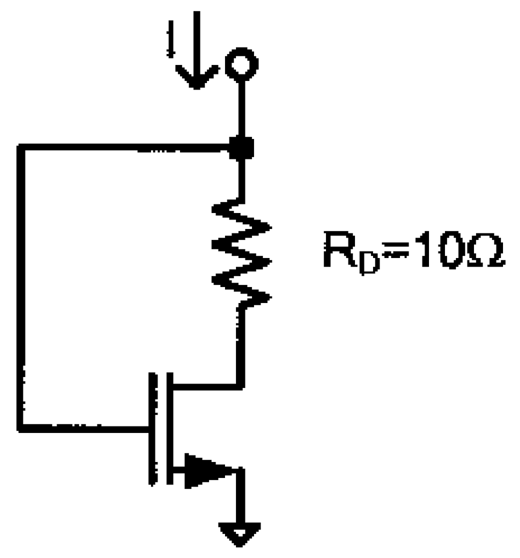
Solutions, Final exam 2010

1. Find the maximum bias current I for which the transistor in the following circuit operates in the saturation region. **[10 marks]**
Assume $\lambda = \gamma = 0$, $V_{TH} = 0.5$ V, $\mu_n C_{ox} = 1$ mA/V^2, $(W/L)_{NMOS} = 10$, $R_D = 10\Omega$.



Sat. condition : $V_{GD} \leq V_{th}$

$$\left.\begin{aligned} V_{GD} &\leq V_{th} \\ V_{GD} &= R_D I \end{aligned}\right\} \rightarrow R_D \cdot I \leq V_{th} \rightarrow I \leq \frac{0.5V}{10\Omega} = 50 \text{ mA}$$

Write your answer in this box

I = 50 mA

2. The magnitude frequency response of a unity-gain closed-loop amplifier shows a peaking of 93% in the vicinity of the gain crossover frequency. What is the phase margin? **[10 marks]**

$$\text{Closed loop transfer function} = \frac{\frac{1}{\beta}}{1 + \text{loop gain}}$$

low freq. gain is $1 \Rightarrow \beta = 1$

$$PM = 180 + \angle \beta H$$

$$\Rightarrow \left| \frac{1}{1 + 1\angle(180 - PM)} \right| = 1.93$$

$$\left| \frac{1}{1 + \cos(180 - PM) + j \sin(180 - PM)} \right| = 1.93 \rightarrow \left| \frac{1}{1 - \cos PM + j \sin PM} \right| = 1.93$$

$$\rightarrow \frac{1}{\sqrt{(1 - \cos PM)^2 + \sin^2 PM}} = 1.93 \rightarrow \frac{1}{1 + \cos^2 PM - 2\cos PM + \sin^2 PM} = (1.93)^2$$

$$\rightarrow \frac{1}{1 + \underbrace{(\cos^2 PM + \sin^2 PM)}_{1} - 2\cos PM} = (1.93)^2 \rightarrow 2 - 2\cos PM = \left(\frac{1}{1.93}\right)^2$$

$$\longrightarrow PM \approx 30^0$$

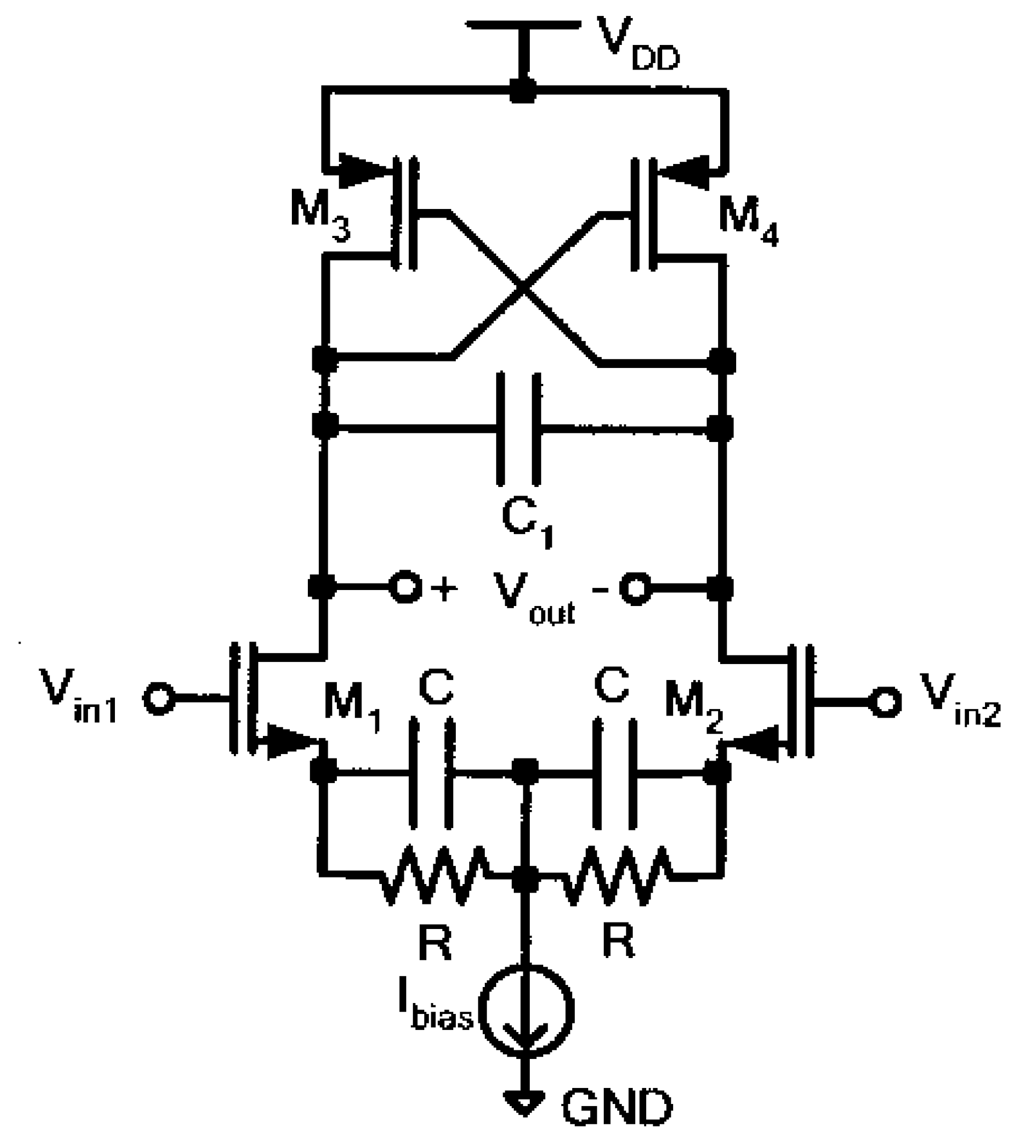
3. Assuming that the following circuit is symmetrical and $\gamma = \lambda = 0$:

