EEC488: Analog CMOS Integrated Circuit Design

3. Single-Stage Amplifiers

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Overview

1. Why Amplifiers?
2. Amplifier Characteristics
3. Amplifier Trade-offs
4. Single-stage Amplifiers
5. Common Source Amplifiers
    1. Resistive Load
    2. Diode-connected Load
    3. Current Source Load
    4. Triode Load
    5. Source Degeneration

Overview

6. Common-Drain (Source-Follower) Amplifiers
    1. Resistive Load
    2. Current Source Load
    3. Voltage Division in Source Followers

7. Common-Gate Amplifiers

6. Cascode Amplifiers

Reading Assignments

- Reading:
  Chapter 3 of Razavi's book

- In this set of slides we will study low-frequency small-signal behavior of single-stage CMOS amplifiers. Although, we assume long-channel MOS models (not a good assumption for deep submicron technologies) the techniques discussed here help us to develop basic circuit intuition and to better understand and predict the behavior of circuits.

Why Amplifiers?

- Amplifiers are essential building blocks of both analog and digital systems.

- Amplifiers are needed for variety of reasons including:

  - To amplify a weak analog signal for further processing

  - To reduce the effects of noise of the next stage

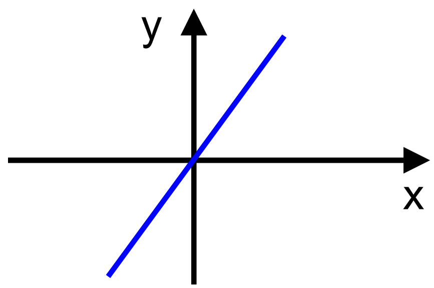
  - To provide a proper logical levels (in digital circuits)

- Amplifiers also play a crucial role in feedback systems

- We first look at the low-frequency performance of amplifiers. Therefore, all capacitors in the small-signal model are ignored!

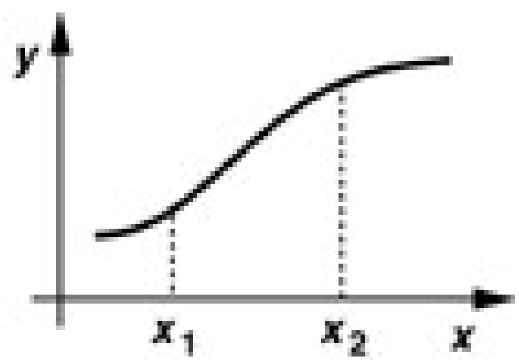
Amplifier Characteristics - 1

- Ideally we would like that the output of an amplifier be a linear function of the input, i.e., the input times a constant gain:



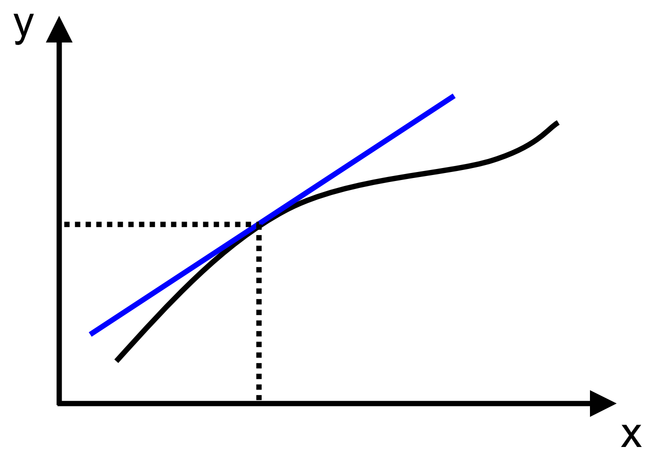
$$y = \alpha_1 x$$

- In real world the input-output characteristics is typically a nonlinear function:



Amplifier Characteristics - 2

- It is more convenient to use a linear approximation of a nonlinear function.
- Use the tangent line to the curve at the given (operating) point.



- The larger the signal changes about the operating point, the worse the approximation of the curve by its tangent line.

- This is why small-signal analysis is so popular!

Amplifier Characteristics - 3

- A well-behaved nonlinear function in the vicinity of a given point can be approximated by its corresponding Taylor series:

$$y \approx f(x_0) + f'(x_0)\cdot(x - x_0) + \frac{f''(x_0)}{2!}\cdot(x - x_0)^2 + \cdots + \frac{f^n(x_0)}{n!}\cdot(x - x_0)^n$$

- Let $\alpha_n = \dfrac{f^n(x_0)}{n!}$ to get:

$$y \approx \alpha_0 + \alpha_1(x - x_0) + \alpha_2(x - x_0)^2 + \cdots + \alpha_n(x - x_0)^n$$

- If $x-x_0=\Delta x$ is small, we can ignore the higher-order terms (hence the name small-signal analysis) to get:

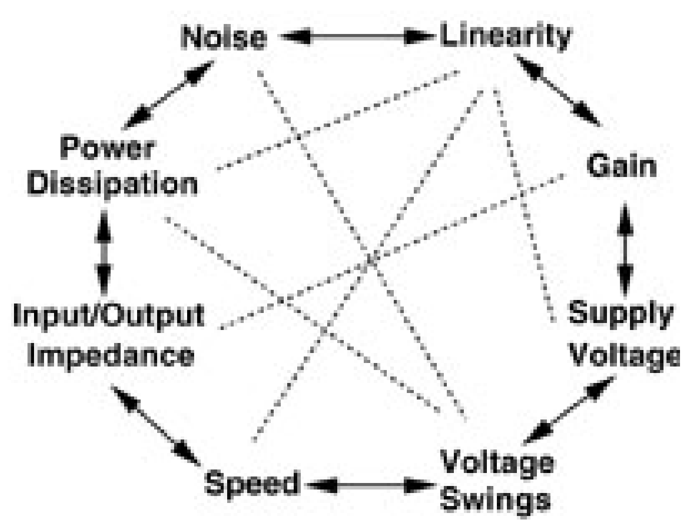
$$y \approx \alpha_0 + \alpha_1(x - x_0)$$

- α_0 is referred to as the operating (bias) point and α_1 is the small-signal gain.

$$\Delta y = y - f(x_0) = y - \alpha_0 \approx \alpha_1 \Delta x$$

Amplifier Trade-offs

- In practice, when designing an amplifier, we need to optimize for some performance parameters. Typically, these parameters trade performance with each other, therefore, we need to choose an acceptable compromise.



