EECE 488 Assignment #2 Solutions

1.a) A DC sweep of the specified MOSFET (nch W=10um, L=0.35um), provides:

Data Point	V_{GS}	V_{DS}	I_D	g_{mo}	V_{th}
1	1	1	448.6638μ	1.6022m	566 mV
2	1	2	483.4163μ	1.6615m	557mV
3	2	2	2.5156m	2.2251m	559 mV

b.) Using the two points with constant V_{GS}, we can find λ as follows:

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2(1+\lambda V_{DS})$$

$$\begin{aligned} I_{D1} &= X(1+\lambda V_{DS1}) \\ I_{D2} &= X(1+\lambda V_{DS2}) \end{aligned} \Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{1+\lambda V_{DS1}}{1+\lambda V_{DS2}}$$

$$I_{D1} + \lambda V_{DS2} I_{D1} = I_{D2} + \lambda V_{DS1} I_{D2}$$

$$\therefore \lambda = \frac{I_{D2}-I_{D1}}{V_{DS2}I_{D1} - V_{DS1}I_{D2}} \approx \underline{\underline{0.084 \ V^{-1}}}$$

Using the two points with constant V_{DS}, we can find V_{th}:

$$\begin{aligned} I_{D2} &= X(V_{GS2}-V_{th})^2 \\ I_{D3} &= X(V_{GS3}-V_{th})^2 \end{aligned} \Rightarrow \frac{I_{D2}}{I_{D3}} = \frac{(V_{GS2}-V_{th})^2}{(V_{GS3}-V_{th})^2}$$

$$(V_{GS3}-V_{th})\sqrt{I_{D2}} = (V_{GS2}-V_{th})\sqrt{I_{D3}}$$

$$\therefore V_{th} = \frac{V_{GS2}\sqrt{I_{D3}} - V_{GS3}\sqrt{I_{D2}}}{\sqrt{I_{D3}} - \sqrt{I_{D2}}} \approx \underline{\underline{0.219 \ V.}}$$

1b.) Continued.

Using any one of the data points and our calculated λ and V_{th}, we can find $\mu_n C_{ox}$ by plugging the values into the long-channel expression for I_D.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2(1+\lambda V_{DS})$$

$$\Rightarrow \mu_n C_{ox} = \frac{2 I_D L}{W(V_{GS}-V_{th})^2(1+\lambda V_{DS})} \approx 4.7558 \times 10^{-5}\ \frac{A}{V^2}$$

c.) g_m is calculated by:

$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})(1+\lambda V_{DS})$$

$$= \frac{2 I_D}{V_{GS}-V_{th}}$$

Relative error is: $\frac{\text{calculated } g_m - \text{spice } g_m}{\text{spice } g_m} \times 100\%$.

Results:

V_{GS}	V_{DS}	hspice g_m	calculated g_m	% Error
1	1	1.602m	1.15m	-28
1	2	1.662m	1.24m	-25
2	2	2.225m	2.83m	27

Problem 2-

Since $\lambda \neq 0 \Rightarrow r_o$ is finite. Also $\gamma \neq 0 \Rightarrow g_{mb} \neq 0$. But the bulk of all transistors are connected to source (it is not shown in picture). So $V_{BS} = 0$. As a result, we do not need to consider g_{mb} at all. For all problems, a small signal model has been drawn. To find the output resistance, we only need to find the ratio of V_X to I_X.

(d) In this problem, R1 has no effect in the circuit operation from the small signal view. Since gate of transistors are open in the mid-band frequencies and $I_{G1}=I_{G2}=0$, there is no voltage drop across R1. So $V_{R1}=0$ and we can replace it with a wire.