i) Find the expression for the small-signal differential voltage gain ($\frac{V_{out}}{V_{in1} - V_{in2}}$) of the circuit at very low frequencies. **[10 marks]**

ii) What is the gain of the circuit at very high frequencies? **[5 marks]**

iii) Repeat part (ii) assuming $\lambda \neq 0$. **[5 marks]**

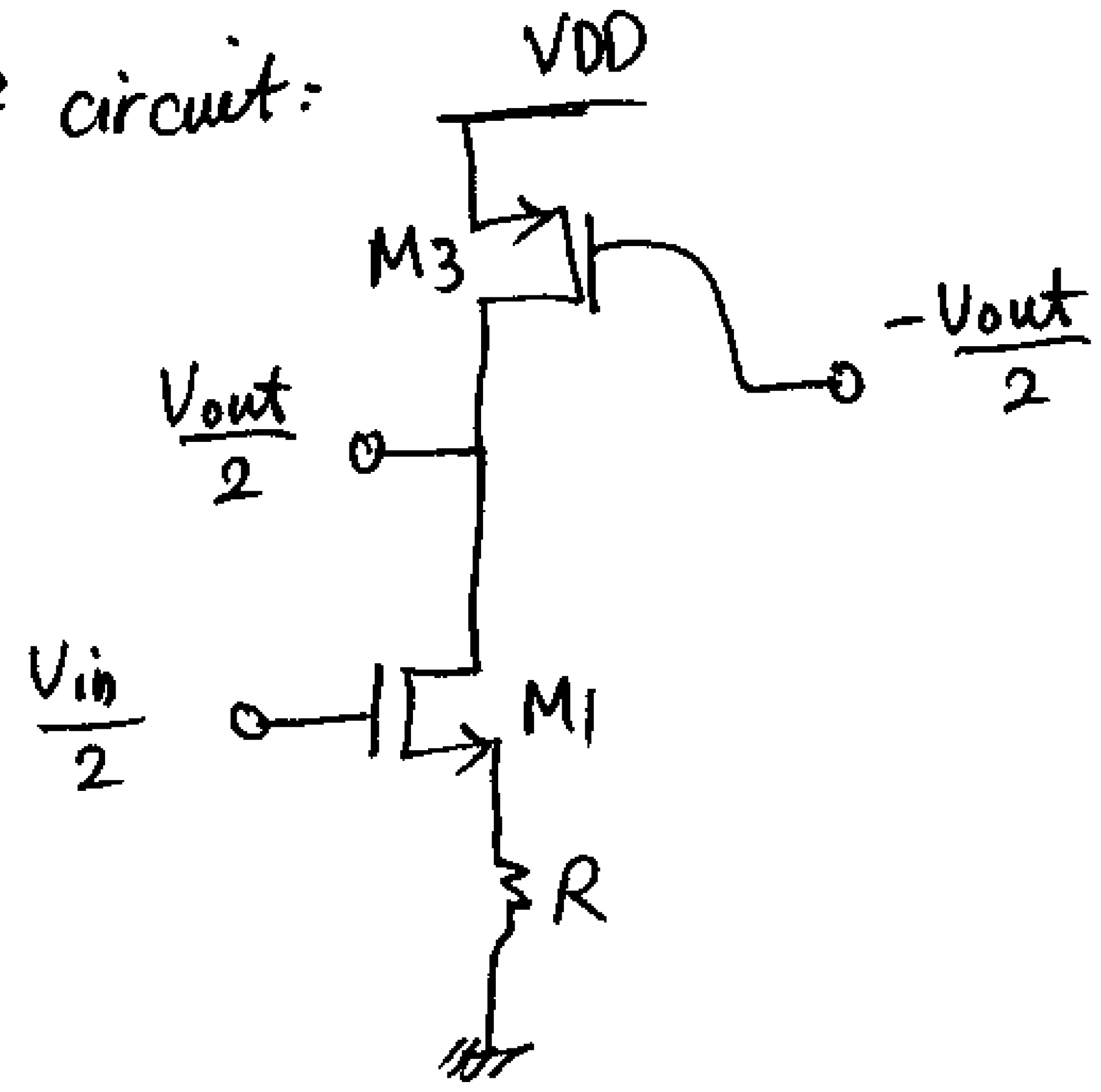
Note: In this question neglect all other capacitances that are not shown in the circuit.



