EECE488: Analog CMOS Integrated Circuit Design

Set 2: Background

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Technical contributions of Pedram Lajevardi in revising the slides is greatly acknowledged.

Overview

- 1. Reading Assignments
- 2. Structure of MOS Transistors
- 3. Threshold Voltage
- 4. Long-Channel Current Equations
- 5. Regions of Operation
- 6. Transconductance
- 7. Second-Order Effects
- 8. Short-Channel Effects
- 9. MOS Layout
- 10. Device Capacitances
- 11. Small-signal Models
- 12. Circuit Impedance
- 13. Equivalent Transconductance

Reading Assignments

Reading:

Chapter 2 of the textbook

Section 16.2 of the textbook

Chapter 17

All the figures in the lecture notes are © *Design of Analog CMOS Integrated Circuits*, McGraw-Hill, 2001, unless otherwise noted.

Transistor

- Transistor stands for ...
- Transistor are semiconductor devices that can be classified as
 - Bipolar Junction Transistors (BJTs)
 - Field Effect Transistors (FETs)
 - Depletion-Mode FETs or (e.g., JFETs)
 - Enhancement-Mode FETs (e.g., MOSFETs)

Simplistic Model

MOS transistors have three terminals: Gate, Source, and Drain

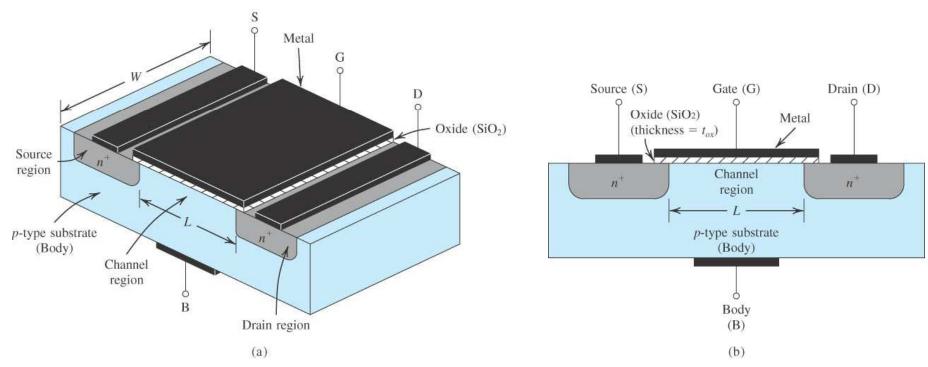


- The voltage of the Gate terminal determines the type of connection between Source and Drain (Short or Open).
- Thus, MOS devices behave like a switch

	NMOS	PMOS
V _G high	Device is ON	Device is OFF
	D is shorted to S	D & S are disconnected
V_{G} low	Device is OFF	Device is ON
	D & S are disconnected	D is shorted to S

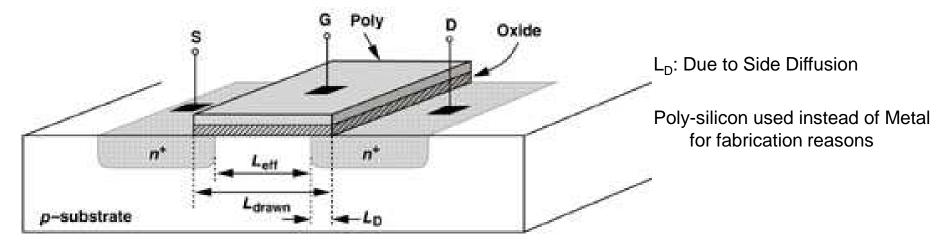
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- Source and Drain terminals are identical except that Source provides charge carriers, and Drain receives them.
- MOS devices have in fact 4 terminals:
 - Source, Drain, Gate, Substrate (bulk)



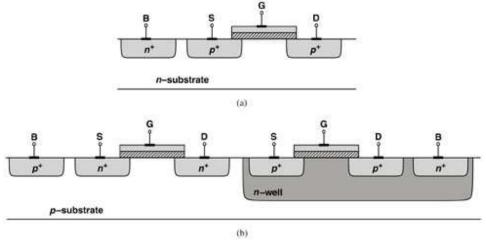
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- Charge Carriers are electrons in NMOS devices, and holes in PMOS devices.
- Electrons have a higher mobility than holes
- So, NMOS devices are faster than PMOS devices
- We rather to have a p-type substrate?!



Actual length of the channel (L_{eff}) is less than the length of gate

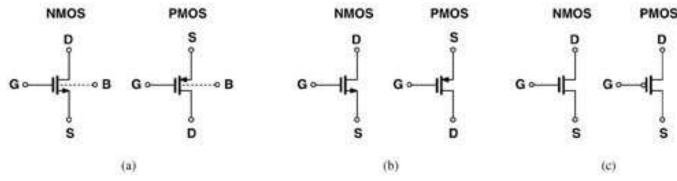
 N-wells allow both NMOS and PMOS devices to reside on the same piece of die.



- As mentioned, NMOS and PMOS devices have 4 terminals: Source, Drain, Gate, Substrate (bulk)
- In order to have all PN junctions reverse-biased, substrate of NMOS is connected to the most negative voltage, and substrate of PMOS is connected to the most positive voltage.

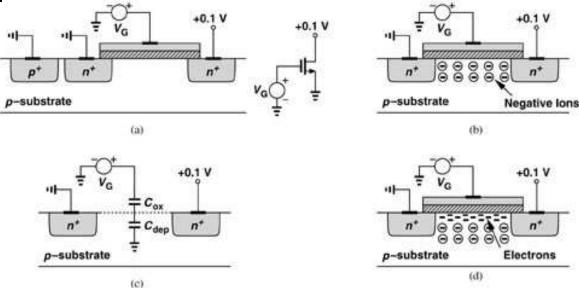
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MOS transistor Symbols:



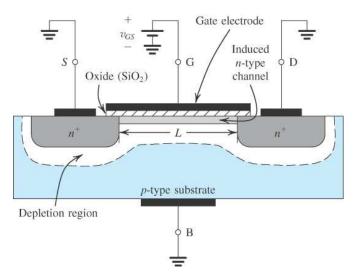
- In NMOS Devices: Source
 — electron
 — Drain
 Current flows from Drain to Source
- In PMOS Devices: $Source \xrightarrow{hole} Drain$ Current flows from Source to Drain
- Current flow determines which terminal is Source and which one is Drain. Equivalently, source and drain can be determined based on their relative voltages.

 Consider an NMOS: as the gate voltage is increased, the surface under the gate is depleted. If the gate voltage increases more, free electrons appear under the gate and a conductive channel is formed.



- (a) An NMOS driven by a gate voltage, (b) formation of depletion region, (c) onset of inversion, and (d) channel formation
- As mentioned before, in NMOS devices charge carriers in the channel under the gate are electrons.

- Intuitively, the threshold voltage is the gate voltage that forces the interface (surface under the gate) to be completely depleted of charge (in NMOS the interface is as much n-type as the substrate is p-type)
- Increasing gate voltage above this threshold (denoted by V_{TH} or V_t) induces an inversion layer (conductive channel) under the gate.