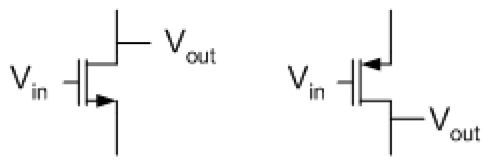
Single-Stage Amplifiers

- We will examine the following types of amplifiers:
  1. Common Source
  2. Common Drain (Source Follower)
  3. Common Gate
  4. Cascode and Folded Cascode

- Each of these amplifiers have some advantages and some disadvantages. Often, designers have to utilize a cascade combination of these amplifiers to meet the design requirements.

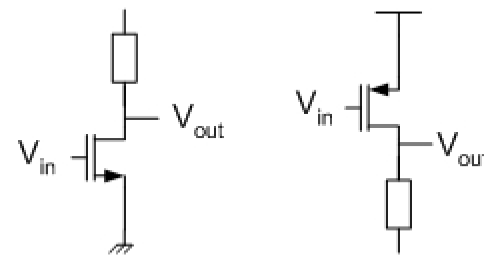
Common Source Basics - 1

- In common-source amplifiers, the input is (somehow!) connected to the gate and the output is (somehow!) taken from the drain.

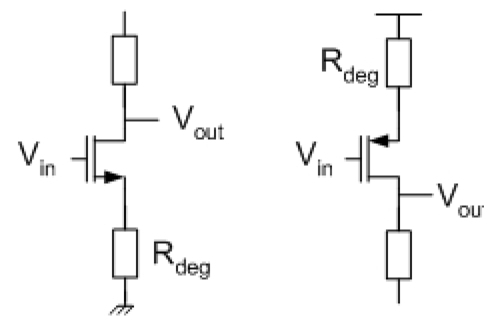


- We can divide common source amplifiers into two groups:

1. Without source degeneration (no body effect for the main transistor):

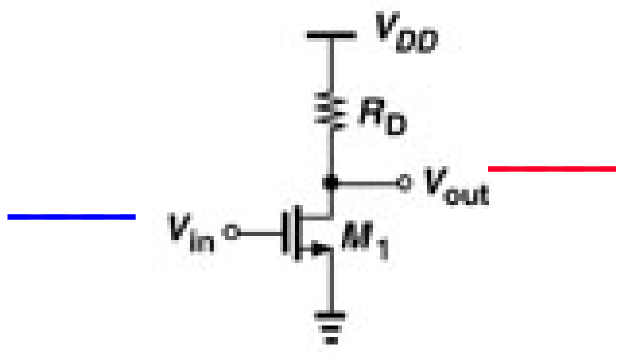


2. With source degeneration (have to take body effect into account for the main transistor):



Common Source Basics - 2

In a simple common source amplifier:



- gate voltage variations times g_m gives the drain current variations,
- drain current variations times the load gives the output voltage variations.
- Therefore, one can expect the small-signal gain to be:

$$|A_v| = g_m \cdot R_D$$

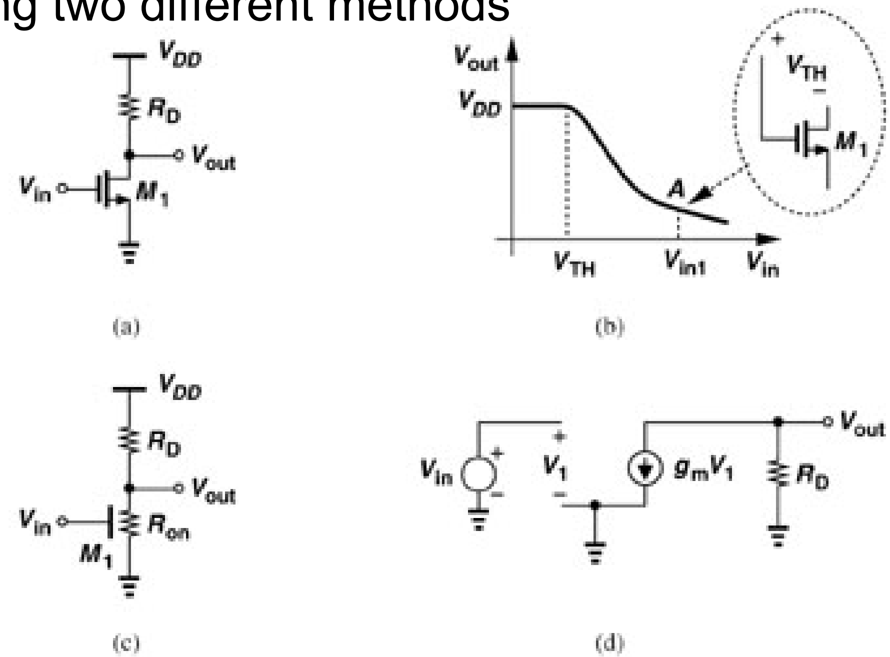
Common Source Basics - 3

- Different types of loads can be used in an amplifier:
  1. Resistive Load
  2. Diode-connected Load
  3. Current Source Load
  4. Triode Load