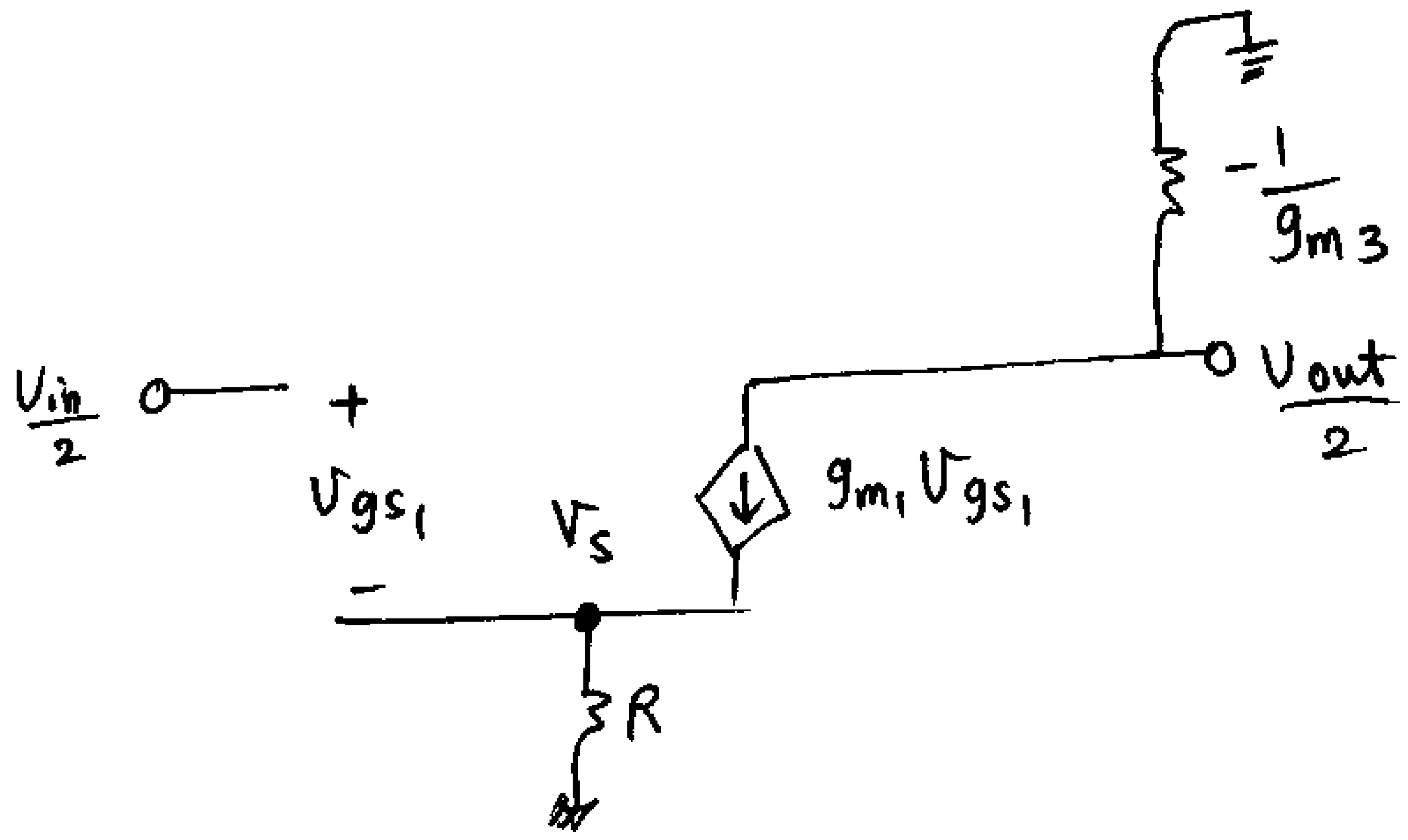
$r_o = \infty$

i) at very low freq. Caps are open.

Half circuit:



small signal model:

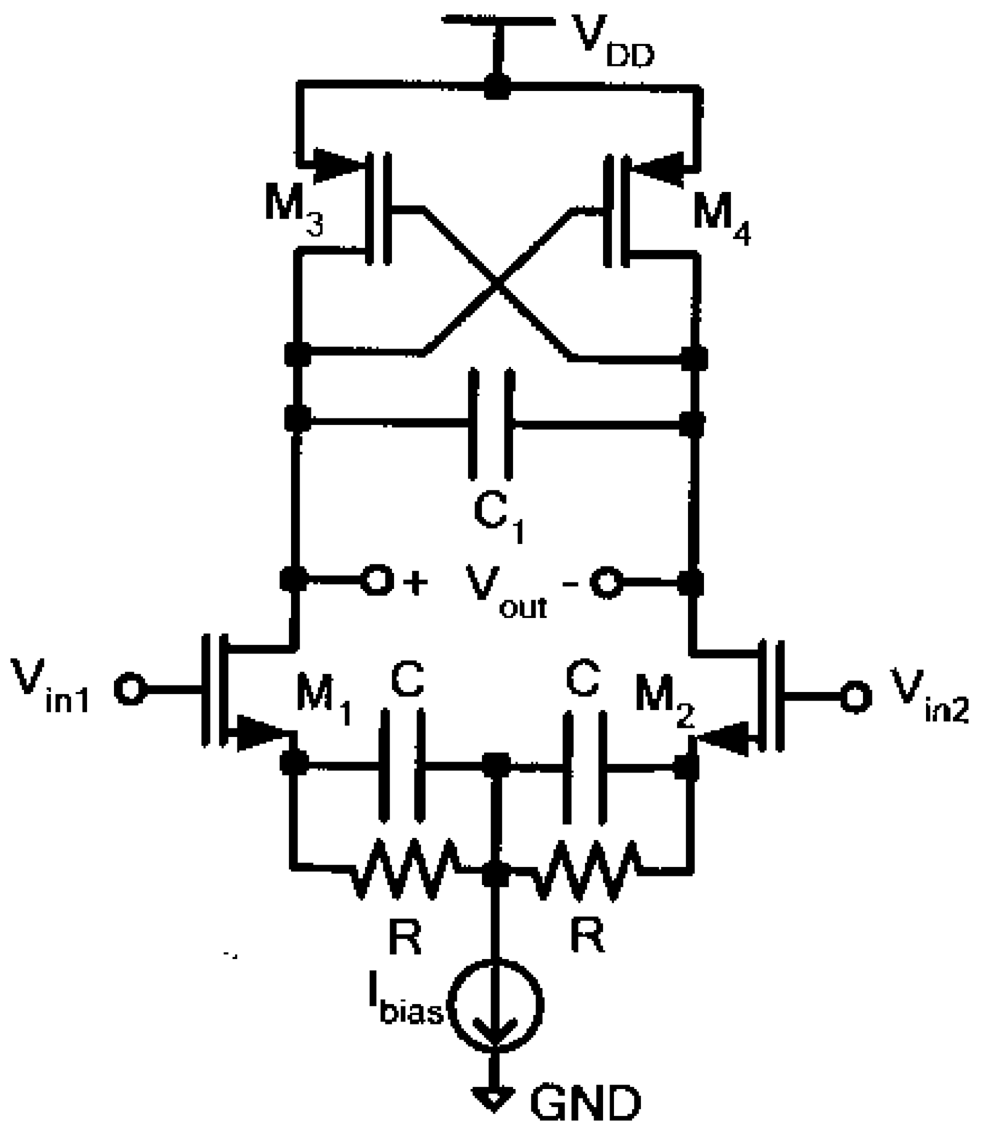


$$\frac{V_{out}}{2} = \left(\frac{-1}{g_{m3}}\right)\left(-g_{m_1} V_{gs_1}\right) = \frac{g_{m1}}{g_{m3}} V_{gs_1} \quad \text{I}$$

$$\frac{V_{in}}{2} = V_S + V_{gs_1} = \left(g_{m_1} V_{gs_1} \cdot R\right) + V_{gs_1} = \left(1 + g_{m_1} R\right) V_{gs_1} \quad \text{II}$$

$$\text{I, II} \rightarrow \frac{\frac{V_{out}}{2}}{\frac{V_{in}}{2}} = \frac{\frac{g_{m1}}{g_{m3}} V_{gs_1}}{\left(1 + g_{m_1} R\right) V_{gs_1}} \rightarrow A_{v_{\text{Low freq.}}} = \frac{g_{m1}}{g_{m3}\left(1 + g_{m_1} R\right)}$$

For your convenience the circuit diagram is replicated here:



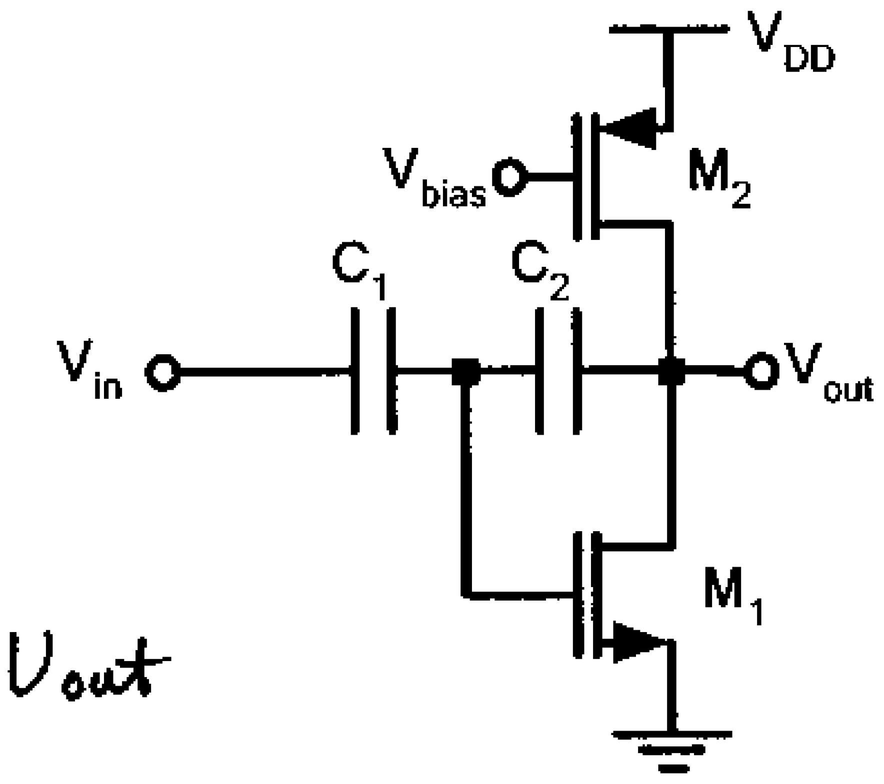
ii) C_1 is short at very high frequencies,

$$A_{v_{\text{high freq.}}} = 0$$

iii) again the high freq. gain is zero.