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Analytically:

$$V_{TH} = \Phi_{MS} + 2 \cdot \left| \Phi_F \right| + \frac{\left| Q_{dep} \right|}{C_{ox}}$$

Where:

SM

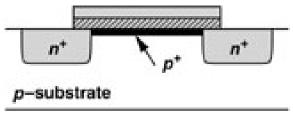
 Φ_{MS} = Built - in Potential = $\Phi_{gate} - \Phi_{Silicon}$

= the difference between the work functions of the polysilicon gate and the silicon substrate

$$\Phi_{F} = \text{Work Function (electrostatic potential)} = \frac{K \cdot T}{q} \cdot \ln \left(\frac{N_{sub}}{n_{i}} \right)$$

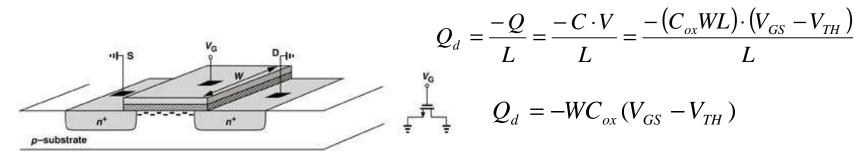
$$Q_{\tiny dep}$$
 = Charge in the depletion region = $\sqrt{4 \cdot q \cdot \mathcal{E}_{\tiny si} \cdot |\Phi_{\tiny F}| \cdot N_{\tiny sub}}$

- In practice, the "native" threshold value may not be suited for circuit design, e.g., V_{TH} may be zero and the device may be on for any positive gate voltage.
- Typically threshold voltage is adjusted by ion implantation into the channel surface (doping P-type material will increase V_{TH} of NMOS devices).

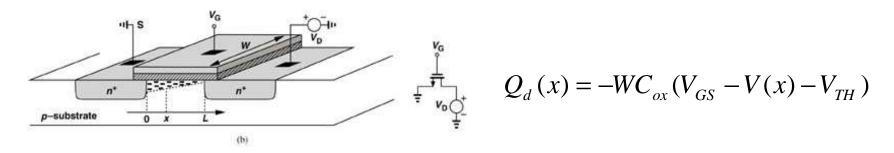


- When V_{DS} is zero, there is no horizontal electric field present in the channel, and therefore no current between the source to the drain.
- When V_{DS} is more than zero, there is some horizontal electric field which causes a flow of electrons from source to drain.

- The voltage of the surface under the gate, V(x), depends on the voltages of Source and Drain.
- If V_{DS} is zero, $V_D = V_S = V(x)$. The charge density Q_d (unit C/m) is uniform.



• If V_{DS} is not zero, the channel is tapered, and V(x) is not constant. The charge density depends on x.



Current:

$$I = \frac{dQ}{dt} = \frac{dQ}{dx} \times \frac{dx}{dt} = Q_d \cdot velocity$$

 \triangleright Velocity in terms of V(x):

velocity =
$$\mu \cdot E$$
, $E = -\frac{dV}{dt}$
 $\rightarrow velocity = (\mu \cdot \frac{-dV(x)}{dx})$

 \triangleright Q_d in terms of V(x):

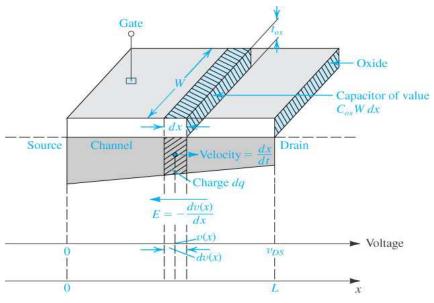
$$Q_d(x) = -WC_{ox}(V_{GS} - V(x) - V_{TH})$$

Current in terms of V(x):

$$\begin{split} I_{D} &= WC_{ox}[V_{GS} - V(x) - V_{TH}]\mu_{n} \frac{dV(x)}{dx} \\ &\int\limits_{x=0}^{L} I_{D} dx = \int\limits_{V=0}^{V_{DS}} WC_{ox} \mu_{n} [V_{GS} - V(x) - V_{TH}] dV \end{split}$$

Long-channel current equation:

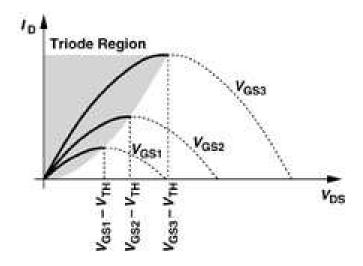
$$I_{D} = \mu_{n} C_{ox} \frac{W}{L} [(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^{2}]$$



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- If $V_{DS} \le V_{GS} V_{TH}$ we say the device is operating in triode (or linear) region.
- Current in Triode Region:

$$I_D = \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot \left[\left(V_{GS} - V_{TH} \right) \cdot V_{DS} - \frac{1}{2} \cdot V_{DS}^2 \right]$$



• Terminology:

$$Aspect\ Ratio = \frac{W}{L}$$

 $Overdrive \quad Voltage = Effective \quad Voltage = V_{GS} - V_{TH} = V_{eff}$

• For very small V_{DS} (deep Triode Region):

 I_D can be approximated to be a linear function of V_{DS} .

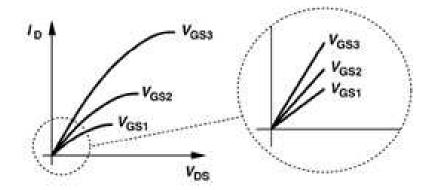
The device resistance will be independent of V_{DS} and will only depend on V_{eff} .

The device will behave like a variable resistor

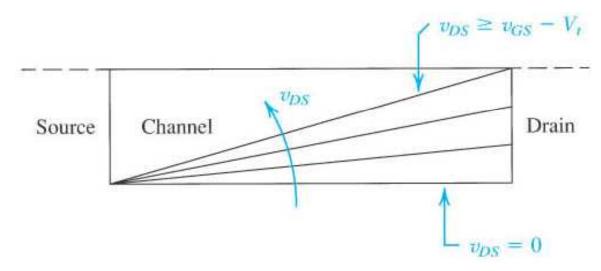
$$s \stackrel{\stackrel{G}{\longrightarrow}}{\longrightarrow} D \Leftrightarrow s \stackrel{V_{GS}}{\longrightarrow} D$$

If
$$V_{DS} \ll 2(V_{GS} - V_{TH})$$
:
$$I_D = \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH}) \cdot V_{DS}$$

$$R_{ON} = \frac{V_{DS}}{I_D} = \frac{1}{\mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH})}$$



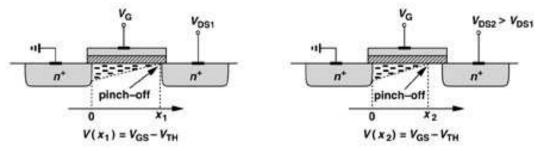
• Increasing V_{DS} causes the channel to acquire a tapered shape. Eventually, as V_{DS} reaches $V_{GS} - V_{TH}$ the channel is pinched off at the drain. Increasing V_{DS} above $V_{GS} - V_{TH}$ has little effect (ideally, no effect) on the channel's shape.



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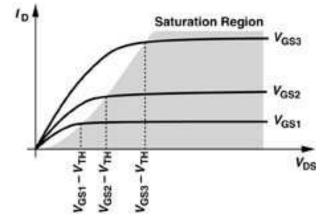
• When V_{DS} is more than $V_{GS} - V_{TH}$ the channel is pinched off, and the horizontal electric field produces a current.

• If $V_{DS} > V_{GS} - V_{TH}$, the transistor is in saturation (active) region, and the channel is pinched off.



$$\int_{x=0}^{L'} I_D dx = \int_{V=0}^{V_{GS} - V_{TH}} W C_{ox} \mu_n [V_{GS} - V(x) - V_{TH}] dV$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{U} (V_{GS} - V_{TH})^2$$



- Let's, for now, assume that L'=L. The fact that
 L' is not equal to L is a second-order effect
 known as channel-length modulation.
- Since I_D only depends on V_{GS}, MOS transistors in saturation can be used as current sources.

