Year 2011 ECE 488 Assignment 2 Solution.

Data point	V_{gs}	V_{ds}	V_{bs}	I_D	g_{m0}	V_{th}
1	1	1	0	312u	1.12m	565m
2	1	2	0	336u	1.16m	556m
3	2	2	0	1.76m	1.56m	558m
4	2	2	-0.5	1.6m		675m
5	2	2	-1	1.45m		769m

(b) Note the data points are taken when transistor is in saturation region since $V_{ds} > V_{gs} - V_t$.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs}-V_t)^2(1+\lambda V_{ds})$$

To find V_{th0}, we use data point 2 and 3 since they have 0V V_{bs} and constant V_{ds}.

$V_{bs} = 0 \Rightarrow V_{th} = V_{th0}$

$$\Rightarrow \frac{I_{D2}}{I_{D3}} = \frac{(V_{gs2}-V_{th0})^2}{(V_{gs3}-V_{th0})^2}$$

$$V_{th0} = \frac{V_{gs2}\sqrt{I_{D3}} - V_{gs3}\sqrt{I_{D2}}}{\sqrt{I_{D3}}-\sqrt{I_{D2}}}$$

$= 0.224$ V or $1.3\cancel{04}$ V

↑ quite off too. ↑ too off

To find λ, we use constant V_{gs}, data point 1 and 2

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{1+\lambda V_{ds1}}{1+\lambda V_{ds2}}$$

$$\lambda = \frac{I_{D2} - I_{D1}}{V_{ds2} I_{D1} - V_{ds1} I_{D2}} = 0.0833 V^{-1}$$

To find $\mu_n C_{ox}$, just use data point 1 and know V_{th0} and λ

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$$

$$\Rightarrow \mu_n C_{ox} = \frac{2 I_D L}{W (V_{gs} - V_t)^2 (1 + \lambda V_{ds})}$$

$$= 4.783 \times 10^{-5} A/V^2$$

For γ, we know

$$V_{th} = V_{th0} + \gamma \left(\sqrt{|2\phi_F + V_{sb}|} - \sqrt{|2\phi_F|} \right)$$

we know V_{th} and V_{th0} from data points and previous calculation. We know V_{sb}. But we don't know ϕ_F and γ. So we need 2 equations for 2 unknowns.

Use data point 4 and 5.

$$\#4. \quad 675\,mV = 224\,mV + \gamma \left(\sqrt{|2\phi_F + 0.5|} - \sqrt{|2\phi_F|} \right)$$

$$\#5 \quad 769\,mV = 224\,mV + \gamma \left(\sqrt{|2\phi_F + 1|} - \sqrt{|2\phi_F|} \right)$$

No solution

If we use $V_{th0} = 558\,mV$ directly off data point 3 where $V_{bs} = 0\,V$. Then,

$$675\,mV = 558\,mV + \gamma\left(\sqrt{|2\phi_F + 0.5|} - \sqrt{|2\phi_F|}\right)$$
$$769\,mV = 558\,mV + \gamma\left(\sqrt{|2\phi_F + 1|} - \sqrt{|2\phi_F|}\right)$$

use matlab $\Rightarrow$

$$\gamma = 0.4492\ V^{-1}$$

$$2\phi_F = 0.6883\ V$$

Yes, the question is tricky.

Another way to do it is to plot I_D vs V_{gs} and estimate V_{th0} at $V_{bs} = 0$. That'll work too.

(c). $g_m = \dfrac{2 I_D}{V_{gs} - V_t}$ plug in from data points.

	data point	V_{gs}	V_{ds}	V_{bs}	g_{m0}	Calculated g_m
error -28%	1	1	1	0	1.12m	0.804m
error -25%	2	1	2	0	1.16m	0.87m
error 28%	3	2	2	0	1.56m	2m

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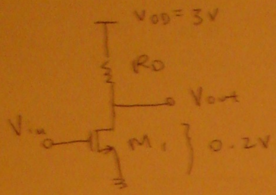
Q2.



