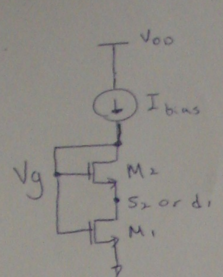
Year 2011 EECE 488 Assignment 1 Solution

Q1.



(a) M_2 is diode connected, so it's operating in the saturation region.

$\Rightarrow V_{gs_2} - V_{t2} > 0$

Since S_2 is the same node as d_1

$\Rightarrow V_{gd_1} - V_{t2} > 0$

Now, the bulk of M_1 is connected to ground, so $V_{s_1b_1} = 0$. The bulk of M_2 is connected to ground as well, so $V_{s_2b_2} > 0 \Rightarrow V_{t2} > V_{t1}$

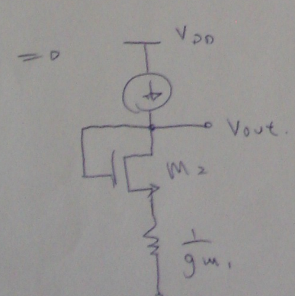
There fore, $V_{gd_1} > V_{t2} > V_{t1} \Rightarrow V_{gd_1} - V_{t1} > 0$

$\Rightarrow V_{gs_1} - V_{t1} > V_{d_1s_1}$, so M_1 operates in the triode region. Note that $V_{gs_1} - V_{d_1s_1} = V_{gd_1}$.

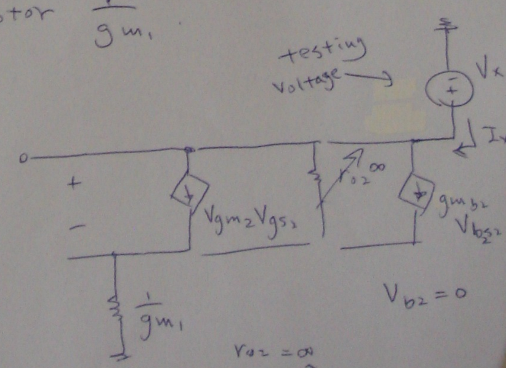
(b) No, it doesn't. M_1 is always in triode region as derived in (a).

(c) $V_{d_1s_1} \approx 0 \Rightarrow M_1$ in deep triode region, and can be modeled as a linear resistor $\frac{1}{g_{m_1}}$

$\Rightarrow$



small sig $\rightarrow$



$$I_x = -g_{mb2} V_{s2} + g_{m2}(V_x - V_{s2})$$

$$I_x = V_{s2}\, g_{m1}$$

$$V_{s2} = \frac{I_x}{g_{m1}}$$

$$I_x = \frac{-g_{mb2}}{g_{m1}} I_x + g_{m2} V_x - \frac{g_{m2}}{g_{m1}} I_x$$

$$I_x + \frac{g_{mb2}}{g_{m1}} I_x + \frac{g_{m2}}{g_{m1}} I_x = g_{m2} V_x$$

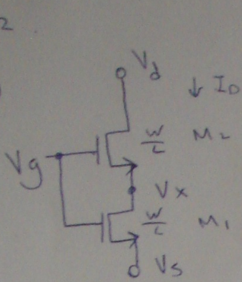
$$I_x\left(1 + \frac{g_{mb2}}{g_{m1}} + \frac{g_{m2}}{g_{m1}}\right) = g_{m2} V_x$$

$$\frac{V_x}{I_x} = R_{out} = \frac{1 + \frac{g_{mb2}}{g_{m1}} + \frac{g_{m2}}{g_{m1}}}{g_{m2}}$$

$$\boxed{= \frac{g_{m1} + g_{mb2} + g_{m2}}{g_{m1}\, g_{m2}}.}$$

Q2.

(a)



First, M_1 and M_2 both need to be on to have I_D flow through.

Let's first assume M_1 is in saturation,

$\Rightarrow V_{gs} - V_t < V_{xs}$

$\Rightarrow V_g - V_t < V_x \quad \cdots\cdots (*)$

Since M_2 is on, $V_{gx} - V_t > 0$

$\Rightarrow V_g - V_x - V_t > 0$ which contradicts $(*)$, so M_1 is not in saturation, it has to be in triode.