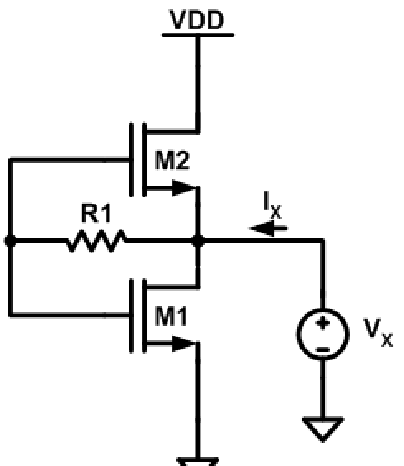

Figure 2: Problem 2.d

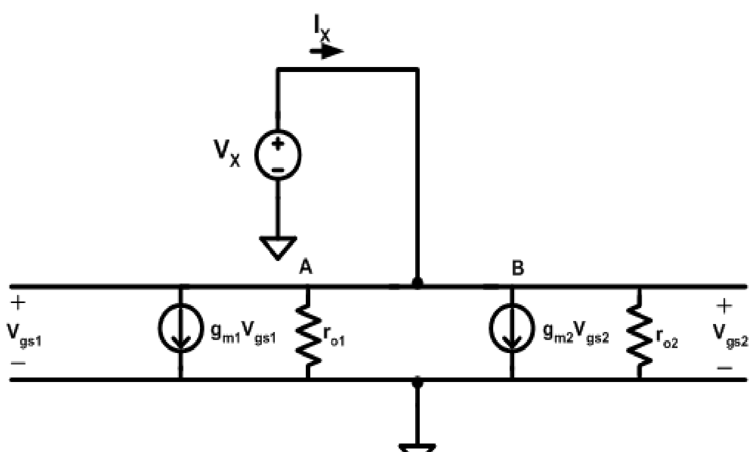

Figure 3: Small signal model of the problem 2.d

$$I_X = \frac{V_X}{r_{o1}} + \frac{V_X}{r_{o2}} + (g_{m1} + g_{m2})V_X \Rightarrow R_{OUT} = \frac{V_X}{I_X} = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m1}} \parallel \frac{1}{g_{m2}}$$

Note (1): A diode-connected transistor seen from the drain terminal is equivalent to $r_o \parallel \frac{1}{g_m}$.

(e) The gates of both transistors are connected to a constant voltage. Therefore, from the small signal viewpoint they are both grounded. As shown in the small signal model (Fig. 5), we can ignore the dependent current sources, as they have no effect.

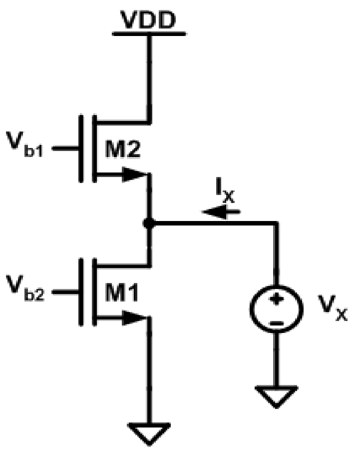

Figure 4: Problem 2.e

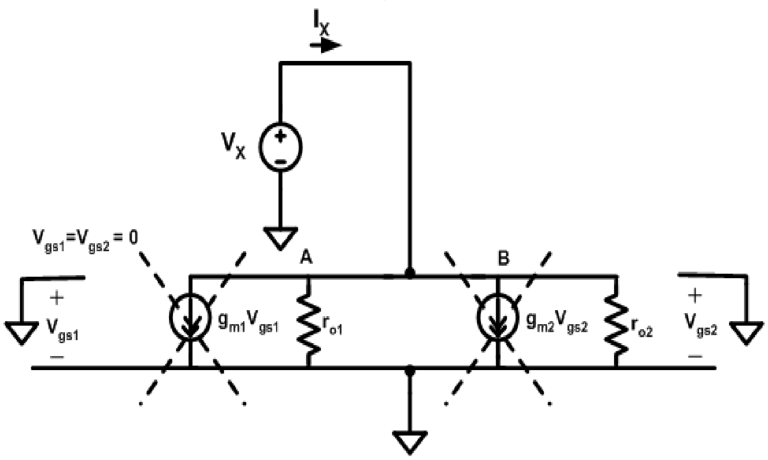

Figure 5: Small signal model of the problem 2.e

$$I_X = \frac{V_X}{r_{o1}} + \frac{V_X}{r_{o2}} \Rightarrow R_{OUT} = \frac{V_X}{I_X} = r_{o1} \parallel r_{o2}$$

Note (2): A MOSFET biased as a current source (a constant voltage at its gate) can be replaced by r_o.