(a) Assume M_1 has the minimum V_{ds} to keep it in sat. so it's given $V_{ds} = 0.2V$

and $V_{ds} = V_{gs} - V_t$

$V_t = 0.5V$. so $V_{gs} = 0.7V \Rightarrow V_g = 0.7V$

(b) It's given $P = 3mW$. so current I_D flowing through the circuit is $1mA$. since $P = IV$ $L = 0.5\mu m$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs} - V_t)^2$$

$$1mA = \frac{1}{2}\,\frac{1mA}{V^2}\,\frac{W}{0.5\mu m}\,(0.2)^2$$

$$W = 25\mu m.$$

(c) For common source amplifier configured in this question, gain $|A_v| = |-g_m R_D|$

$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{gs} - V_t) = 10mS$$

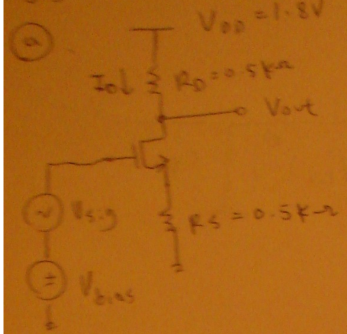
$$15 = 10mS\, R_D \Rightarrow R_D = 1.5k\Omega$$

(d) $V_{out,DC} = V_{DD} - I_D R_D = 1.5V$

(e) As long as M1 has 0.2V across then the circuit will operate correctly. So, it's from 0.2V to V_{DD}

Q3.

(a)



Find the drain current first to figure out dc node voltages.

We know $V_{out} = 1.44V$

$$I_D = \frac{V_{DD} - V_{out}}{R_D} = \frac{1.8V - 1.44V}{0.5k\Omega}$$

$$= 0.72\ mA.$$

$V_D = V_{out} = 1.44V$ $\qquad$ $V_S = R_S \cdot I_D = 0.36\ V.$

$V_{DS} = 1.08V$

For transistor to be in saturation, the drain current satisfy the following:

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{gs} - V_t)^2$$

$$0.72\ mA = \frac{1}{2} \frac{800\mu A}{V^2} 20\ (V_{gs} - V_t)^2 \Rightarrow V_{gs} - V_t = 0.3\ V.$$

Assume V_b is connected to ground,

$$V_t = V_{th0} + \gamma\left(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F}\right)$$

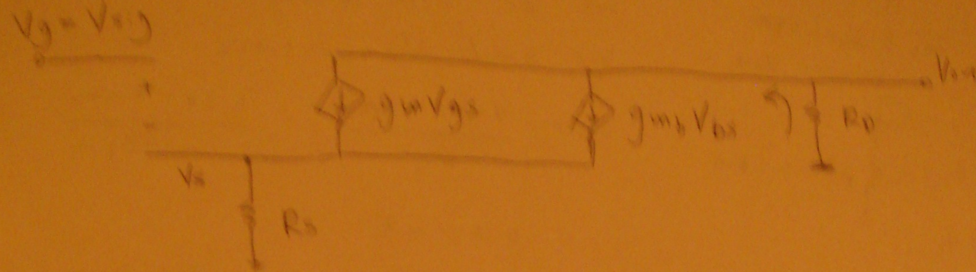
$$= 0.4V + 1V^{1/2}\left(\sqrt{0.36 + 0.64} - \sqrt{0.64}\right)$$

$$= 0.6V.$$

$V_{gs} - V_t = 0.3V$ $\qquad$ $V_{gs} = 0.9V.$ $\qquad$ $\boxed{V_g = 0.9V + V_s = 1.26V.}$

(b) Yes, because $V_{ds} = 1.08V > \underbrace{V_{gs} - V_t}_{0.3V}$

② Small signal Model



$$\frac{-V_{out}}{R_D} = g_m V_{gs} - g_{mb} V_s$$

sub

$$\frac{-V_{out}}{R_D} = \frac{V_s}{R_s} \qquad V_s = -\frac{R_s}{R_D} V_{out}$$

$$\frac{-V_{out}}{R_D} = g_m \left(V_{sig} + \frac{R_s}{R_D} V_{out} \right) + g_{mb} \frac{R_s}{R_D} V_{out}$$

$$= g_m V_{sig} + \frac{g_m R_s}{R_D} V_{out} + g_{mb} \frac{R_s}{R_D} V_{out}$$

$$\Rightarrow -g_m V_{sig} = V_{out} \left(\frac{1}{R_D} + \frac{g_m R_s}{R_D} + g_{mb} \frac{R_s}{R_D} \right)$$

$$\frac{V_{out}}{V_{sig}} = \frac{-g_m}{\frac{1}{R_D} + g_m \frac{R_s}{R_D} + g_{mb} \frac{R_s}{R_D}}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_t) = 4.8 \text{ mS}$$

$$g_{mb} = \frac{\gamma}{2\sqrt{|2\phi_F + V_{SB}|}} g_m = 2.4 \text{ mS}$$

$$\Rightarrow \frac{V_{out}}{V_{sig}} = -0.52$$

Q4. Very low frequency, capacitors are like open circuit

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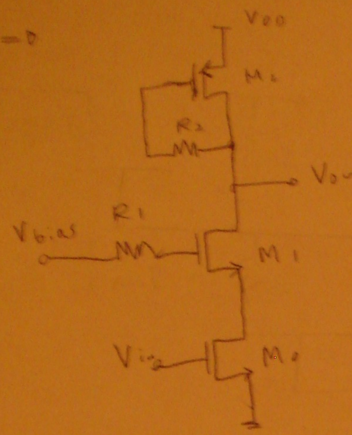

No current flow into gate so R_2 and R_1 can be replaced by a wire.