- The following parameters of amplifiers are very important:
  1. Small-signal gain
  2. Voltage swing

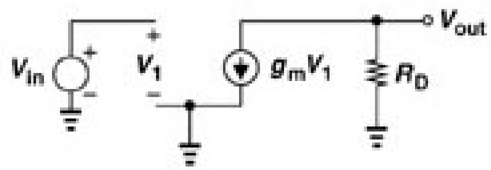
Resistive Load - 1

- Let's use a resistor as the load.
- The region of operation of M_1 depends on its size and the values of V_{in} and R.
- We are interested in the small-signal gain and the headroom (which determines the maximum voltage swing).
- We will calculate the gain using two different methods
  1. Small-signal model
  2. Large-signal analysis



Resistive Load - 2

Gain – Method 1: Small-Signal Model



(d)

- This is assuming that the transistor is in saturation, and channel length modulation is ignored.
- The current through R_D:

$$i_D = g_m \cdot v_{IN}$$

- Output Voltage:

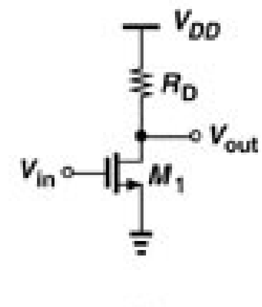
$$v_{OUT} = -i_D \cdot R_D = -g_m \cdot v_{IN} \cdot R_D$$

- Small-signal Gain:

$$A_v = \frac{v_{OUT}}{v_{IN}} = -g_m \cdot R_D$$

Resistive Load - 3

Gain – Method 2: Large-Signal Analysis



- If $V_{IN} < V_{TH}$, M1 is off, and $V_{OUT} = V_{DD} = V_{DS}$.

$$V_{OUT} = V_{DD} - R_D \cdot i_D = V_{DD}$$

$$A_v = \frac{\partial V_{OUT}}{\partial V_{IN}} = 0$$

- As V_{IN} becomes slightly larger than V_{TH}, M_1 turns on and goes into saturation ($V_{DS} \approx V_{DD} > V_{GS} - V_{TH} \approx 0$).

$$V_{OUT} = V_{dd} - R_D \cdot i_D = V_{dd} - R_D \cdot \frac{1}{2} \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH})^2$$

$$A_v = \frac{\partial V_{OUT}}{\partial V_{IN}} = -R_D \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH}) = -R_D \cdot g_m$$

- As V_{IN} increases, V_{DS} decreases, and M_1 goes into triode when $V_{IN} - V_{TH} = V_{OUT}$. We can find the value of V_{IN} that makes M_1 switch its region of operation.

$$V_{OUT} = V_{dd} - R_D \cdot i_D = V_{dd} - R_D \cdot \frac{1}{2} \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH})^2 = (V_{IN} - V_{TH})$$

Resistive Load - 4

Gain – Method 2: Large-Signal Analysis (Continued)

- As V_{IN} increases, V_{DS} decreases, and M_1 goes into triode.

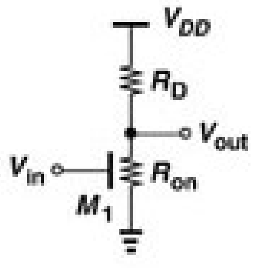
$$V_{OUT} = V_{DD} - R_D \cdot i_D = V_{DD} - R_D \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot \left[(V_{IN} - V_{TH}) \cdot V_{OUT} - \frac{V_{OUT}^2}{2} \right]$$

$$\frac{\partial V_{OUT}}{\partial V_{IN}} = -R_D \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot \left[(V_{IN} - V_{TH}) \cdot \frac{\partial V_{OUT}}{\partial V_{IN}} + V_{OUT} - V_{OUT} \cdot \frac{\partial V_{OUT}}{\partial V_{IN}} \right]$$

- We can find A_v from above. It will depend on both V_{IN} and V_{OUT}.
- If V_{IN} increases further, M_1 goes into deep triode if $V_{OUT} << 2(V_{IN} - V_{TH})$.

$$V_{OUT} = V_{DD} - R_D \cdot i_D = V_{DD} - R_D \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH}) \cdot V_{OUT}$$

$$V_{OUT} = \frac{V_{DD}}{1 + R_D \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH})} = \frac{V_{DD}}{1 + R_D \cdot \frac{1}{R_{ON}}} = V_{DD} \cdot \frac{R_{ON}}{R_{ON} + R_D}$$



