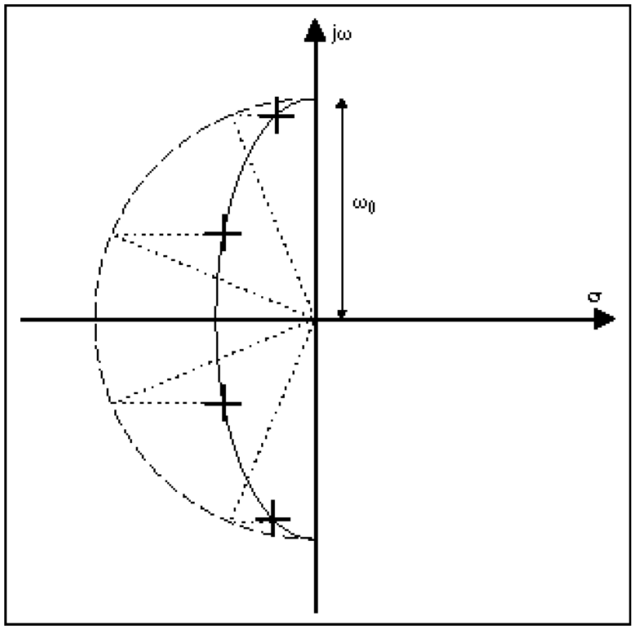
$$H(s) = \frac{1}{\prod_{i=1}^{n}(s - p_i)}$$

- One can find the poles and/or polynomial coefficients from:
  - geometric considerations
  - using MATLAB's **`buttap`** command
  - from a table:

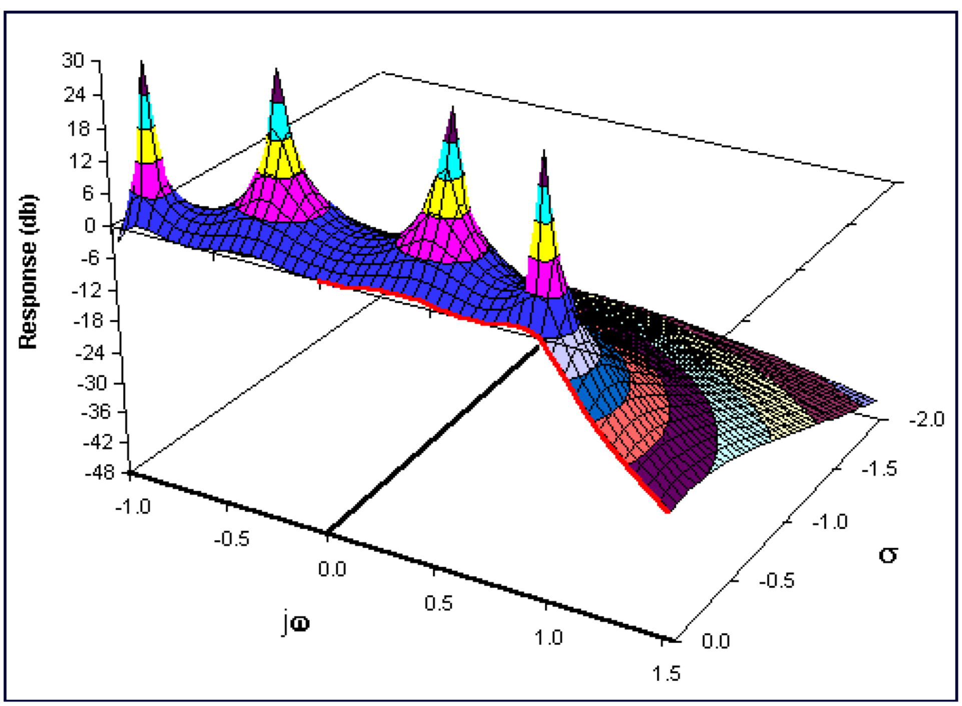
n	a_1	a_2	a_3	4
2	1.414			
3	2.000	2.000		
4	2.6131	3.414	2.613	