So, M_1 has to be in triode, and M_2 can be in either triode or saturation.

Case #1, M_1 & M_2 both in triode.

$$I_{D1} = \mu_n C_{ox} \frac{W}{L}\left[(V_{gs}-V_t)V_{xs} - \frac{1}{2}V_{xs}^2\right] = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{gs}-V_t)V_{xs} - V_{xs}^2\right]$$

$$I_{D2} = \mu_n C_{ox}\frac{W}{L}\left[(V_{gs}-V_{xs}-V_t)(V_{ds}-V_{xs}) - \frac{1}{2}(V_{ds}-V_{xs})^2\right]$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{gs}-V_{xs}-V_t)(V_{ds}-V_{xs}) - (V_{ds}-V_{xs})^2\right]$$

Note that $V_{gs} - V_{xs} = V_{gx}$ $\quad V_{ds} - V_{xs} = V_{dx}$

I_{D1} has to be equal to I_{D2}

$$\Rightarrow 2(V_{gs}-V_t)V_{xs} - V_{xs}^2 = 2(V_{gs}-V_{xs}-V_t)(V_{ds}-V_{xs}) - (V_{ds}-V_{xs})^2$$

$$\Rightarrow 2\left[2(V_{gs}-V_t)V_{xs} - V_{xs}^2\right] = 2(V_{gs}-V_t)V_{ds} - V_{ds}^2$$

$$\Rightarrow I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\cdot\frac{1}{2}\right)\left[2(V_{gs}-V_t)V_{ds} - V_{ds}^2\right]$$

$$\downarrow$$

$$\frac{W}{2L}$$

Case #2 M_1 triode M_2 Saturation

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 (V_{gs} - V_t) V_{xs} - V_{xs}^2 \right]$$

$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{gx} - V_t)^2$$

$I_{D1} = I_{D2}$

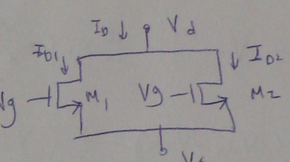
$$\Rightarrow 2 (V_{gs} - V_t) V_{xs} - V_{xs}^2 = (V_{gs} - V_{xs} - V_t)^2$$

$$= (V_{gs} - V_t)^2 - 2 V_{xs} (V_{gs} - V_t) + V_{xs}^2$$

$$\Rightarrow 2 \left[2 (V_{gs} - V_t) V_{xs} - V_{xs}^2 \right] = \frac{1}{2} (V_{gs} - V_t)^2$$

$$\Rightarrow I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{2L} \right) (V_{gs} - V_t)^2$$

(b) Figure can be redrawn :



$$I_D = I_{D1} + I_{D2}$$

since $V_{d1} = V_{d2}$, $V_{s1} = V_{s2}$, $V_{g1} = V_{g2}$, the transistors are always in the same operating region.

case #1 M_1 M_2 both in triode :

$$I_{D1} = I_{D2} = \mu_n C_{ox} \frac{W}{L} \left[(V_{gs} - V_t) V_{ds} - \frac{1}{2} V_{ds}^2 \right]$$

$$I_D = I_{D1} + I_{D2} = \mu_n C_{ox} \left(\frac{2W}{L} \right) \left[(V_{gs} - V_t) V_{ds} - \frac{1}{2} V_{ds}^2 \right]$$

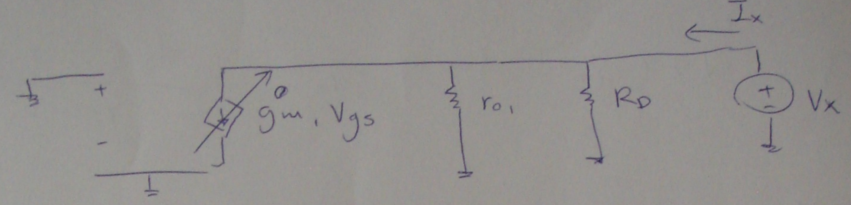
Case #2 M_1 M_2 both in Saturation

$$I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_t)^2$$

$$I_D = I_{D1} + I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{2W}{L} \right) (V_{gs} - V_t)^2$$