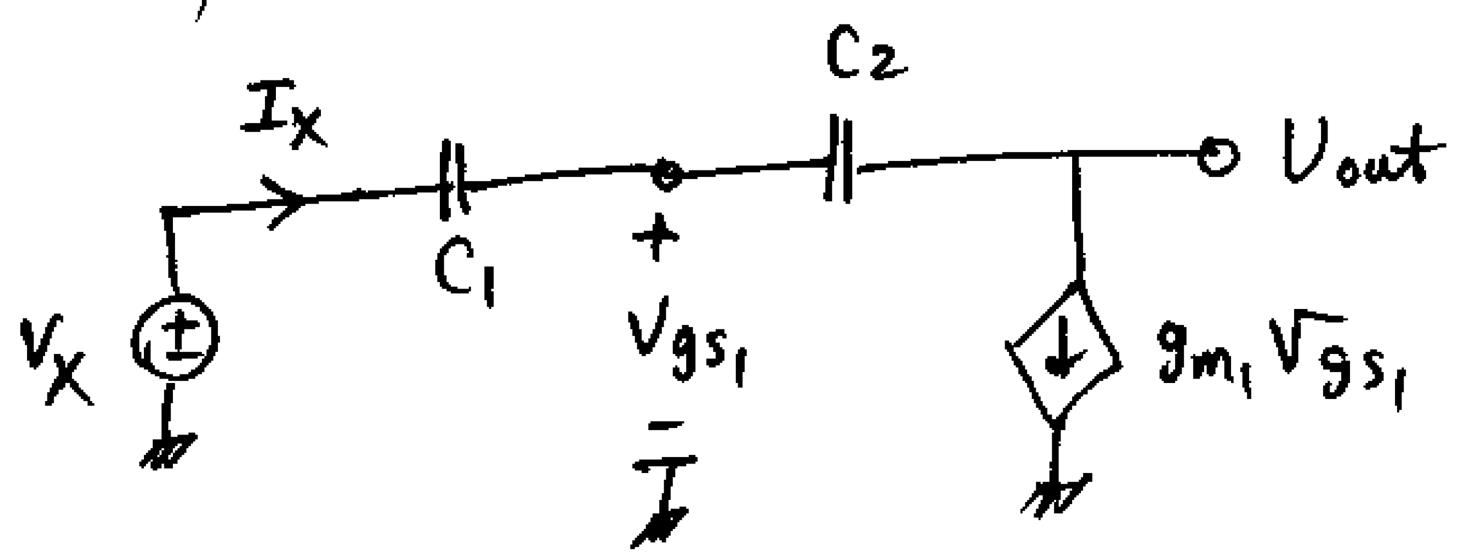
4. Neglecting all other capacitances and assuming $\lambda = 0$, for the following circuit:

i) Calculate the input impedance. **[8 marks]**
ii) Calculate the output impedance. **[8 marks]**
iii) How would the input impedance change if C_2 is replaced with a resistor. **[4 marks]**



$\lambda = 0 \longrightarrow r_o = \infty$

i)

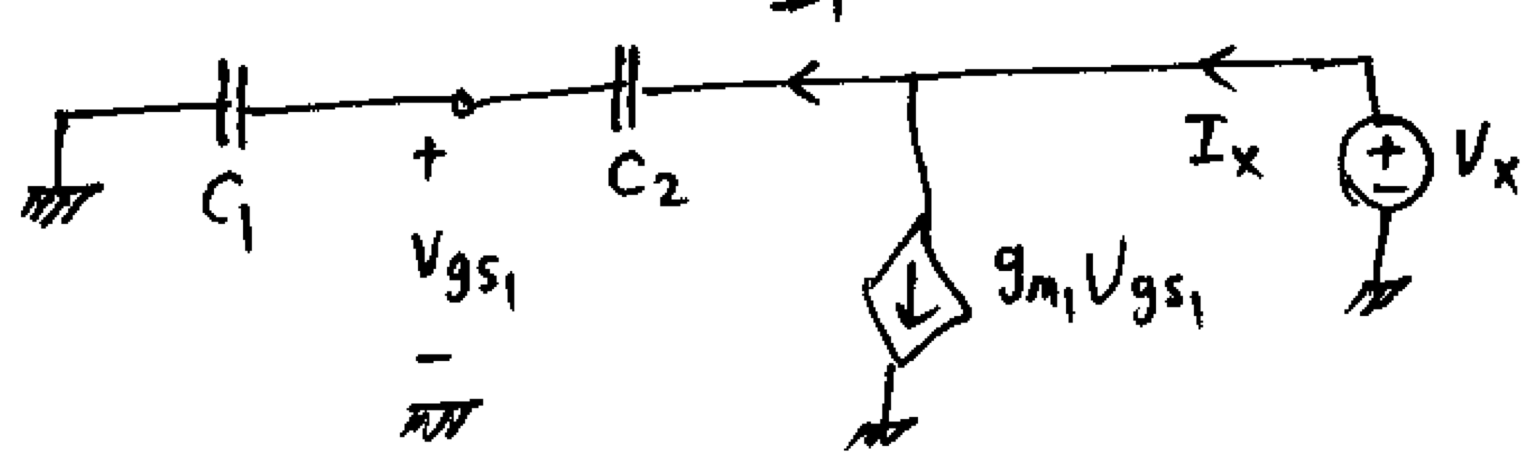


$$I_X = g_{m_1} V_{gs_1} \longrightarrow V_{gs_1} = \frac{I_X}{g_{m_1}}$$

$$I_X = C_1 s \left(V_X - V_{gs_1}\right) = C_1 s \left(V_X - \frac{I_X}{g_{m_1}}\right) \longrightarrow I_X\left(1 + \frac{C_1 s}{g_{m_1}}\right) = C_1 s V_X$$

$$\longrightarrow Z_{in} = \frac{V_X}{I_X} = \frac{1 + \frac{C_1 s}{g_{m_1}}}{C_1 s} = \frac{1}{g_{m_1}} + \frac{1}{C_1 s}$$

ii)