Current Equation for NMOS:

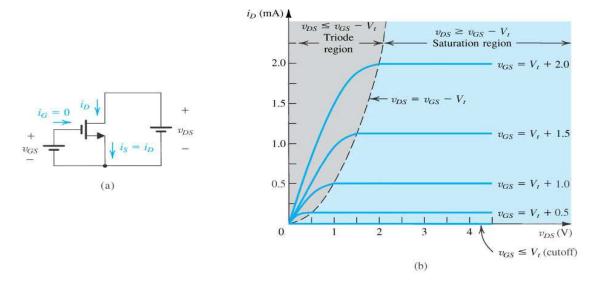
$$I_{D} = I_{DS} = \begin{cases} 0 & ; if \ V_{GS} < V_{TH} \ (Cut - off) \\ \\ \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot \left(V_{GS} - V_{TH}\right) \cdot V_{DS} & ; if \ V_{GS} > V_{TH} \ , V_{DS} << 2(V_{GS} - V_{TH}) \ (Deep \ Triode) \\ \\ \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot \left[\left(V_{GS} - V_{TH}\right) \cdot V_{DS} - \frac{1}{2} \cdot V_{DS}^{2}\right] \ ; if \ V_{GS} > V_{TH} \ , V_{DS} < V_{GS} - V_{TH} \ (Triode) \\ \\ \frac{1}{2} \cdot \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH})^{2} \ ; if \ V_{GS} > V_{TH} \ , V_{DS} > V_{GS} - V_{TH} \ (Saturation) \end{cases}$$

Current Equation for PMOS:

$$I_{D} = I_{SD} = \begin{cases} 0 & ; if \ V_{SG} < \left|V_{TH}\right| \ (Cut - off) \\ \\ \mu_{p} \cdot C_{ox} \cdot \frac{W}{L} \cdot \left(V_{SG} - \left|V_{TH}\right|\right) \cdot V_{SD} & ; if \ V_{SG} > \left|V_{TH}\right|, V_{SD} << 2(V_{SG} - \left|V_{TH}\right|) \ (Deep \ Triode) \\ \\ \mu_{p} \cdot C_{ox} \cdot \frac{W}{L} \cdot \left(\left(V_{SG} - \left|V_{TH}\right|\right) \cdot V_{SD} - \frac{1}{2} \cdot V_{SD}^{2}\right) & ; if \ V_{SG} > \left|V_{TH}\right|, V_{SD} < V_{SG} - \left|V_{TH}\right| \ (Triode) \\ \\ \frac{1}{2} \cdot \mu_{p} \cdot C_{ox} \cdot \frac{W}{L} \cdot \left(V_{SG} - \left|V_{TH}\right|\right)^{2} & ; if \ V_{SG} > \left|V_{TH}\right|, V_{SD} > V_{SG} - \left|V_{TH}\right| \ (Saturation) \end{cases}$$

Regions of Operation:

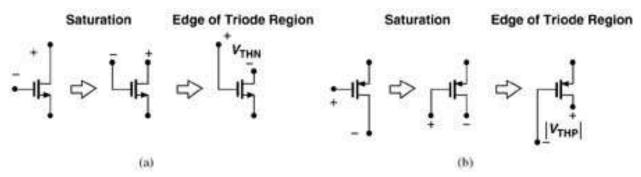
Cut-off, triode (linear), and saturation (active or pinch-off)



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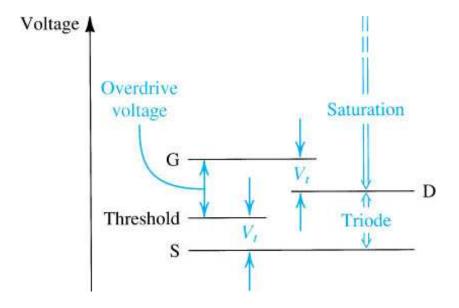
 Once the channel is pinched off, the current through the channel is almost constant. As a result, the I-V curves have a very small slope in the pinch-off (saturation) region, indicating the large channel resistance.

 The following illustrates the transition from pinch-off to triode region for NMOS and PMOS devices.



- For NMOS devices:
 - If V_D increases (V_G Const.), the device will go from Triode to Pinch-off. If V_G increases (V_D Const.), the device will go from Pinch-off to Triode.
- ** In NMOS, as V_{DG} increases the device will go from Triode to Pinch-off.
- For PMOS devices:
 - If V_D decreases (V_G Const.), the device will go from Triode to Pinch-off. If V_G decreases (V_D Const.), the device will go from Pinch-off to Triode.
- ** In PMOS, as V_{GD} increases the device will go from Pinch-off to Triode.

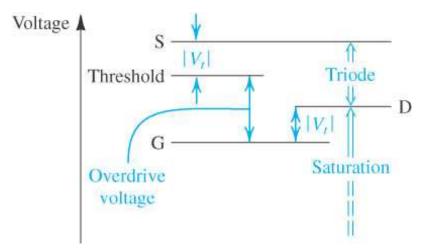
NMOS Regions of Operation:



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 Relative levels of the terminal voltages of the enhancement-type NMOS transistor for different regions of operation.

PMOS Regions of Operation:



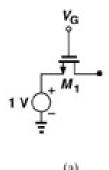
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 The relative levels of the terminal voltages of the enhancement-type PMOS transistor for different regions of operation.

Example:

For the following circuit assume that $V_{TH}=0.7V$.

When is the device on?



What is the region of operation if the device is on?

Sketch the on-resistance of transistor M₁ as a function of V_G.

Transconductance - 1

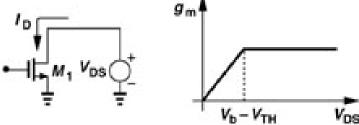
- The drain current of the MOSFET in saturation region is ideally a function of gate-overdrive voltage (effective voltage). In reality, it is also a function of V_{DS}.
- It makes sense to define a figure of merit that indicates how well the device converts the voltage to current.
- Which current are we talking about?
- What voltage is in the designer's control?
- What is this figure of merit?

$$g_{m} = \frac{\partial I_{D}}{\partial V_{GS}} \bigg|_{V_{DS}} = Const.$$

Transconductance - 2

Example:

Plot the transconductance of the following circuit as a function of V_{DS} (assume V_b is a constant voltage).



Transconductance in triode:

$$g_{m} = \frac{\partial}{\partial V_{GS}} \left(\mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot \left[\left(V_{GS} - V_{TH} \right) \cdot V_{DS} - \frac{1}{2} \cdot V_{DS}^{2} \right] \right) \Big|_{V_{DS}} = Const.$$

$$= \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot V_{DS}$$

Transconductance in saturation:

$$g_{m} = \frac{\partial}{\partial V_{GS}} \left(\frac{1}{2} \cdot \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH})^{2} \right) \Big|_{V_{DS}} = Const.$$

$$= \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH})$$

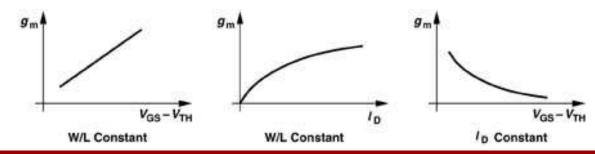
Moral: Transconductance drops if the device enters the triode region.

Transconductance - 3

• Transconductance, g_m, in saturation:

$$g_{m} = \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH}) = \sqrt{2\mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot I_{D}} = \frac{2 \cdot I_{D}}{V_{GS} - V_{TH}}$$

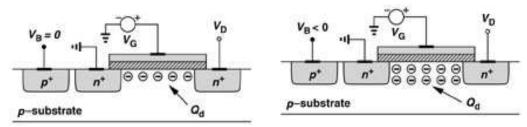
- If the aspect ratio is constant: g_m depends linearly on (V_{GS} V_{TH}).
 Also, g_m depends on square root of I_D.
- If I_D is constant: g_m is inversely proportional to (V_{GS} V_{TH}).
 Also, g_m depends on square root of the aspect ratio.
- If the overdrive voltage is constant: g_m depends linearly on I_D.
 Also, g_m depends linearly on the aspect ratio.



Second-Order Effects (Body Effect)

Substrate Voltage:

- So far, we assumed that the bulk and source of the transistor are at the same voltage (V_B=V_S).
- If V_B > V_s, then the bulk-source PN junction will be forward biased, and the device will not operate properly.
- If V_B < V_s,
 - the bulk-source PN junction will be reverse biased.
 - $-\,$ the depletion region widens, and Q_{dep} increases.
 - V_{TH} will be increased (Body effect or Backgate effect).