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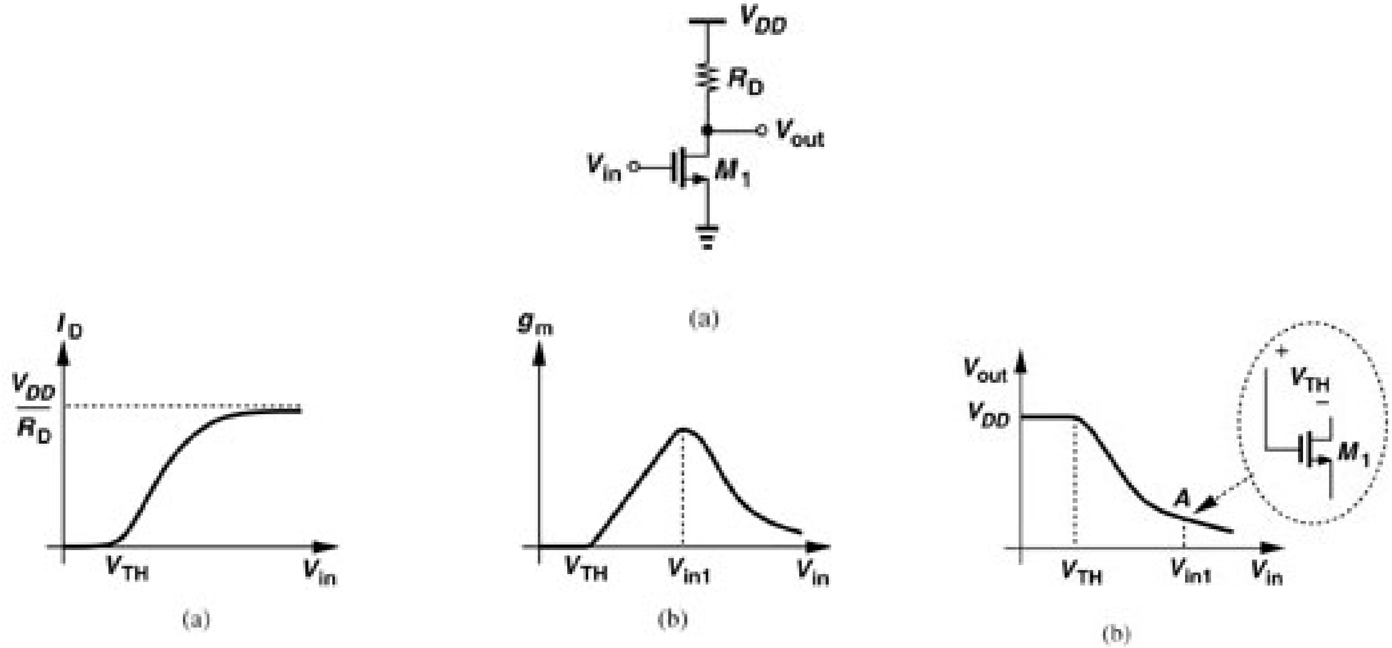
Resistive Load - 5

Example: Sketch the drain current and g_m of M_1 as a function of V_{IN}.



- g_m depends on V_{IN}, so if V_{IN} changes by a large amount the small-signal approximation will not be valid anymore.
- In order to have a linear amplifier, we don't want gain to depend on parameters like g_m which depend on the input signal.

Resistive Load - 6

- Gain of common-source amplifier:

$$A_v = -g_m \cdot R_D = -\mu_n C_{ox} \frac{W}{L}(V_{IN} - V_{TH}) \cdot \frac{V_{RD}}{I_D} = -\sqrt{2\mu_n C_{ox} \frac{W}{L}} \cdot \frac{V_{RD}}{\sqrt{I_D}} = \frac{-2 \cdot V_{RD}}{V_{eff}}$$

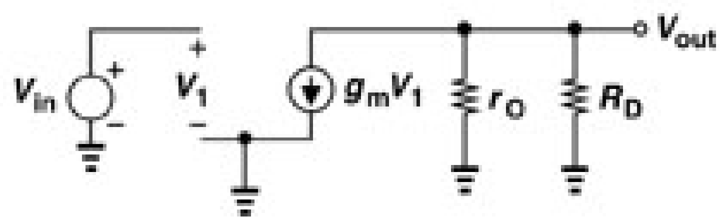
- To increase the gain:
  1. Increase g_m by increasing W or V_{IN} (DC portion or bias). Either way, I_D increases (more power) and V_{RD} increases, which limits the voltage swing.
  2. Increase R_D and keep I_D constant (g_m and power remain constant). But, V_{RD} increases which limits the voltage swing.
  3. Increase R_D and reduce I_D so V_{RD} remains constant.
     - ➢ If I_D is reduced by decreasing W, the gain will not change.
     - ➢ If I_D is reduced by decreasing V_{IN} (bias), the gain will increase. Since R_D is increased, the bandwidth becomes smaller (why?).
- Notice the trade-offs between gain, bandwidth, and voltage swings.

Resistive Load - 7

- Now let's consider the simple common-source circuit with channel length modulation taken into account.
- Channel length modulation becomes more important as R_D increases (in the next slide we will see why!).
- Again, we will calculate the gain in two different methods
  1. Small-signal Model
  2. Large Signal Analysis

Resistive Load - 8

Gain – Method 1: Small-Signal Model



- This is assuming that the transistor is in saturation.
- The current through R_D:

$$i_D = g_m \cdot v_{IN}$$

- Output Voltage:

$$v_{OUT} = -i_D \cdot (R_D \| r_o) = -g_m \cdot v_{IN} \cdot (R_D \| r_o)$$

- Small-signal Gain:

$$A_v = \frac{v_{OUT}}{v_{IN}} = -g_m \cdot (R_D \| r_o)$$

Resistive Load - 9

Gain – Method 2: Large-Signal Analysis

- As V_{IN} becomes slightly larger than V_{TH}, M_1 turns on and goes into saturation ($V_{DS} \approx V_{DD} > V_{GS} - V_{TH} \approx 0$).

$$V_{OUT} = V_{DD} - R_D \cdot I_D = V_{DD} - R_D \cdot \frac{1}{2} \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH})^2 \cdot (1 + \lambda \cdot V_{OUT})$$

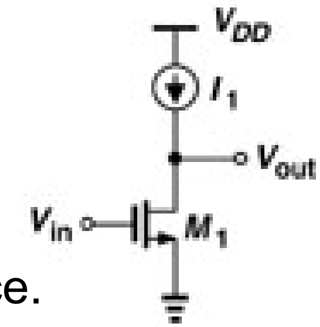
$$\frac{\partial V_{OUT}}{\partial V_{IN}} = -R_D \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH}) \cdot \left[(1 + \lambda \cdot V_{OUT}) + \frac{1}{2} \cdot (V_{IN} - V_{TH}) \cdot \lambda \cdot \frac{\partial V_{OUT}}{\partial V_{IN}} \right]$$

$$A_v = \frac{-R_D \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH}) \cdot (1 + \lambda \cdot V_{OUT})}{1 + \frac{1}{2} \cdot R_D \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH})^2 \cdot \lambda} = \frac{-R_D \cdot g_m}{1 + R_D \cdot I_D \cdot \lambda} = \frac{-R_D \cdot g_m}{1 + R_D \cdot \frac{1}{r_o}}$$

$$= \frac{-r_o \cdot R_D \cdot g_m}{r_o + R_D} = -g_m \cdot (R_D \| r_o)$$

Resistive Load - 10

Example:



- Assuming M_1 is biased in active region, what is the small-signal gain of the following circuit?
- I_1 is a current source and ideally has an infinite impedance.

$$A_v = \frac{v_{OUT}}{v_{IN}} = -g_m \cdot (\infty \| r_o) = -g_m \cdot r_o$$

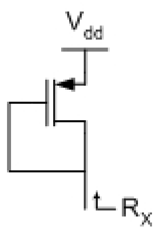
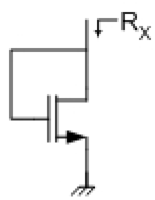
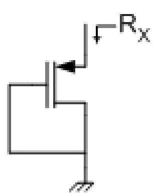
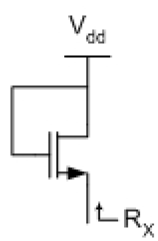
- This is the maximum gain of this amplifier (why?), and is known as the **intrinsic gain**.
- How can V_{IN} change if I_1 is constant?

$$I_D = \frac{1}{2} \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH})^2 \cdot (1 + \lambda \cdot V_{OUT})$$

- Here we have to take channel-length modulation into account. As V_{IN} changes, V_{OUT} also changes to keep I_1 constant.

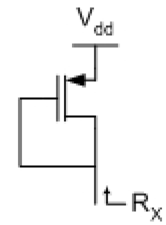
Diode Connected Load - 1

- Often, it is difficult to fabricate tightly controlled or reasonable size resistors on chip. So, it is desirable to replace the load resistor with a MOS device.
- Recall the diode connected devices:

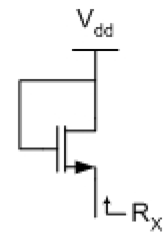
	Body Effect	R_X (when $\lambda \neq 0$)	R_X (when $\lambda = 0$)
 	NO	$R_X = r_o \Vert \dfrac{1}{g_m}$	$R_X = \dfrac{1}{g_m}$
 	YES	$R_X = r_o \Vert \dfrac{1}{g_m + g_{mb}}$	$R_X = \dfrac{1}{g_m + g_{mb}}$

Diode Connected Load - 2

- Now consider the common-source amplifier with two types of diode connected loads:
  1. PMOS diode connected load:
     (No body effect)



  2. NMOS diode connected load:
     (Body effect has to be taken into account)

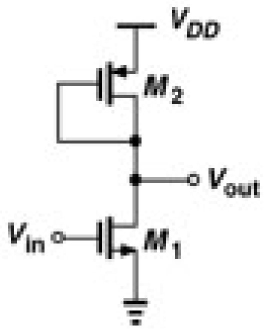


Diode Connected Load - 3

PMOS Diode Connected Load:

- Note that this is a common source configuration with M_2 being the load. We have:

$$A_v = \frac{v_{OUT}}{v_{IN}} = -g_{m1} \cdot \left(R_X \| r_{o1} \right) = -g_{m1} \cdot \left(\frac{1}{g_{m2}} \| r_{o2} \| r_{o1} \right)$$



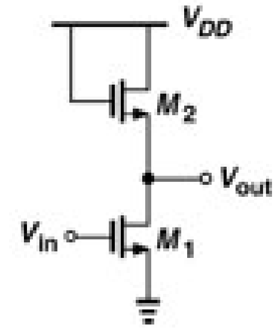
- Ignoring the channel length modulation ($r_{o1}=r_{o2}=\infty$), we can write:

$$A_v = -g_{m1} \cdot \left(\frac{1}{g_{m2}} \| \infty \| \infty \right) = -\frac{g_{m1}}{g_{m2}} = -\frac{\sqrt{2\mu_n \cdot C_{ox} \cdot \left(\frac{W}{L}\right)_1 \cdot I_{D1}}}{\sqrt{2\mu_p \cdot C_{ox} \cdot \left(\frac{W}{L}\right)_2 \cdot I_{D2}}} = -\sqrt{\frac{\mu_n \cdot \left(\frac{W}{L}\right)_1}{\mu_p \cdot \left(\frac{W}{L}\right)_2}}$$

$$A_v = -\frac{g_{m1}}{g_{m2}} = -\frac{\frac{2 \cdot I_{D1}}{V_{GS1} - V_{TH1}}}{\frac{2 \cdot I_{D2}}{V_{SG2} - |V_{TH2}|}} = -\frac{V_{SG2} - |V_{TH2}|}{V_{GS1} - V_{TH1}}$$

Diode Connected Load - 4

NMOS Diode Connected Load:



- Again, note that this is a common source configuration with M_2 being the load. We have:

$$A_v = \frac{v_{OUT}}{v_{IN}} = -g_{m1} \cdot \left(R_X \| r_{o1}\right) = -g_{m1} \cdot \left(\frac{1}{g_{m2} + g_{mb2}} \| r_{o2} \| r_{o1}\right)$$

- Ignoring the channel length modulation ($r_{o1}=r_{o2}=\infty$), we can write:

$$A_v = -g_{m1} \cdot \left(\frac{1}{g_{m2} + g_{mb2}} \| \infty \| \infty\right) = -\frac{g_{m1}}{g_{m2} + g_{mb2}} = -\frac{g_{m1}}{g_{m2} \cdot (1+\eta)}$$

$$A_v = -\frac{1}{1+\eta}\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} = -\frac{1}{1+\eta} \cdot \frac{V_{GS2} - V_{TH}}{V_{GS1} - V_{TH}}$$

Diode Connected Load - 5

- For a diode connected load we observe that (to the first order approximation):
  1. The amplifier gain is not a function of the bias current. So, the change in the input and output levels does not affect the gain, and the amplifier becomes more linear.