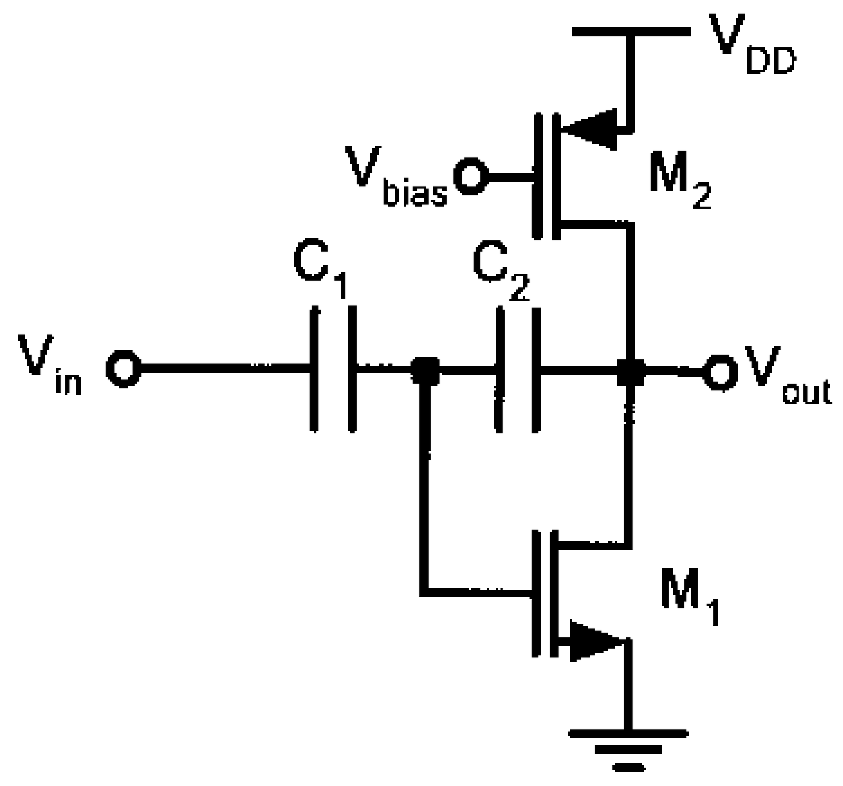
$$\left.\begin{aligned} I_1 &= I_X - g_{m_1} V_{gs_1} \\ I_1 &= C_2 s \left(V_X - V_{gs_1}\right) \\ I_1 &= C_1 s \left(V_{gs_1} - 0\right) \end{aligned}\right\} \longrightarrow C_1 s V_{gs_1} = C_2 s \left(V_X - V_{gs_1}\right) \longrightarrow V_{gs_1}\left(C_1 s + C_2 s\right) = C_2 s V_X$$

$$\longrightarrow V_{gs_1} = \frac{C_2 s}{C_1 s + C_2 s} V_X = \frac{C_2}{C_1 + C_2} V_X$$

$$I_X - g_{m_1} V_{gs_1} = C_1 s V_{gs_1} \longrightarrow I_X = \left(g_{m_1} + C_1 s\right) V_{gs_1} = \left(g_{m_1} + C_1 s\right) \frac{C_2}{C_1 + C_2} V_X$$

$$\longrightarrow Z_{out} = \frac{V_X}{I_X} = \frac{\left(C_1 + C_2\right)}{C_2\left(g_{m_1} + C_1 s\right)}$$

For your convenience the circuit diagram is replicated here:



iii) Z_{in} will not change as it does not depend on the value of C_2.