$$H(s) = \frac{1}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + 1}$$

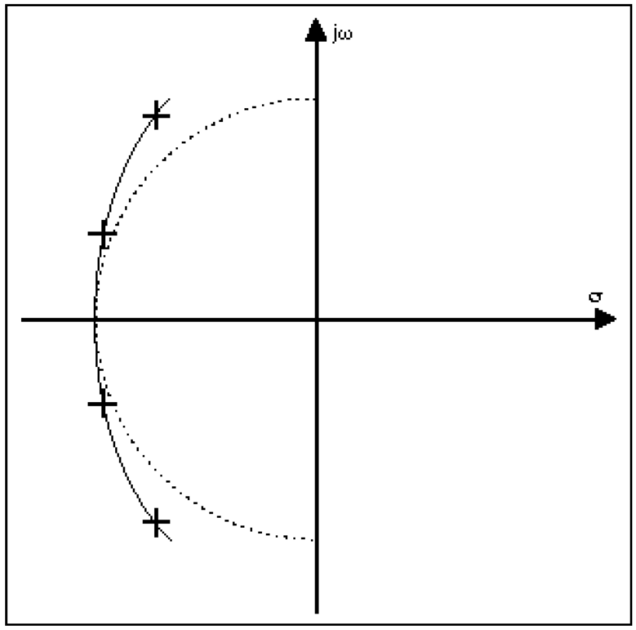
A pole-zero diagram of a fourth-order Chebychev low-pass filter



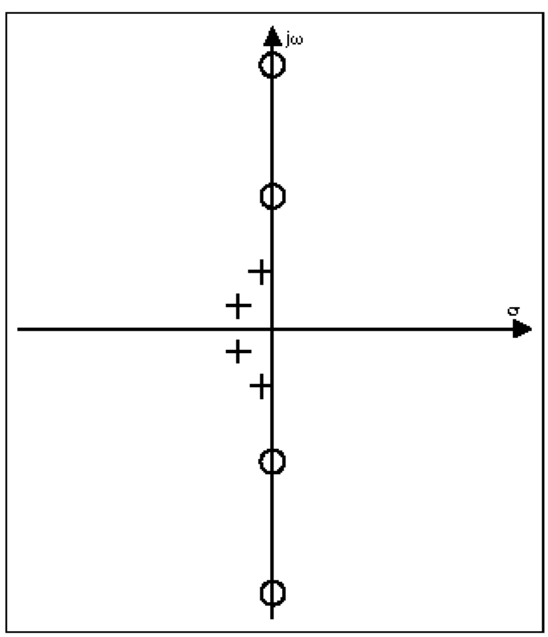
The transfer function of a fourth-order Chebychev low-pass filter



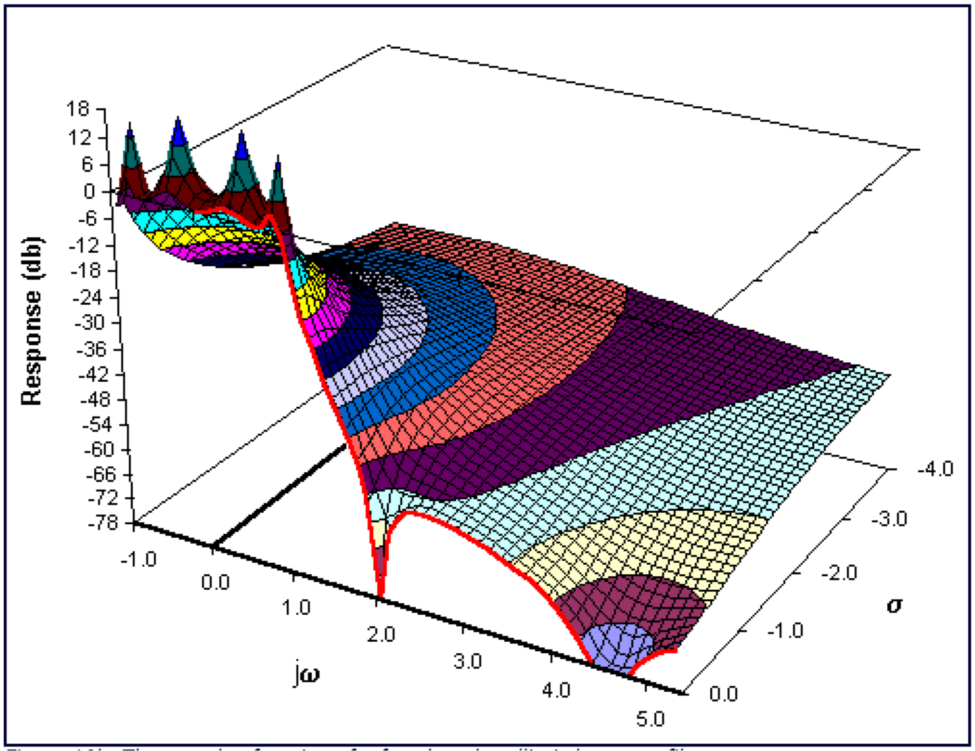
A pole-zero diagram of a fourth-order Bessel low-pass filter