Q3. Assume all bulks are connected to soures

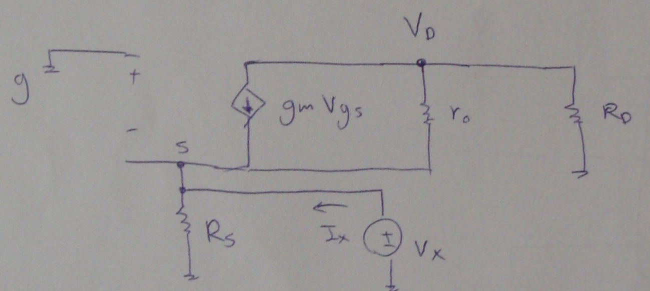
circuit 1. small signal model.



$$I_x = \frac{V_x}{r_{o1}} + \frac{V_x}{R_D}$$

$$\Rightarrow \frac{V_x}{I_x} = R_{OUT} = r_o \parallel R_D.$$

circuit 2. small signal model



$g_m V_{gs} = -g_m V_x$ because $V_g = 0$ $V_s = V_x$.

$$I_x = \frac{V_x}{R_S} + g_m V_x + \frac{V_x - V_D}{r_o} \quad \cdots \quad (**)$$

$$\frac{V_D}{R_D} = \frac{V_x}{r_o} - \frac{V_D}{r_o} + g_m V_x$$

$$V_D\left(\frac{1}{R_D} + \frac{1}{r_o}\right) = V_x\left(\frac{1}{r_o} + g_m\right)$$

$$V_D = \frac{V_x\left(\frac{1}{r_o} + g_m\right)}{\frac{1}{R_D} + \frac{1}{r_o}} = \frac{V_x\left(\frac{1}{r_o} + g_m\right) R_D r_o}{r_o + R_D}$$

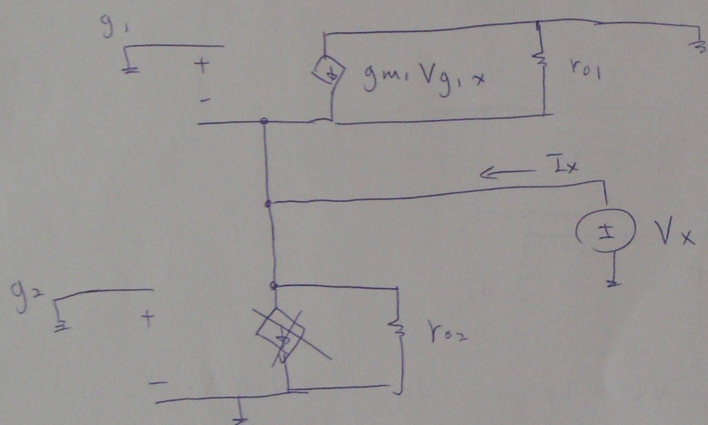
plug in (**)

$$I_x = \frac{V_x}{R_S} + g_m V_x + \frac{V_x}{r_o} - \frac{V_x\left(\frac{1}{r_o} + g_m\right)R_D}{r_o + R_D}$$

$$I_x = V_x\left(\frac{1}{R_S} + g_m + \frac{1}{r_o} - \frac{R_D\left(\frac{1}{r_o} + g_m\right)}{r_o + R_D}\right)$$

$$\frac{V_x}{I_x} = R_{OUT} = \frac{1}{\frac{1}{R_S} + g_m + \frac{1}{r_o} - \frac{R_D\left(\frac{1}{r_o} + g_m\right)}{r_o + R_D}}$$

circuit 3. small signal.



$$\frac{-V_x}{r_{o1}} - g_m V_x + I_x = \frac{V_x}{r_{o2}}$$

$$I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{r_{o1}} + g_m V_x$$

$$I_x = V_x\left(\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_m\right)$$

$$\frac{V_x}{I_x} = R_{OUT} = \frac{1}{\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_m}$$

$$= r_{o1} \parallel r_{o2} \parallel \frac{1}{g_m}.$$