• It can be shown that (what is the unit for γ ?):

$$V_{TH} = V_{TH\,0} + \gamma \cdot \left(\sqrt{\left| 2 \cdot \Phi_F + V_{SB} \right|} - \sqrt{\left| 2 \cdot \Phi_F \right|} \right) \text{ where } \gamma = \frac{\sqrt{2 \cdot q \cdot \varepsilon_{si} \cdot N_{sub}}}{C_{ox}}$$

Body Effect - 2

Example:

Consider the circuit below (assume the transistor is in the active region):

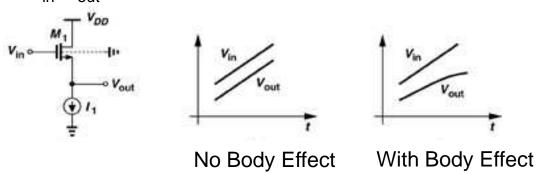
If body-effect is ignored, V_{TH} will be constant, and I₁ will only depend on V_{GS1}=V_{in}-V_{out}. Since I₁ is constant, V_{in}-V_{out} remains constant.

$$V_{in} - V_{out} - V_{TH} = C = Const. \rightarrow V_{in} - V_{out} = V_{TH} + C = D = Conts.$$

• In general, I_1 depends on V_{GS1} - $V_{TH} = V_{in}$ - V_{out} - V_{TH} (and with body effect V_{TH} is not constant). Since I_1 is constant, V_{in} - V_{out} - V_{TH} remains constant:

$$V_{in} - V_{out} - V_{TH} = C = Const. \rightarrow V_{in} - V_{out} = V_{TH} + C$$

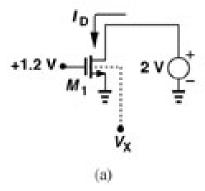
• As V_{out} increases, V_{SB1} increases, and as a result V_{TH} increases. Therefore, V_{in} - V_{out} Increases.



Body Effect - 3

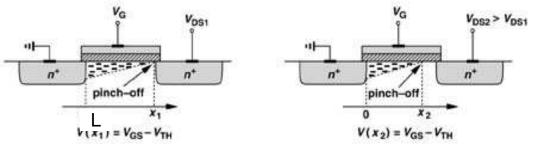
Example:

For the following Circuit sketch the drain current of transistor M_1 when V_X varies from - ∞ to 0. Assume V_{TH0} =0.6V, γ =0.4V^{1/2}, and $2\Phi_F$ =0.7V.



Channel Length Modulation - 1

• When a transistor is in the saturation region $(V_{DS} > V_{GS} - V_{TH})$, the channel is pinched off.



• The drain current is $I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L'} (V_{GS} - V_{TH})^2$ where $L' = L - \Delta L$

$$\frac{1}{L'} = \frac{1}{L - \Delta L} = \frac{1}{L} \cdot \frac{1}{1 - \Delta L/L} \approx \frac{1}{L} \cdot \left(1 + \frac{\Delta L}{L}\right)$$

- Assuming $\Delta L/_{L} = \lambda \cdot V_{DS}$ we get: $\frac{1}{L'} \approx \frac{1}{L} \cdot \left(1 + \Delta L/_{L}\right) = \frac{1}{L} \cdot \left(1 + \lambda \cdot V_{DS}\right)$
- The drain current is $I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L'} (V_{GS} V_{TH})^2 \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} V_{TH})^2 \cdot (1 + \lambda \cdot V_{DS})$
- As I_D actually depends on both V_{GS} and V_{DS}, MOS transistors are not ideal current sources (why?).

Channel Length Modulation - 2

- λ represents the relative variation in effective length of the channel for a given increment in V_{DS} .
- For longer channels λ is smaller, i.e., $\lambda \propto 1/L$
- Transconductance: $g_m = \frac{\partial I_D}{\partial V_{GS}} \Big|_{V_{DS} = Const.}$

In Triode:

$$g_m = \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot V_{DS}$$

In Saturation (ignoring channel length modulation):

$$g_m = \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH}) = \sqrt{2\mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot I_D} = \frac{2 \cdot I_D}{V_{GS} - V_{TH}}$$

In saturation with channel length modulation:

$$g_{m} = \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH}) \cdot (1 + \lambda \cdot V_{DS}) = \sqrt{2\mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot I_{D} \cdot (1 + \lambda \cdot V_{DS})} = \frac{2 \cdot I_{D}}{V_{CS} - V_{TH}}$$

The dependence of I_D on V_{DS} is much weaker than its dependence on V_{GS}.

Channel Length Modulation - 3

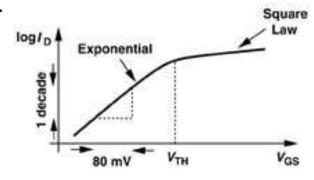
Example:

Given all other parameters constant, plot I_D - V_{DS} characteristic of an NMOS for $L=L_1$ and $L=2L_1$

- In Triode Region: $I_D \approx \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot \left[\left(V_{GS} V_{TH} \right) \cdot V_{DS} \frac{1}{2} \cdot V_{DS}^2 \right]$ Therefore: $\frac{\partial I_D}{\partial V_{DS}} \approx \frac{W}{L}$
- In Saturation Region: $I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} V_{TH})^2 \cdot (1 + \lambda \cdot V_{DS})$ So we get: $\frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \cdot \lambda$ Therefore: $\frac{\partial I_D}{\partial V_{DS}} \approx \frac{W \cdot \lambda}{L} \approx \frac{W}{L^2}$
- Changing the length of the device from L₁ to 2L₁ will flatten the I_D-V_{DS} curves (slope will be divided by two in triode and by four in saturation).
- Increasing L will make a transistor a better current source, while degrading its current capability.
- Increasing W will improve the current capability.

Sub-threshold Conduction

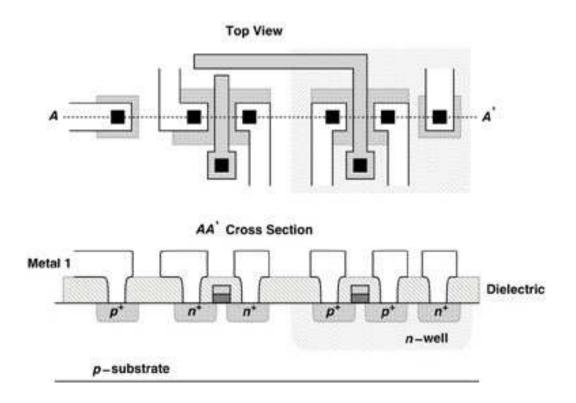
- If V_{GS} < V_{TH}, the drain current is not zero.
- The MOS transistors behave similar to BJTs.
- In BJT: $I_C = I_S \cdot e^{\frac{V_{BE}}{V_T}}$



- In MOS: $I_D = I_0 \cdot e^{\frac{V_{GS}}{\zeta \cdot V_T}}$
- As shown in the figure, in MOS transistors, the drain current drops by one decade for approximately each 80mV of drop in V_{GS}.
- In BJT devices the current drops faster (one decade for approximately each 60mv of drop in V_{GS}).
- This current is known as sub-threshold or weak-inversion conduction.