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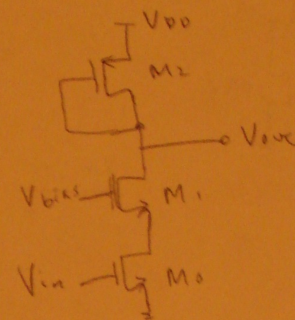
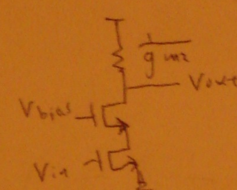
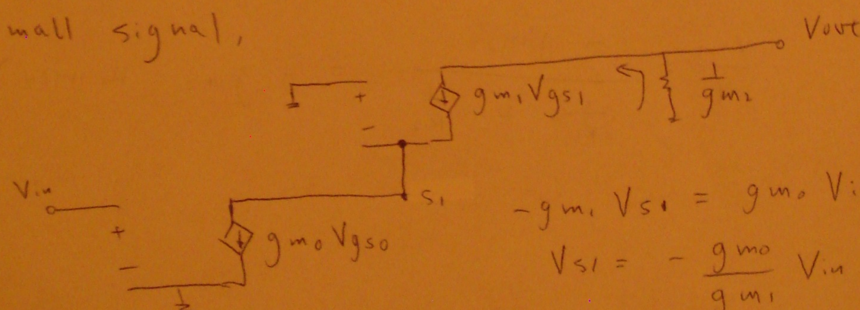


M_2 is diode connected, so impedance looking up to M_2 is $\frac{1}{g_{m2}}$.



small signal,



$$-g_{m1} V_{S1} = g_{m0} V_{in}$$

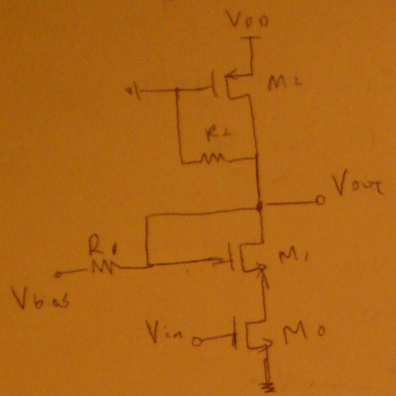
$$V_{S1} = -\frac{g_{m0}}{g_{m1}} V_{in}$$

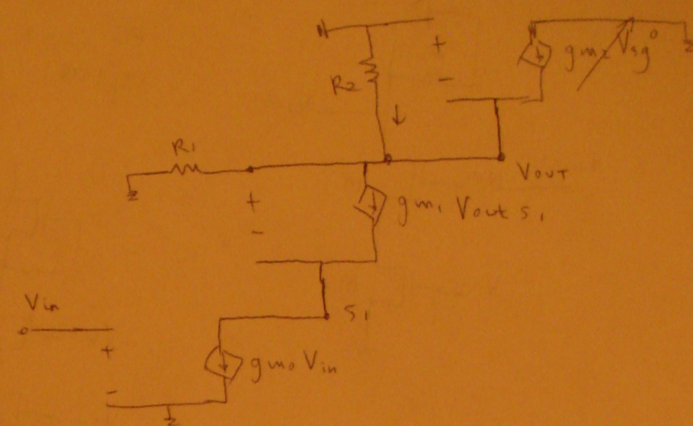
$$-\frac{V_{out}}{\frac{1}{g_{m2}}} = g_{m0} V_{in}$$

$$-g_{m2} V_{out} = g_{m0} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m0}}{g_{m2}}$$

Very high frequency, caps are short





$$\frac{-V_{out}}{R_2} = \frac{V_{out}}{R_1} + g_{m1}(V_{out} - V_{S1})$$

$$g_{m1}(V_{out} - V_{S1}) = g_{m0}V_{in}$$

$$g_{m0}V_{in} = -V_{out}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m0}}{\frac{1}{R_1} + \frac{1}{R_2}} = -g_{m0}(R_1 \parallel R_2)$$