A pole-zero diagram of a fourth-order elliptic low-pass filter



The transfer function of a fourth-order elliptic low-pass filter



5. Frequency Modulation

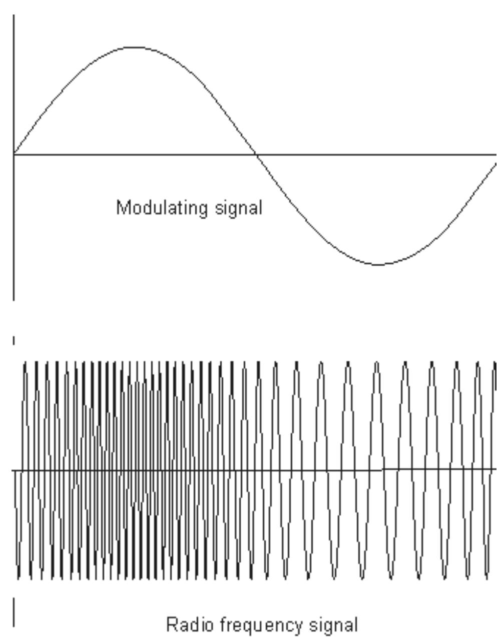
- An FM modulated signal is given by

$$s(t) = A\cos\left[\omega_c t + \phi(t)\right]$$

- The instantaneous frequency is proportional to the amplitude of the modulating signal.
- The transfer function of an FM modulator is

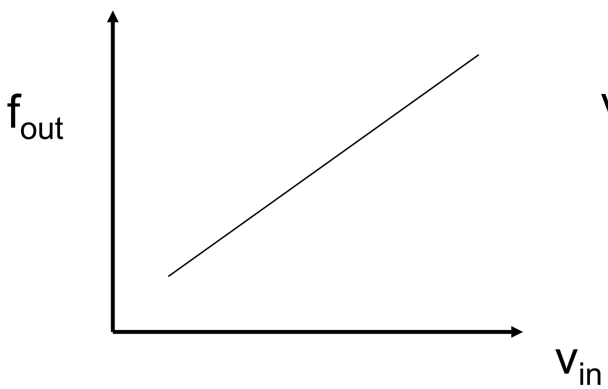
$$\omega_i = \omega_c + k_f m(t) = \omega_c + \frac{d\phi(t)}{dt}$$

where k_f is the frequency deviation constant.

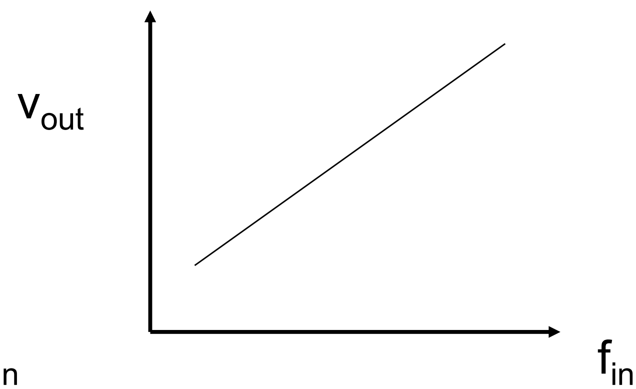


- An FM **modulator** can be realized using a Voltage Controlled Oscillator or VCO
- An FM **demodulator** can be realized using a filter with a slope proportional to frequency (*i.e.,* a discriminator) or a phase locked loop.

FM Modulator Transfer Function



FM Demodulator Transfer Function



Sinusoidal Modulation of an FM Signal

- Fourier analysis of frequency modulated signals is, in general, very complicated.
- For the special case of a sinusoidal modulating signal,

$$m(t) = a_m \cos \omega_m t$$

where $\beta = \dfrac{k_f a_m}{\omega_m}$, we can derive a simple result.