(f) This circuit is the combination of circuits shown in Fig.2 and Fig.4. So using *note (1)* and *note (2)*, we can conclude that the output resistance is $r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}}$.

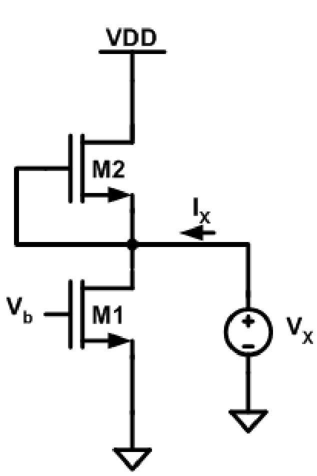

Figure 6: Problem 2.f

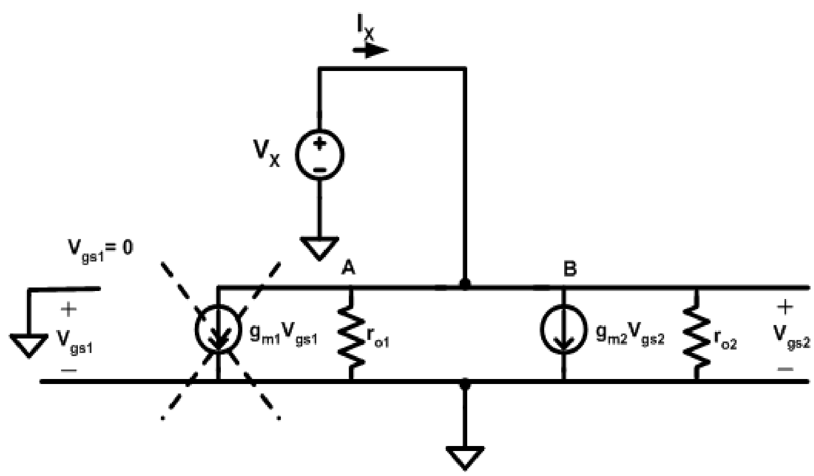

Figure 7: Small signal model of the problem 2.f

$$I_X = \frac{V_X}{r_{o1}} + \frac{V_X}{r_{o2}} + g_{m2}V_X \Rightarrow R_{OUT} = \frac{V_X}{I_X} = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

Problem 3- For each of the following problems, we ignore the bulk effect (g_{mb}=0). However, we have to consider r_o as $\lambda \neq 0$. Also, we try to use the results we found before. Instead of drawing the small signal model, we substitute the diode-connected, or current source transistors with their equivalent resistors we found in the previous problem (wherever possible).

(b) This is a common-source amplifier with source degeneration. The equivalent circuit of Fig. 8 has been shown in Fig. 9 where:
$R_S = r_{o1} \parallel \frac{1}{g_{m1}}$ and $R_D = r_{o3} \parallel \frac{1}{g_{m3}}$. According to the textbook (page 67), the voltage gain (A_V) of this type of amplifier is:
$A_V = -G_m R_{out}$ where:
$G_m = \frac{g_m r_o}{R_S + (1 + g_m R_S) r_o}$ and $R_{OUT} = R_D \parallel [(1 + g_m r_o) R_S + r_o]$. Therefore the voltage gain is:

$$A_V = -\frac{g_{m2} r_{o2}}{R_S + (1 + g_{m2} R_S) r_{o2}} \cdot R_D \parallel [(1 + g_{m2} r_{o2}) R_S + r_{o2}]$$