5. Design a two-stage op amp based on the topology shown below with the following design specifications:

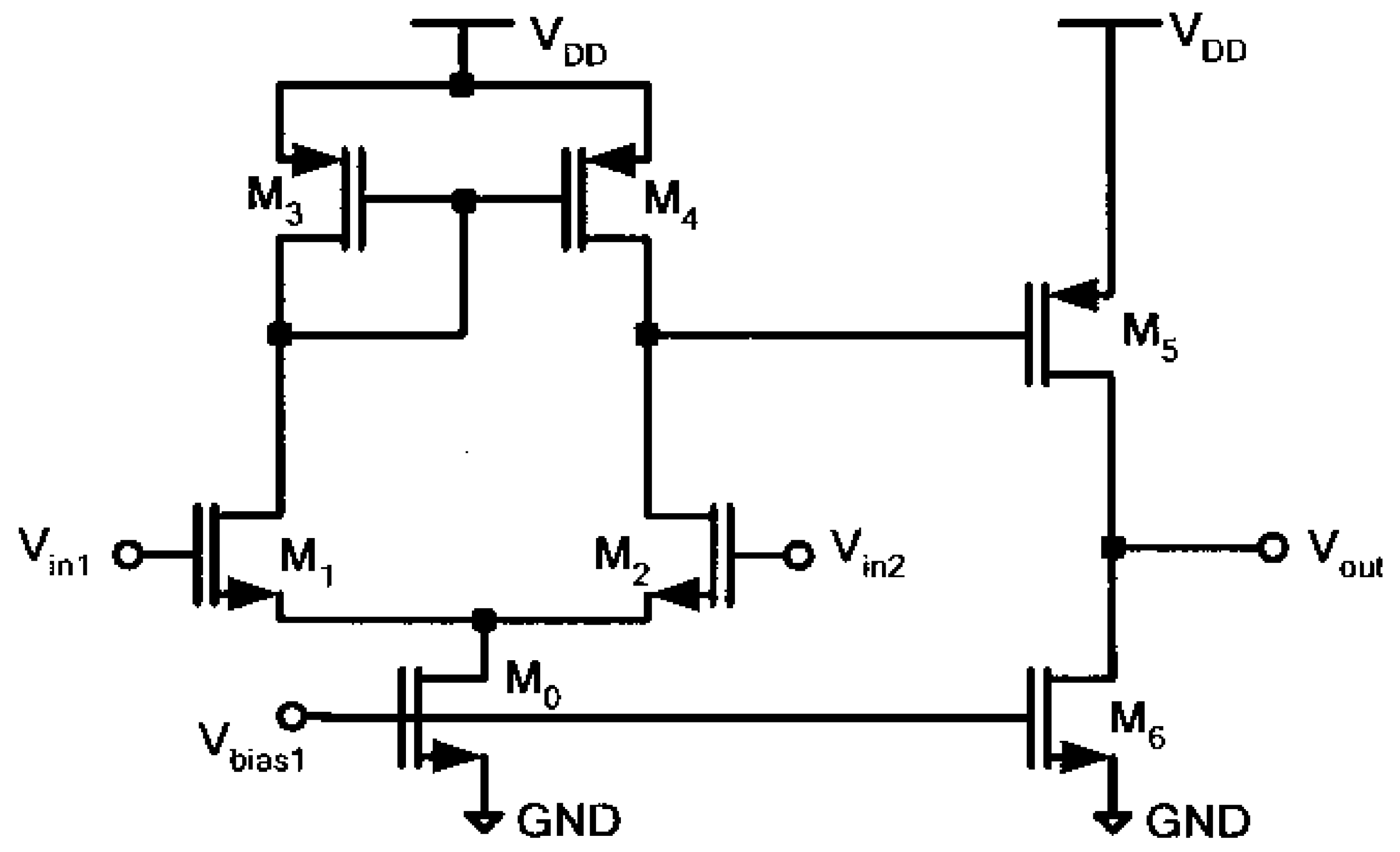
- V_{DD}=3 V
- Total power consumption of 3 mW
- Output swing of 2.6 V
- Total gain of 1000
- L = 0.4μm for all the device

Use the following assumptions for your design

- Allocate equal overdrive voltages to M_5 and M_6
- Assume the bias currents of the first stage and the second stage are equal.
- V_{SG3}=V_{SG5}

The technology parameters are:
$\lambda_{(NMOS)} = \lambda_{(PMOS)} = 0.1\ V^{-1}$, $\gamma = 0$, $V_{DD} = 3$ V, $V_{TH(NMOS)} = |V_{TH(PMOS)}| = 0.5$ V, $\mu_n C_{ox} = 1\ mA/V^2$, $\mu_p C_{ox} = 0.5\ mA/V^2$.

Note: Use the parameter λ only for calculating the r_o of the transistors. **Do not** use λ in any other calculation including your bias currents.



Find V_{bias1}, and all the transistor widths (i.e., $W_0, W_1, W_2, W_3, W_4, W_5, W_6$). **[20 marks]**