  2. The amplifier gain is not a function of the input signal (amplifier becomes more linear).

  3. The amplifier gain is a weak function (square root) of the transistor sizes. So, we have to change the dimensions by a considerable amount so as to increase the gain.

Diode Connected Load - 6

4. The gain of the amplifier is reduced when body effect should be considered.

5. We want M_1 to be in saturation, and M_2 to be on (M_2 cannot be in triode (why?)):

6. The voltage swing is constrained by both the required overdrive voltages and the threshold voltage of the diode connected device.

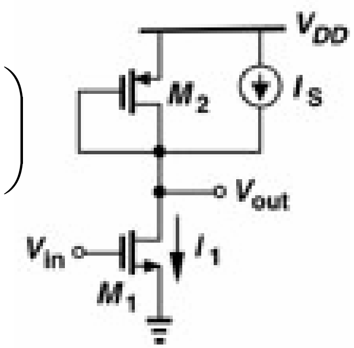
$$M1: V_{OUT} > V_{GS1} - V_{TH1} = V_{eff1}, \quad M2: V_{OUT} < V_{DD} - |V_{TH2}|$$

7. A high amplifier gain leads to a high overdrive voltage for the diode connected device which limits the voltage swing.

Diode Connected Load - 6

Example:

- Find the gain of the following circuit if M1 is biased in saturation and $I_s=0.75I_1$.



$$A_v=\frac{v_{OUT}}{v_{IN}}=-g_{m1}\cdot\left(R_X\|r_{Is}\|r_{o1}\right)=-g_{m1}\cdot\left(\frac{1}{g_{m2}}\|r_{o2}\|\infty\|r_{o1}\right)=-g_{m1}\cdot\left(\frac{1}{g_{m2}}\|r_{o2}\|r_{o1}\right)$$

- Ignoring the channel length modulation ($r_{o1}=r_{o2}=\infty$) we get:

$$A_v=-g_{m1}\cdot\left(\frac{1}{g_{m2}}\|\infty\|\infty\right)=-\frac{g_{m1}}{g_{m2}}=-\frac{\sqrt{2\mu_n\cdot C_{ox}\cdot\left(\frac{W}{L}\right)_1\cdot I_{D1}}}{\sqrt{2\mu_p\cdot C_{ox}\cdot\left(\frac{W}{L}\right)_2\cdot I_{D2}}}=-\frac{\frac{2\cdot I_{D1}}{V_{GS1}-V_{TH1}}}{\frac{2\cdot I_{D2}}{V_{SG2}-|V_{TH2}|}}$$

$$A_v=-2\cdot\sqrt{\frac{\mu_n\cdot\left(\frac{W}{L}\right)_1}{\mu_p\cdot\left(\frac{W}{L}\right)_2}}=-4\cdot\frac{V_{SG2}-|V_{TH2}|}{V_{GS1}-V_{TH1}}$$

Diode Connected Load - 7

Example (Continued):

- We observe for this example that:

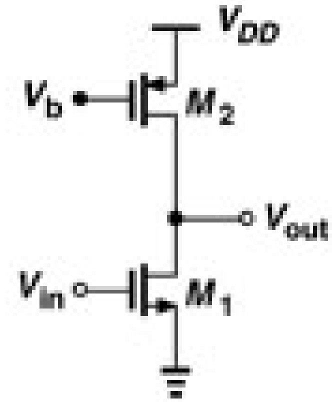
  1. For fixed transistor sizes, using the current source increases the gain by a factor of 2.

  2. For fixed overdrive voltages, using the current source increases the gain by a factor of 4.

  3. For a given gain, using the current source allows us to make the diode connected load 4 times smaller.

  4. For a given gain, using the current source allows us to make the overdrive voltage of the diode connected load 4 times smaller. This increases the headroom for voltage swing.

Current Source Load - 1



- Note that current source M_2 is the load.
-
- Recall that the output impedance of M_2 seen from V_{out}:

$$R_X = \frac{v_X}{i_X} = r_{o2}$$

$$A_v = \frac{v_{OUT}}{v_{IN}} = -g_{m1} \cdot (R_X \| r_{o1}) = -g_{m1} \cdot (r_{o2} \| r_{o1})$$

- For large gain at given power, we want large r_o and $r_o = \frac{1}{\lambda \cdot I_D} \propto \frac{1}{\frac{1}{L} \cdot \frac{W}{L}} = \frac{L^2}{W}$

  Increase L and W keeping the aspect ratio constant (so r_o increases and I_D remains constant). However, this approach increases the capacitance of the output node.
- We want M_2 to be in saturation so

$$V_{SD2} = V_{DD} - V_{OUT} > V_{SG2} - |V_{TH}| = V_{eff\,2} \rightarrow V_{OUT} < V_{DD} - V_{eff\,2}$$

Current Source Load - 2

- We also want M_1 to be in saturation:

$$V_{DS1} = V_{OUT} > V_{GS1} - V_{TH} = V_{eff1} \rightarrow V_{OUT} > V_{eff1}$$

- Thus, we want V_{eff1} and V_{eff2} to be small, so that there is more headroom for output voltage swing. For a constant I_D, we can increase W_1 and W_2 to reduce V_{eff1} and V_{eff2}.