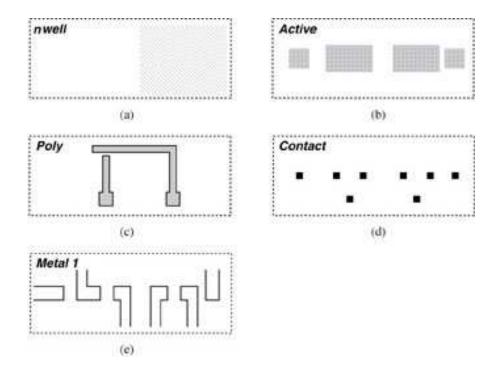
CMOS Processing Technology

Top and side views of a typical CMOS process



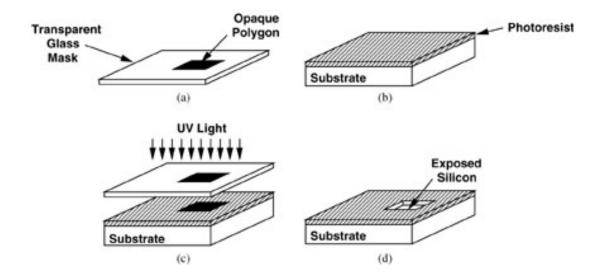
CMOS Processing Technology

Different layers comprising CMOS transistors



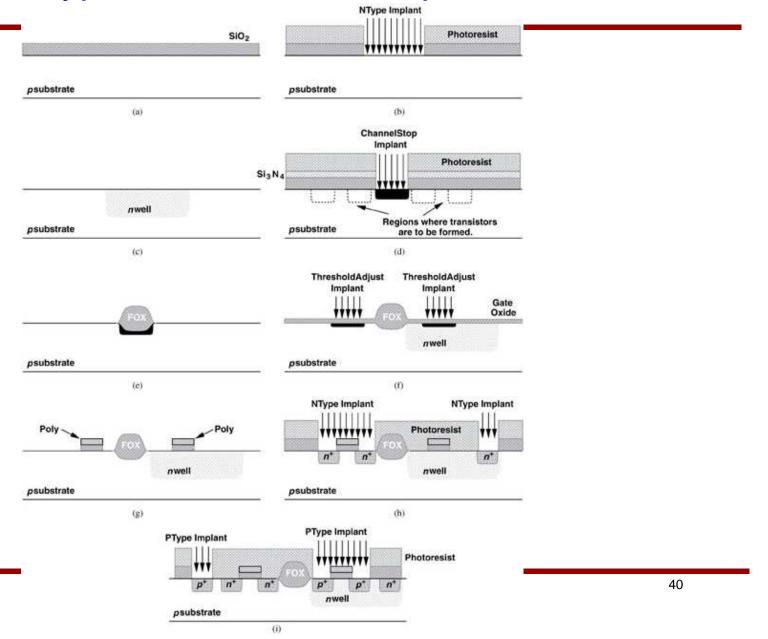
Photolithography (Lithography)

Used to transfer circuit layout information to the wafer



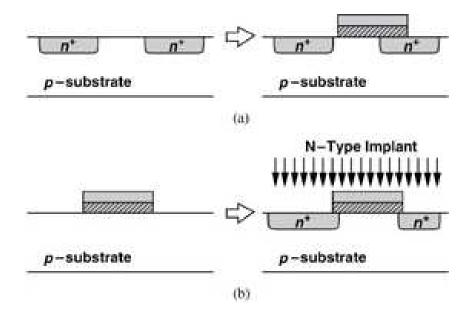
Typical Fabrication Sequence

SM



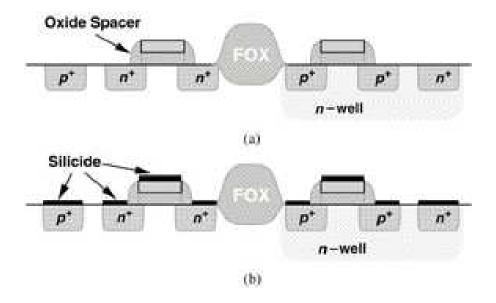
Self-Aligned Process

 Why source and drain junctions are formed after the gate oxide and polysilicon layers are deposited?



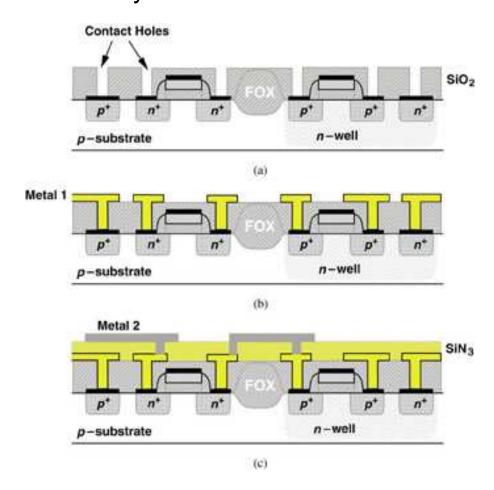
Back-End Processing

Oxide spacers and silicide



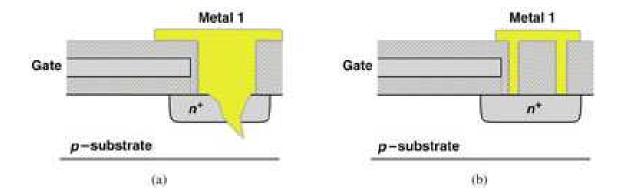
Back-End Processing

Contact and metal layers fabrication



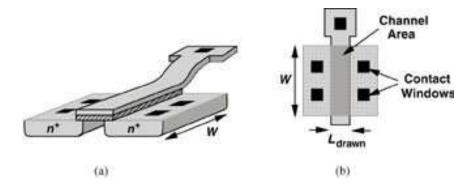
Back-End Processing

Large contact areas should be avoided to minimize the possibility of spiking



MOS Layout - 1

It is beneficial to have some insight into the layout of the MOS devices.

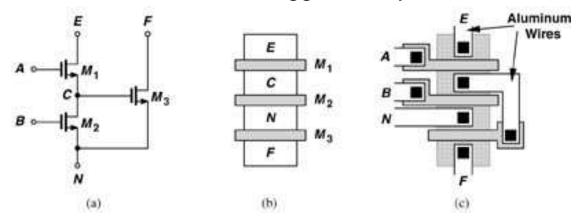


- When laying out a design, there are many important parameters we need to pay attention to such as: drain and source areas, interconnects, and their connections to the silicon through contact windows.
- Design rules determine the criteria that a circuit layout must meet for a given technology. Things like, minimum length of transistors, minimum area of contact windows, ...

MOS Layout - 2

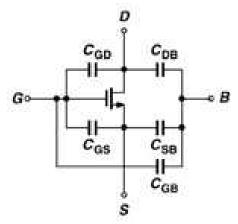
Example:

Figures below show a circuit with a suggested layout.



• The same circuit can be laid out in different ways, producing different electrical parameters (such as different terminal capacitances).

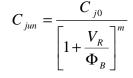
- The quadratic model determines the DC behavior of a MOS transistor.
- The capacitances associated with the devices are important when studying the AC behavior of a device.
- There is a capacitance between any two terminals of a MOS transistor.
 So there are 6 Capacitances in total.
- The Capacitance between Drain and Source is negligible (C_{DS}=0).

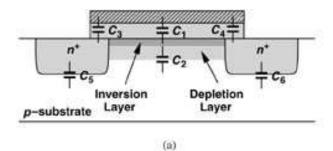


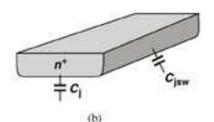
These capacitances will depend on the region of operation (Bias values).