$V_{OD5} = V_{OD6}$

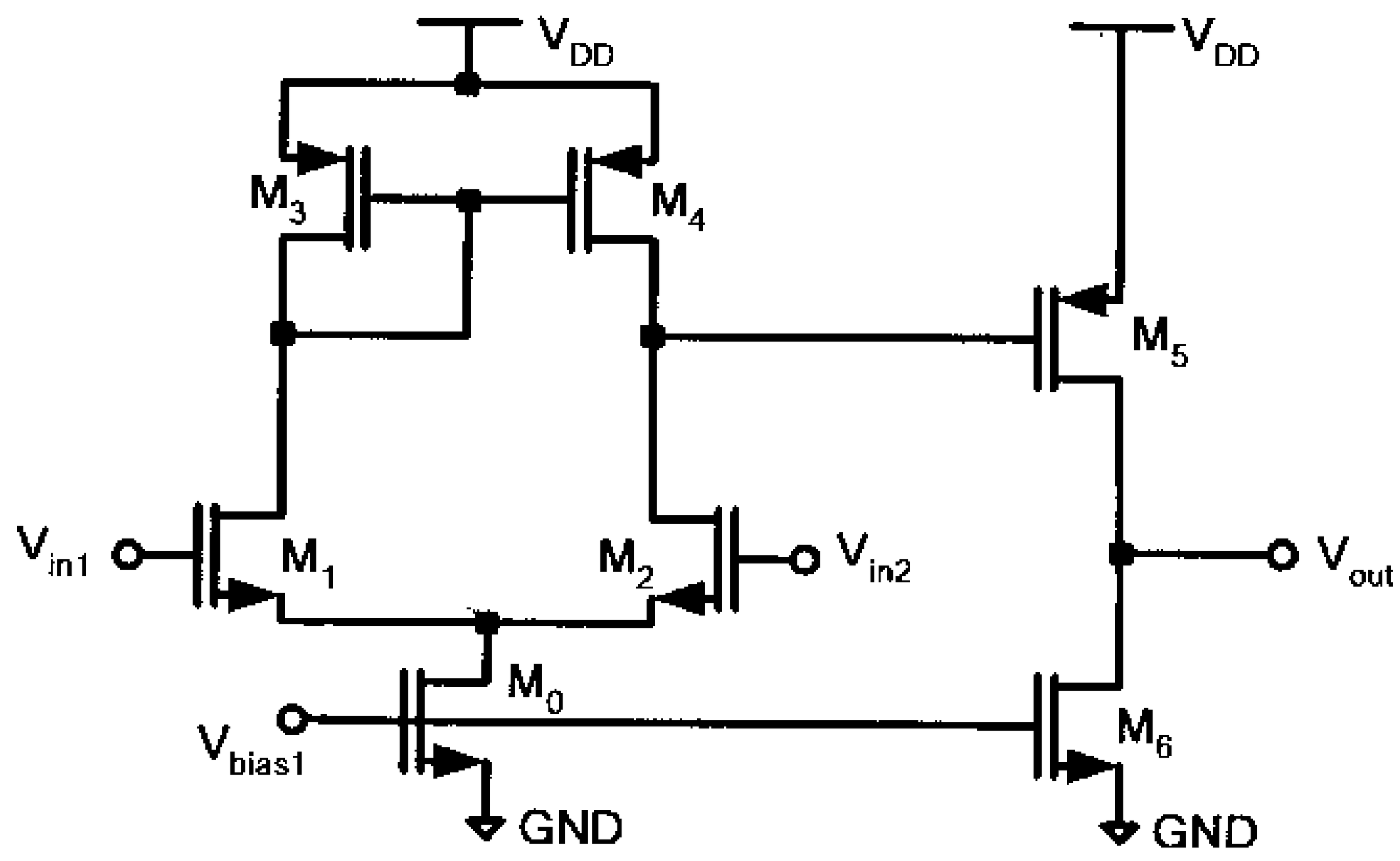
Output swing $= V_{DD} - |V_{OD5}| - V_{OD6} = 3 - 2|V_{OD5}| = 2.6\ V \rightarrow |V_{OD5}| = V_{OD6} = 0.2\ V$

$V_{SG3} = V_{SG5} \rightarrow |V_{OD3}| = |V_{OD5}| = 0.2\ V$

$I_0 = I_5 \longrightarrow I_0 = I_5 = \frac{1}{2} \times \frac{3\ mW}{3\ V} = 0.5\ mA$

$I_5 = 0.5\ mA = \frac{1}{2} \times 0.5\ \frac{mA}{V^2} \times \left(\frac{W}{L}\right)_5 \times V_{OD5}^2 \rightarrow \left.\begin{array}{l}\left(\frac{W}{L}\right)_5 = 50 \\ L = 0.4\mu m\end{array}\right\} \rightarrow \boxed{W_5 = 20\ \mu m}$

For your convenience the circuit diagram and transistor parameters are replicated here:
$\lambda_{(NMOS)} = \lambda_{(PMOS)} = 0.1\ V^{-1}$, $\gamma = 0$, $V_{DD} = 3\ V$, $V_{TH(NMOS)} = |V_{TH(PMOS)}| = 0.5\ V$, $\mu_n C_{ox} = 1\ mA/V^2$, $\mu_p C_{ox} = 0.5\ mA/V^2$.



$$I_6 = 0.5\ mA = \frac{1}{2} \times 1 \times \frac{W6}{0.4} \times V_{OD6}^2 = \frac{1}{2}\frac{W6}{0.4}(0.2)^2 \longrightarrow \boxed{W_6 = 10\ \mu m}$$

$$I_3 = 0.25\ mA = \frac{1}{2} \times 0.5 \times \frac{W_3}{0.4}(0.2)^2 \longrightarrow \boxed{W_3 = W_4 = 10\ \mu m}$$

$$g_{m5} = \frac{2 I_{D5}}{|V_{OD5}|} = \frac{2 \times 0.5}{0.2} = 5\ \frac{mA}{V} \qquad r_{o2} = r_{o4} = \frac{1}{\lambda I_{D2}} = \frac{1}{0.1\,(0.25m)} = 40\ K\Omega$$

$$r_{o5} = r_{o6} = \frac{1}{0.1 \times 0.5 mA} = 20\ K\Omega$$

$$|A_V| = g_{m1}(r_{o2} \| r_{o4}) \cdot g_{m5}(r_{o5} \| r_{o6}) = 1000 \rightarrow g_{m1} \times 20K\Omega \times 5 \times 10K\Omega = 1000$$

$$\rightarrow g_{m1} = 1\ \frac{mA}{V} = \sqrt{2 k_n I_{D1}} \rightarrow 1 = \sqrt{2 \times 1 \times \frac{W_1}{0.4} \times 0.25m} \rightarrow \boxed{W_1 = 0.8\mu m}$$

$$W_1 = W_2 = 0.8\ \mu m$$

$$V_{OD0} = 0.2V \rightarrow V_{bias1} - V_{th} = 0.2 \rightarrow V_{bias1} = 0.2 + 0.5 = 0.7\ V$$

$$I_0 = 1\ mA = \frac{1}{2} \times 1 \times \frac{W_0}{0.4} \times (0.2)^2 \rightarrow \boxed{W_0 = 20\ \mu m}$$