- The intrinsic gain of this amplifier is: $A_v = -g_m \cdot r_o$
- In general, we have:

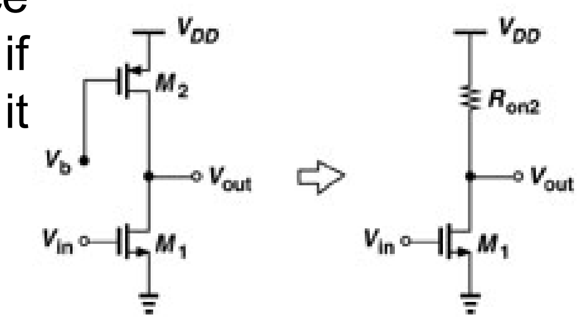
$$g_m \propto \frac{W}{L} \quad , \quad r_o \propto \frac{L^2}{W} \quad \rightarrow \quad A_v \propto L$$

- But since current in this case is roughly constant:

$$g_m = \sqrt{2\mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot I_D} \propto \sqrt{\frac{W}{L}} \quad , \quad r_o = \frac{1}{\lambda \cdot I_D} \propto L \quad \rightarrow \quad A_v \propto \sqrt{LW}$$

Triode Load

- We recognize that this is a common source configuration with M2 being the load. Recall that if M_2 is in deep triode, i.e., $V_{SD} << 2(V_{SG}-|V_{TH}|)$, it behaves like a resistor.



$$If\ V_{SD} << 2\left(V_{SG} - |V_{TH}|\right):$$

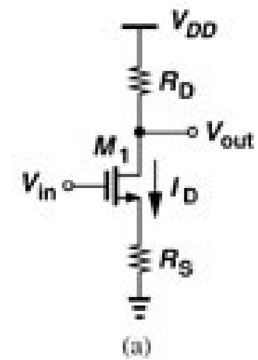
$$R_{ON2} = \frac{1}{\mu_p \cdot C_{ox} \cdot \frac{W}{L} \cdot \left(V_{SG} - |V_{TH}|\right)} = \frac{1}{\mu_p \cdot C_{ox} \cdot \frac{W}{L} \cdot \left(V_{dd} - V_b - |V_{TH}|\right)} \quad , \quad A_v = -g_{m1} \cdot \left(R_{ON2} \| r_{o1}\right)$$

- V_b should be low enough to make sure that M_2 is in deep triode region and usually requires additional complexity to be precisely generated.
- R_{ON2} depends on μ_p, C_{ox}, and V_{TH} which in turn depend on the technology being used.
- In general, this amplifier with triode load is difficult to design and use!

- However, compared to diode-connected load, triode load consumes less headroom: $M_1 : V_{OUT} > V_{GS1} - V_{TH} = V_{eff1}$, $M_2 : V_{OUT} \approx V_{DD}$

Source Degeneration - 1

- The following circuit shows a common source configuration with a degeneration resistor in the source.



- We will show that this configuration makes the common source amplifier more linear.

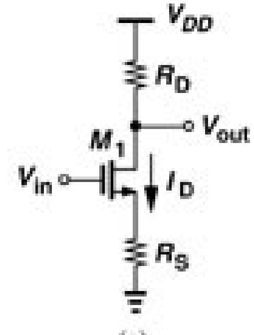
- We will use two methods to derive the gain of this circuit:
  1. Small-signal Model
  2. Using the following Lemma

Lemma:

In linear systems, the voltage gain is equal to $-G_m R_{out}$.

Source Degeneration - 2

Gain – Method 1: Small Signal Model



(a)

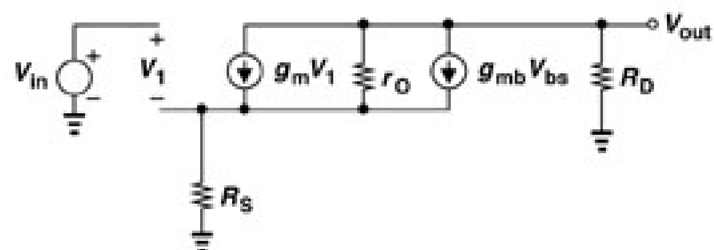
$$i_{OUT} = g_m \cdot v_1 + g_{mb} \cdot v_{BS} + \frac{v_{OUT} - i_{OUT} \cdot R_S}{r_O} \quad , \quad i_{OUT} = \frac{-v_{OUT}}{R_D}$$

$$v_1 = v_{IN} - i_{OUT} \cdot R_S = v_{IN} + \frac{v_{OUT}}{R_D} \cdot R_S \quad , \quad v_{BS} = -i_{OUT} \cdot R_S = \frac{v_{OUT}}{R_D} \cdot R_S$$

$$\frac{-v_{OUT}}{R_D} = g_m \cdot \left(v_{IN} + \frac{v_{OUT}}{R_D} \cdot R_S \right) + g_{mb} \cdot \left(\frac{v_{OUT}}{R_D} \cdot R_S \right) + \frac{v_{OUT} + \frac{v_{OUT}}{R_D} \cdot R_S}{r_O}$$

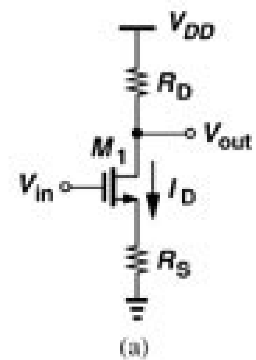
$$v_{OUT} \cdot \left(1 + g_m \cdot R_S + g_{mb} \cdot R_S + \frac{R_D}{r_O} + \frac{R_S}{r_O} \right) = -g_m \cdot v_{IN} \cdot R_D$$

$$A_v = \frac{v_{OUT}}{v_{IN}} = \frac{-g_m \cdot r_O \cdot R_D}{r_O \cdot \left(1 + \left(g_m + g_{mb}\right) \cdot R_S\right) + R_D + R_S}$$