- The following will be used to calculate the capacitances between terminals:
 - 1. Oxide Capacitance: $C_1 = W \cdot L \cdot C_{ox}$, $C_{ox} = \frac{\mathcal{E}_{ox}}{t_{ox}}$
 - 2. Depletion Capacitance: $C_2 = C_{dep} = W \cdot L \cdot \sqrt{\frac{q \cdot \varepsilon_{si} \cdot N_{sub}}{4 \cdot \Phi_F}}$
 - 3. Overlap Capacitance: $C_3 = C_4 = C_{ov} = W \cdot L_D \cdot C_{ox} + C_{fringe}$
 - 4. Junction Capacitance:
 - Sidewall Capacitance: C_{jsw}
 - \triangleright Bottom-plate Capacitance: C_j

$$C_5 = C_6 = C_i + C_{isw}$$





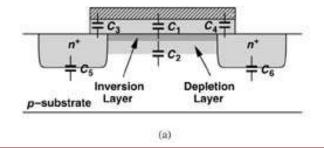


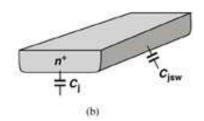
In Cut-off:

- 1. C_{GS} : is equal to the overlap capacitance. $C_{GS} = C_{ov} = C_3$
- 2. C_{GD} : is equal to the overlap capacitance. $C_{GD} = C_{ov} = C_4$
- 3. C_{GB} : is equal to $C_{gate-channel} = C_1$ in series with $C_{channel-bulk} = C_2$.
- 4. C_{SB}: is equal to the junction capacitance between source and bulk.
- C_{DB}: is equal to the junction capacitance between source and bulk.

$$C_{SB} = C_5$$

$$C_{DB} = C_6$$

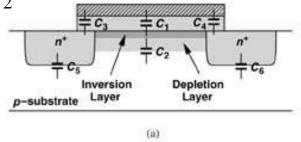


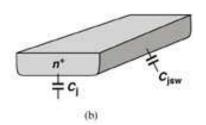


In Triode:

- The channel isolates the gate from the substrate. This means that if V_{G} changes, the charge of the inversion layer are supplied by the drain and source as long as V_{DS} is close to zero. So, C₁ is divided between gate and drain terminals, and gate and source terminals, and C₂ is divided between bulk and drain terminals, and bulk and source terminals.
 - 1. C_{GS} : $C_{GS} = C_{ov} + \frac{C_1}{2}$
 - 2. C_{GD} : $C_{GD} = C_{ov} + \frac{C_1}{C_1}$
 - 3. C_{GB} : the channel isolates the gate from the substrate. $C_{GB} = 0$

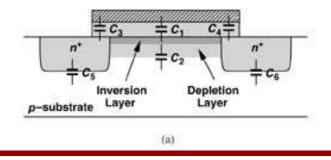
 - 4. C_{SB} : $C_{SB} = C_5 + \frac{C_2}{2}$ 5. C_{DB} : $C_{DB} = C_6 + \frac{C_2}{2}$

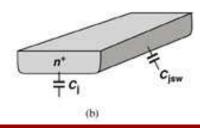




In Saturation:

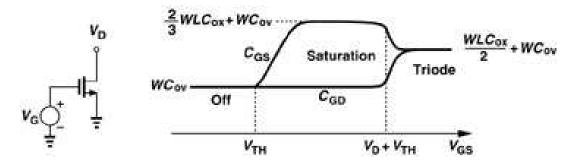
- The channel isolates the gate from the substrate. The voltage across the channel varies which can be accounted for by adding two equivalent capacitances to the source. One is between source and gate, and is equal to two thirds of C₁. The other is between source and bulk, and is equal to two thirds of C_2 .
 - 1. C_{GS} : $C_{GS} = C_{ov} + \frac{2}{3}C_1$
 - 2. C_{GD} : $C_{GD} = C_{ov}$
 - 3. C_{GB} : the channel isolates the gate from the substrate. $C_{GB} = 0$
 - 4. C_{SB} : $C_{SB} = C_5 + \frac{2}{3}C_2$ 5. C_{DB} : $C_{DB} = C_6$





In summary:

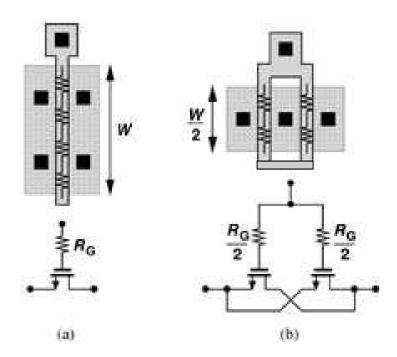
	Cut-off	Triode	Saturation
C _{GS}	C_{ov}	$C_{ov} + \frac{C_1}{2}$	$C_{ov} + \frac{2}{3}C_1$
$C_{\sf GD}$	C_{ov}	$C_{ov} + \frac{C_1}{2}$	C_{ov}
C_{GB}	$\frac{C_1 \cdot C_2}{C_1 + C_2} \langle C_{GB} \langle C_1$	0	0
C_{SB}	C_5	$C_5 + \frac{C_2}{2}$	$C_5 + \frac{2}{3}C_2$
C _{DB}	C_6	$C_6 + \frac{C_2}{2}$	C_6



Importance of Layout

Example (Folded Structure):

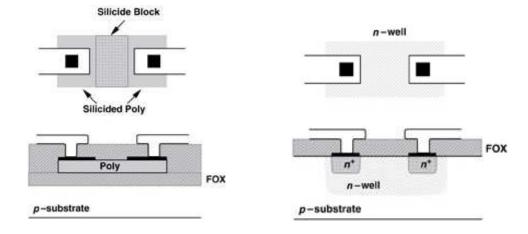
Calculate the gate resistance of the circuits shown below.



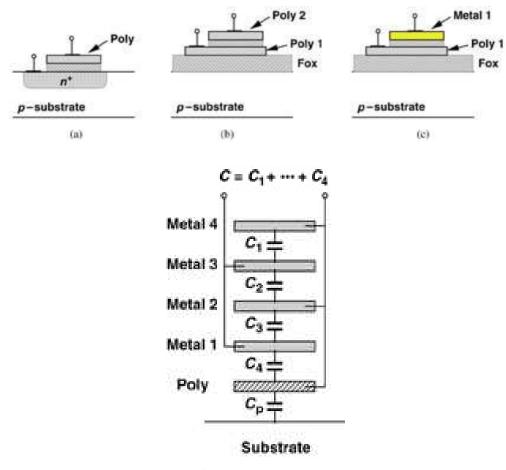
Folded structure:

- Decreases the drain capacitance
- Decreases the gate resistance
- Keeps the aspect ratio the same

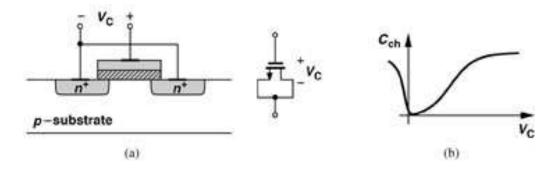
Resistors

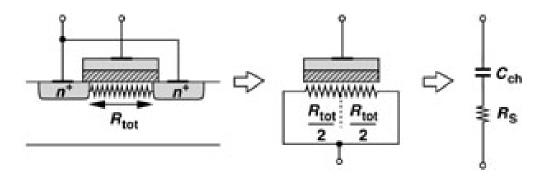


Capacitors:

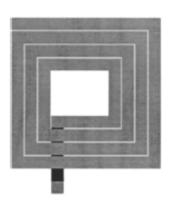


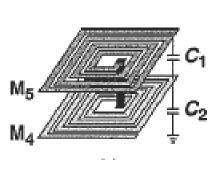
Capacitors

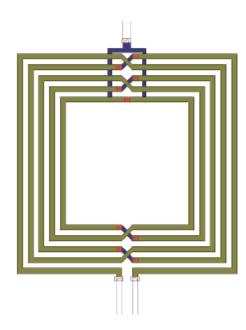




Inductors

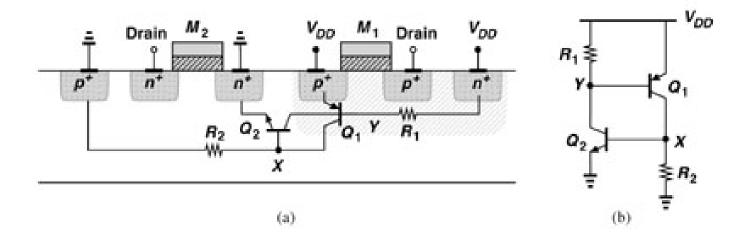






Latch-Up

Due to parasitic bipolar transistors in a CMOS process



- Small signal model is an approximation of the large-signal model around the operation point.
- In analog circuits most MOS transistors are biased in saturation region.
- In general, I_D is a function of V_{GS} , V_{DS} , and V_{BS} . We can use this Taylor series approximation:

$$\begin{aligned} & \text{Taylor Expansion}: I_{D} = I_{D0} + \frac{\partial I_{D}}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial I_{D}}{\partial V_{DS}} \cdot \Delta V_{DS} + \frac{\partial I_{D}}{\partial V_{BS}} \cdot \Delta V_{BS} + \text{second order terms} \\ & \Delta I_{D} \approx \frac{\partial I_{D}}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial I_{D}}{\partial V_{DS}} \cdot \Delta V_{DS} + \frac{\partial I_{D}}{\partial V_{BS}} \cdot \Delta V_{BS} = g_{m} \cdot \Delta V_{GS} + \frac{\Delta V_{DS}}{r_{o}} + g_{mb} \cdot \Delta V_{BS} \end{aligned}$$