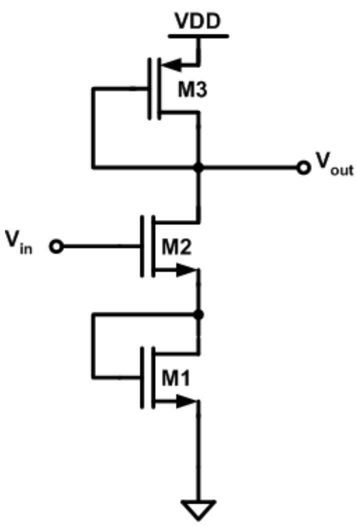

Figure 8: Problem 3.b

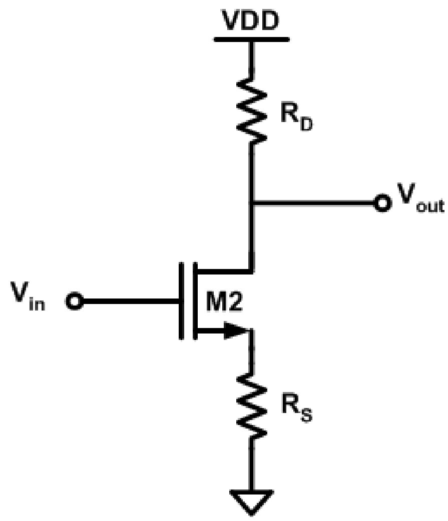

Figure 9: Common-source amplifier with source degeneration

(c) This circuit is a source follower or common-drain amplifier. Again for simplicity, the circuit has been drawn in Fig. 11 with R_S and R_D instead of the actual transistors. From *note (1)* we can see that:

$R_S = r_{o1} \parallel \frac{1}{g_{m1}}$ and $R_D = r_{o3} \parallel \frac{1}{g_{m3}}$.

The small signal model of the circuit is drawn in Fig. 12. We can find the voltage gain by writing two node equations at the output terminal and node A:

$$\left.\begin{array}{l} V_{out} = -\frac{V_A}{R_D} \cdot R_S \Rightarrow V_A = -\frac{V_{out}}{R_S} \cdot R_D \\ V_{gs} = V_{in} - V_{out} \end{array}\right\} \Rightarrow g_m (V_{in} - V_{out}) = \frac{V_{out}}{R_S} + \frac{V_{out} + \frac{V_{out}}{R_S} R_D}{r_o} \Rightarrow$$

$$g_m V_{in} = V_{out} \left(g_m + \frac{1}{R_S} + \frac{R_S + R_D}{R_S r_o} \right) \Rightarrow A_V = \frac{V_{out}}{V_{in}} = \frac{g_m R_S r_o}{(1 + g_m r_o) R_S + R_D + r_o}$$

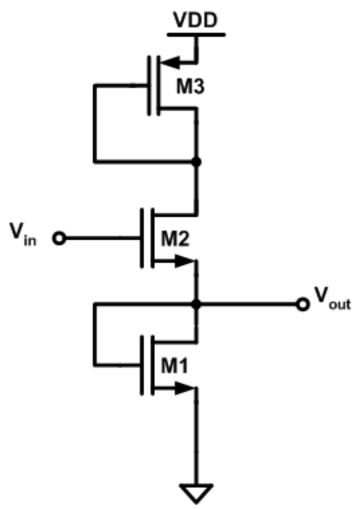

Figure 10: Problem 3.c

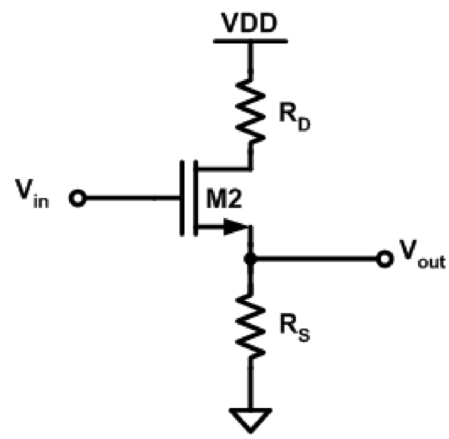

Figure 11: Source follower amplifier (common-drain)