- Here, the parameter β is referred to as the modulation index

$$\beta = \frac{\Delta\omega}{\omega_m}$$

where $\Delta\omega$ is the maximum frequency deviation.

- For the special case of a sinusoidal modulating signal,

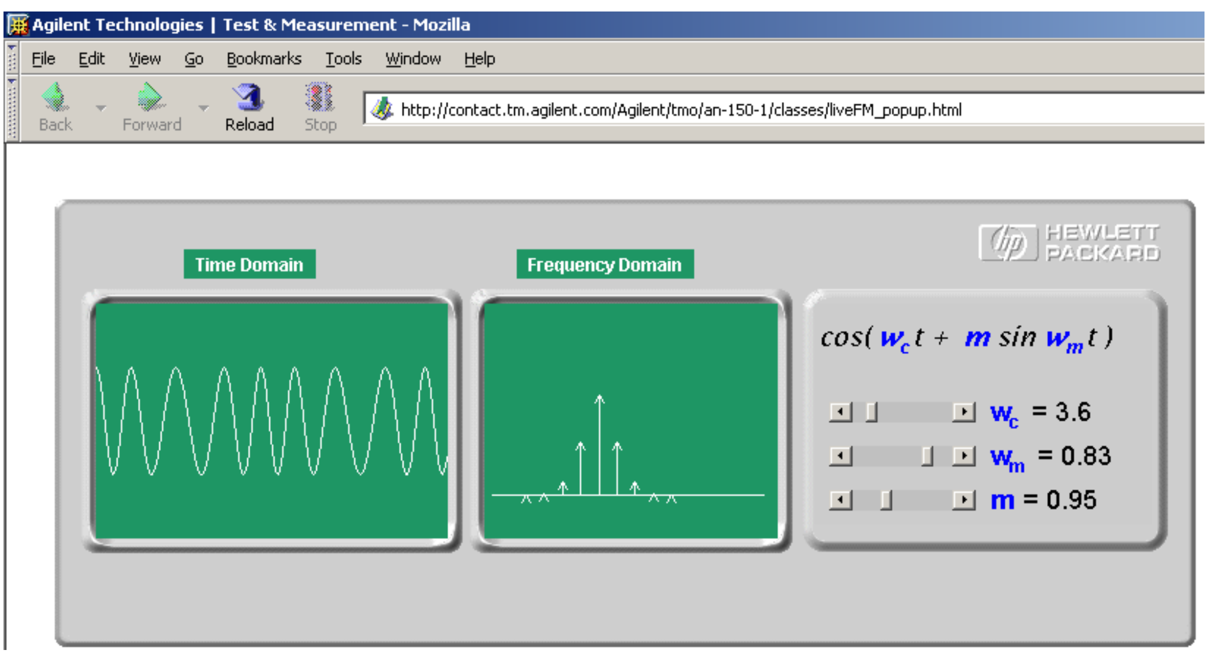
$$s(t) = A\cos(\omega_c t + \beta \sin \omega_m t)$$

- By the use of Fourier series, we can show that

$$s(t) = A\sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

where $J_n(\beta)$ is the Bessel function of the first kind of order n and argument β.

- This allows us to directly evaluate the spectrum of an FM signal for the special case of sinusoidal modulation.



Here m = β = the modulation index.

Observations

- The spectrum consists of a carrier-frequency component plus an infinite number of sideband components at

$$\omega_c \pm n\omega_m \quad (n = 1, 2, 3, \ldots)$$

- The relative amplitudes of the spectral lines depend on the value of $J_n(\beta)$, which becomes very small for large n
- The number of significant spectral lines is a function of the modulation index β
- It can be shown (Carson's rule) that 98% of the total signal power is contained in the bandwidth

$$W_B \approx 2(\beta + 1)\omega_m$$

Supplementary Materials

- “Spectrum Analyzer Basics," AN-150, Keysight Technologies.
  - Available from: http://courses.ece.ubc.ca/elec391/AN150.pdf

- Spectrum Analyzer Multimedia Tutorial for Windows, Agilent Technologies.
  - Available from: http://courses.ece.ubc.ca/elec391/Spectrum.zip
  - Installed on the PCs in MCLD 322

- “dB or not dB? - Everything you ever wanted to know about decibels but were afraid to ask,” AN-1MA98, Rohde & Schwarz.
  - Available from http://courses.ece.ubc.ca/elec391/R&S-dB.pdf