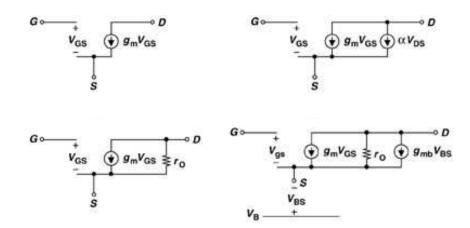
- Current in Saturation: $I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L'} (V_{GS} V_{TH})^2 \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} V_{TH})^2 \cdot (1 + \lambda \cdot V_{DS})$
- Taylor approximation: $\Delta I_D \approx \frac{\partial I_D}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial I_D}{\partial V_{DS}} \cdot \Delta V_{DS} + \frac{\partial I_D}{\partial V_{BS}} \cdot \Delta V_{BS}$
- Partial Derivatives:

$$\begin{split} \frac{\partial I_{D}}{\partial V_{GS}} &= \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH}) \cdot \left(1 + \lambda \cdot V_{DS}\right) = g_{m} \\ \frac{\partial I_{D}}{\partial V_{DS}} &= \frac{1}{2} \cdot \mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH})^{2} \cdot \lambda \approx I_{D} \cdot \lambda = \frac{1}{r_{o}} \\ \frac{\partial I_{D}}{\partial V_{BS}} &= \frac{\partial I_{D}}{\partial V_{TH}} \cdot \frac{\partial V_{TH}}{\partial V_{BS}} = \left[-\mu_{n} \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH}) \cdot \left(1 + \lambda \cdot V_{DS}\right) \right] \cdot \left[-\frac{\gamma}{2\sqrt{|2 \cdot \Phi_{F} + V_{SB}|}} \right] \\ &= -g_{m} \cdot \left[-\frac{\gamma}{2\sqrt{|2 \cdot \Phi_{F} + V_{SB}|}} \right] = g_{m} \cdot \eta = g_{mb} \end{split}$$

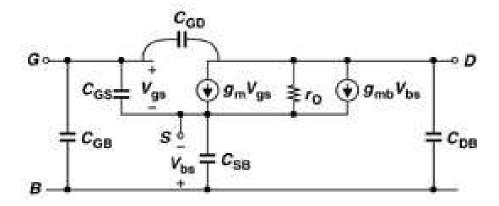
Small-Signal Model:

$$i_D = g_m \cdot v_{GS} + \frac{v_{DS}}{r_o} + g_{mb} \cdot v_{BS}$$

- Terms, $g_m v_{GS}$ and $g_{mb} v_{BS}$, can be modeled by dependent sources. These terms have the same polarity: increasing v_G , has the same effect as increasing v_B .
- The term, v_{DS}/r_o can be modeled using a resistor as shown below.

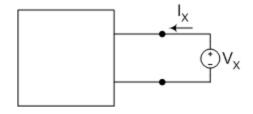


Complete Small-Signal Model with Capacitances:



- Small signal model including all the capacitance makes the intuitive (qualitative) analysis of even a few-transistor circuit difficult!
- Typically, CAD tools are used for accurate circuit analysis
- For intuitive analysis we try to find a simplest model that can represent the role of each transistor with reasonable accuracy.

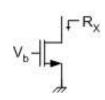
- It is often useful to determine the impedance of a circuit seen from a specific pair of terminals.
- The following is the recipe to do so:
 - 1. Connect a voltage source, V_X , to the port.
 - 2. Suppress all independent sources.
 - 3. Measure or calculate I_X .



$$R_{X} = \frac{V_{X}}{I_{X}}$$

Example:

- Find the small-signal impedance of the following current sources.
- $V_b = \bigcup_{t=R_X}^{V_{dd}}$
- We draw the small-signal model, which is the same for both circuits, and connect a voltage source as shown below:

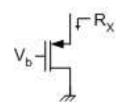


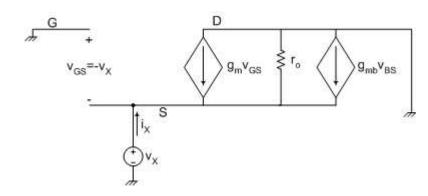
$$i_{x} = \frac{v_{x}}{r_{o}} + g_{m} \cdot v_{GS} = \frac{v_{x}}{r_{o}}$$

$$R_{x} = \frac{v_{x}}{i_{x}} = r_{o}$$

Example:

- Find the small-signal impedance of the following circuits.
- We draw the small-signal model, which is the same for both circuits, and connect a voltage source as shown below:



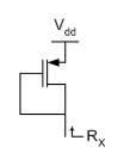


$$i_{X} = \frac{v_{X}}{r_{o}} - g_{m} \cdot v_{GS} - g_{mb} \cdot v_{BS} = \frac{v_{X}}{r_{o}} + g_{m} \cdot v_{X} + g_{mb} \cdot v_{X}$$

$$R_{x} = \frac{v_{x}}{i_{x}} = \frac{1}{\frac{1}{r_{o}} + g_{m} + g_{mb}} = r_{o} \left\| \frac{1}{g_{m}} \right\| \frac{1}{g_{mb}}$$

Example:

Find the small-signal impedance of the following circuit. This
circuit is known as the diode-connected load, and is used
frequently in analog circuits.



 We draw the small-signal model and connect the voltage source as shown below:

$$i_{x} = \frac{v_{x}}{r_{o}} + g_{m} \cdot v_{GS} = \frac{v_{x}}{r_{o}} + g_{m} \cdot v_{x} = v_{x} \cdot \left(\frac{1}{r_{o}} + g_{m}\right)$$

$$R_{x} = \frac{v_{x}}{i_{x}} = \frac{1}{\frac{1}{r} + g_{m}} = r_{o} \left\| \frac{1}{g_{m}} \right\|$$

$$G \downarrow V_{x}$$

$$V_{GS} = v_{x}$$

$$V_{GS} = v_{x}$$

$$V_{GS} = v_{x}$$

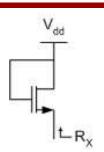
$$V_{GS} = v_{x}$$

• If channel length modulation is ignored $(r_0 = \infty)$ we get:

$$R_{X} = r_{o} \left\| \frac{1}{g_{m}} = \infty \right\| \frac{1}{g_{m}} = \frac{1}{g_{m}}$$

Example:

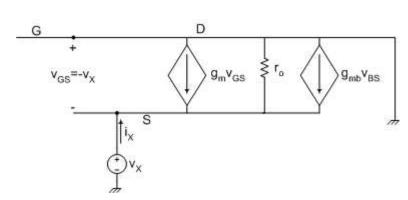
• Find the small-signal impedance of the following circuit. This circuit is a diode-connected load with body effect.



$$i_{X} = \frac{v_{X}}{r_{o}} - g_{m} \cdot v_{GS} - g_{mb} \cdot v_{BS} = \frac{v_{X}}{r_{o}} + g_{m} \cdot v_{X} + g_{mb} \cdot v_{X}$$

$$= v_{X} \cdot \left(\frac{1}{r_{o}} + g_{m} + g_{mb}\right)$$

$$R_{x} = \frac{v_{x}}{i_{x}} = \frac{1}{\frac{1}{r_{o}} + g_{m} + g_{mb}} = r_{o} \left\| \frac{1}{g_{m} + g_{mb}} = r_{o} \right\| \frac{1}{g_{m}} \left\| \frac{1}{g_{mb}} \right\|$$



• If channel length modulation is ignored $(r_0=\infty)$ we get:

$$R_{X} = r_{o} \left\| \frac{1}{g_{m} + g_{mb}} = \infty \right\| \frac{1}{g_{m} + g_{mb}} = \frac{1}{g_{m} + g_{mb}} = \frac{1}{g_{m}} \left\| \frac{1}{g_{mb}} \right\|$$

Equivalent Transconductance - 1

 Recall that the transconductance of a transistor was a a figure of merit that indicates how well the device converts a voltage to current.