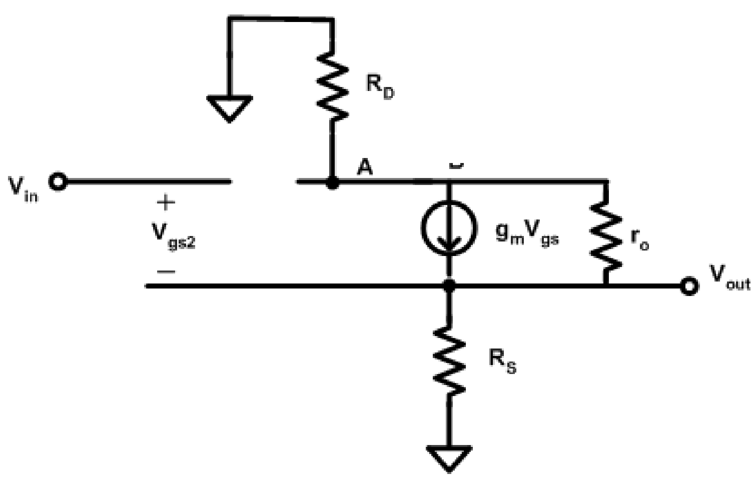

Figure 12: Small signal model of Fig. 11

(d) This problem is similar to 3.b except that diode connected transistors have been replaced by current sources. So we use *note (2)* to find the equivalent resistors for M3 and M1:
$R_S = r_{o1}$ and $R_D = r_{o3}$.

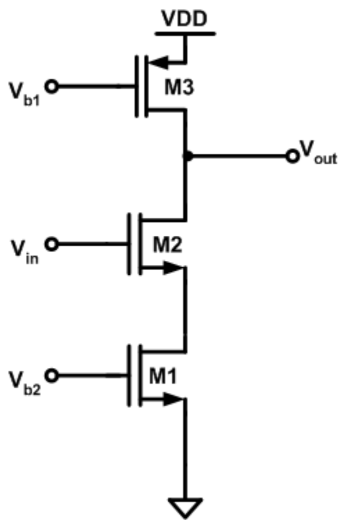

Figure 13: Problem 3.d

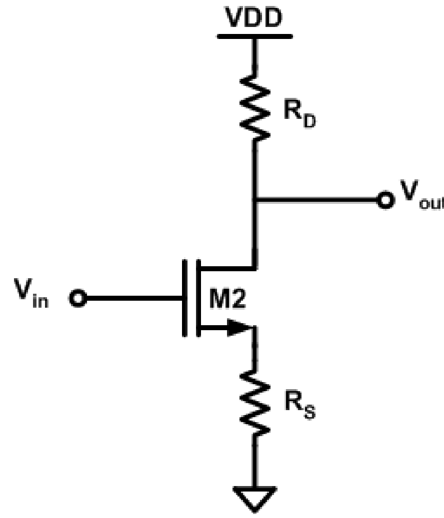

Figure 14:Common-source amplifier with source degeneration

(A_V) of this type of amplifier is:
$A_V = -G_m R_{out}$ where:

$G_m = \dfrac{g_m r_o}{R_S + (1 + g_m R_S) r_o}$ and $R_{OUT} = R_D \parallel \left[(1 + g_m r_o) R_S + r_o\right]$. Therefore the voltage gain is:

$$A_V = -\frac{g_{m2} r_{o2}}{R_S + (1 + g_{m2} R_S) r_{o2}} \cdot R_D \parallel \left[(1 + g_{m2} r_{o2}) R_S + r_{o2}\right]$$