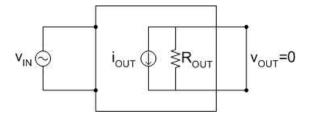
$$g_{m} = \frac{\partial I_{D}}{\partial V_{GS}} \bigg|_{V_{DS}} = Const.$$

 It is sometimes useful to define the equivalent transconductance of a circuit as follows:

$$G_{m} = \frac{\partial I_{OUT}}{\partial V_{IN}} \Big|_{V_{OUT}} = Const.$$

 The following is a small-signal block diagram of an arbitrary circuit with a Norton equivalent at the output port. We notice that: V_{OUT}=Constant so v_{OUT}=0 in the small signal model.

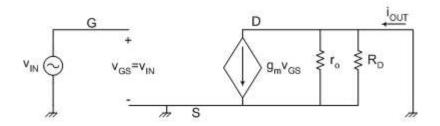
$$G_{\scriptscriptstyle m} = \frac{i_{\scriptscriptstyle OUT}}{v_{\scriptscriptstyle IN}} \bigg| v_{\scriptscriptstyle OUT} = 0$$

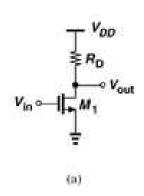


Equivalent Transconductance - 2

Example:

• Find the equivalent transconductance of an NMOS transistor in saturation from its small-signal model.





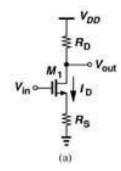
$$i_{OUT} = g_{m} \cdot v_{GS} = g_{m} \cdot v_{IN}$$

$$G_{m} = \frac{i_{OUT}}{v_{IN}} = g_{m}$$

Equivalent Transconductance - 3

Example:

 Find the equivalent transconductance of the following circuit when the NMOS transistor in saturation.



$$v_{IN} = v_{GS} + v_{S} = v_{GS} + i_{OUT} \cdot R_{S}$$

$$i_{OUT} = g_{m} \cdot v_{GS} + g_{mb} \cdot v_{BS} - \frac{v_{S}}{r_{o}} = g_{m} \cdot (v_{IN} - i_{OUT} \cdot R_{S}) + g_{mb} \cdot (-i_{OUT} \cdot R_{S}) - \frac{i_{OUT} \cdot R_{S}}{r_{o}}$$

$$i_{OUT} \cdot \left(1 + g_{m} \cdot R_{s} + g_{mb} \cdot R_{s} + \frac{R_{s}}{r_{o}}\right) = g_{m} \cdot v_{IN}$$

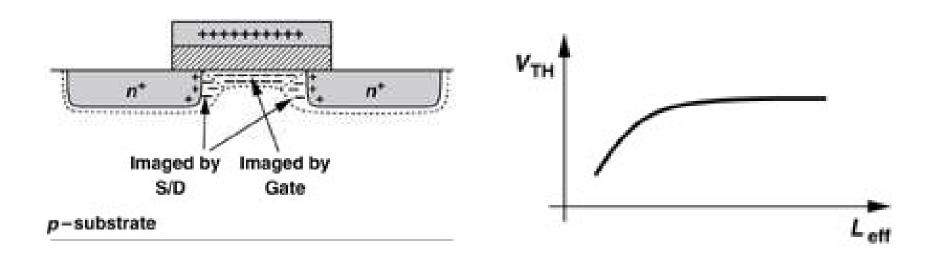
$$G_{m} = \frac{i_{OUT}}{v_{IN}} = \frac{g_{m} \cdot r_{O}}{1 + g_{m} \cdot R_{S} + g_{mb} \cdot R_{S} + \frac{R_{S}}{r_{O}}} = \frac{g_{m} \cdot r_{O}}{r_{O} + r_{O} \cdot (g_{m} \cdot R_{S} + g_{mb} \cdot R_{S}) + R_{S}}$$

Short-Channel Effects

- Threshold Reduction
 - Drain-induced barrier lowering (DIBL)
- Mobility degradation
- Velocity saturation
- Hot carrier effects
 - Substrate current
 - Gate current
- Output impedance variation

Threshold Voltage Variation in Short Channel Devices

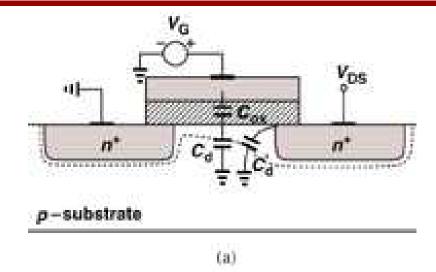
• The Threshold of transistors fabricated on the same chip decreases as the channel length decreases.

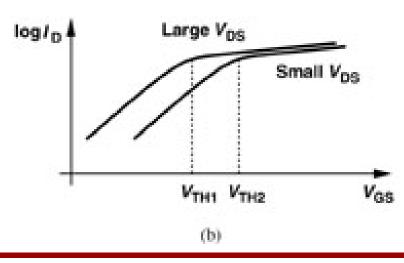


 Intuitively, the extent of depletion regions associated with drain and source in the channel area, reduces the immobile charge that must be imaged by the charge on the gate.

Drain-Induced Barrier Lowering (DIBL)

When the channel is short, the drain voltage increases the channel surface potential, lowering the barrier to flow charge from source (think of increased electric field) and therefore, decreasing the threshold.

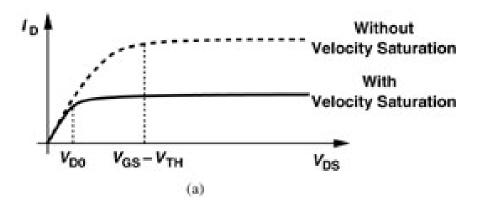


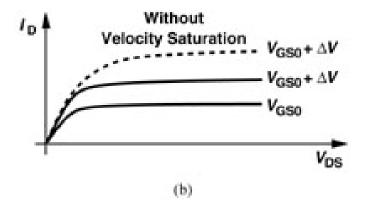


SM

Effects of Velocity Saturation

Due to drop in mobility at high electric fields





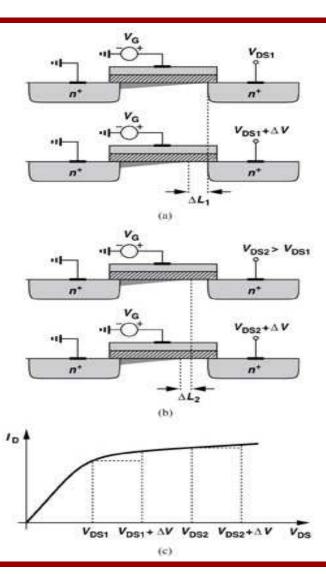
(a) Premature drain current saturation and (b) reduction in g_m

Hot Carrier Effects

- Short channel devices may experience high lateral drain-source electric field
- Some carriers that make it to drain have high velocity (called "hot" carriers)
- "Hot" carriers may "hit" silicon atoms at high speed and cause impact ionization
- The resulting electron and holes are absorbed by the drain and substrate causing extra drain-substrate current
- Really "hot" carriers may be injected into gate oxide and flow out of gate causing gate current!

Output Impedance Variation

Recall the definition of λ .



Output Impedance Variation in Short-Channel Devices