Source Degeneration - 3

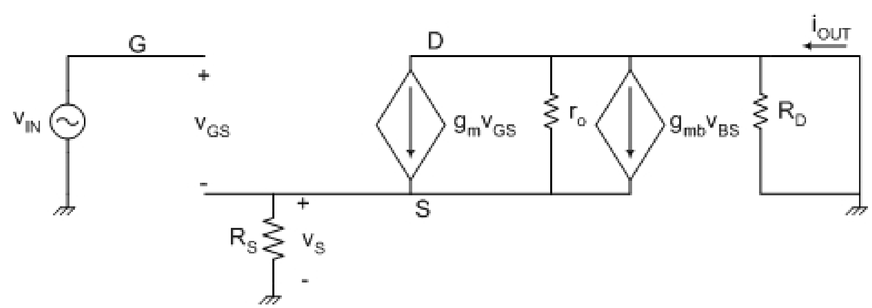
Gain – Method 2: Lemma



- The Lemma states that in linear systems, the voltage gain is equal to $-G_m R_{out}$. So we need to find G_m and R_{out}.

1. G_m:

   Recall that the equivalent transconductance of the above Circuit is:

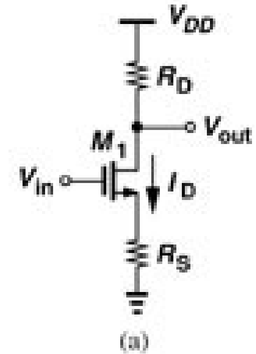


$$G_m = \frac{i_{OUT}}{v_{IN}} = \frac{g_m \cdot r_O}{r_O + r_O \cdot \left(g_m \cdot R_S + g_{mb} \cdot R_S\right) + R_S} = \frac{g_m \cdot r_O}{r_O[1 + \left(g_m + g_{mb}\right) \cdot R_S] + R_S}$$

Source Degeneration - 4

Gain – Method 2: Lemma (Continued)

1. R_{OUT}:
   We use the following small signal model to derive the small signal output impedance of this amplifier:



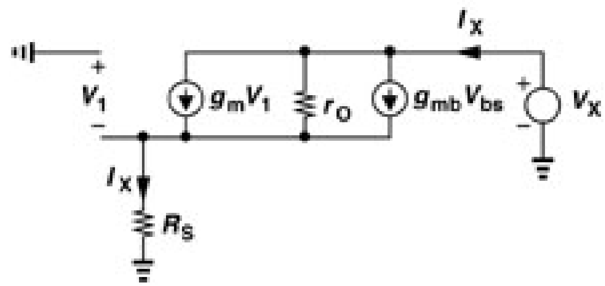
$$v_1 = -i_X \cdot R_S \quad , \quad v_{BS} = -i_X \cdot R_S$$

$$v_X = i_X \cdot R_S + \left(i_X - g_m \cdot v_1 - g_{mb} \cdot v_{BS}\right) \cdot r_O$$

$$= i_X \cdot R_S + \left(i_X - g_m \cdot \left(-i_X \cdot R_S\right) - g_{mb} \cdot \left(-i_X \cdot R_S\right)\right) \cdot r_O$$

$$R_X = \frac{v_X}{i_X} = R_S + \left(1 + g_m \cdot R_S + g_{mb} \cdot R_S\right) \cdot r_O = R_S + \left(1 + (g_m + g_{mb}) \cdot R_S\right) \cdot r_O$$

$$R_{OUT} = R_X \| R_D = \frac{\left(R_S + \left(1 + (g_m + g_{mb}) \cdot R_S\right) \cdot r_O\right) \cdot R_D}{\left(R_S + \left(1 + (g_m + g_{mb}) \cdot R_S\right) \cdot r_O\right) + R_D}$$

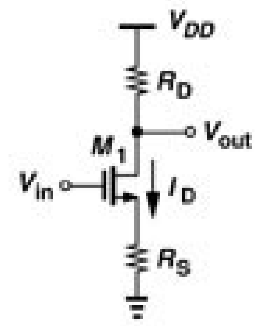


- Since typically $r_O >> R_S$:

$$R_X = R_S + \left(1 + (g_m + g_{mb}) \cdot R_S\right) \cdot r_O = \left(1 + (g_m + g_{mb}) \cdot R_S\right) \cdot r_O = \left(g_m + g_{mb}\right) \cdot R_S \cdot r_O$$

Source Degeneration - 5

Gain – Method 2: Lemma (Continued)



(a)

$$G_m = \frac{g_m \cdot r_O}{r_O(1+\cdot(g_m + g_{mb})\cdot R_S) + R_S}$$

$$R_{OUT} = \frac{(R_S + (1+(g_m + g_{mb})\cdot R_S)\cdot r_O)\cdot R_D}{R_S + (1+(g_m + g_{mb})\cdot R_S)\cdot r_O + R_D}$$

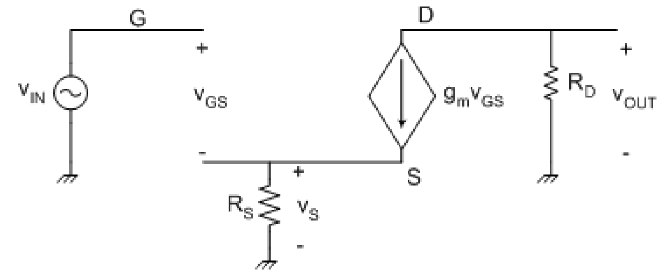
$$A_v = -G_m \cdot R_{OUT} = -\frac{g_m \cdot r_O}{r_O + r_O \cdot (g_m \cdot R_S + g_{mb} \cdot R_S) + R_S} \cdot \frac{(r_O + (1+g_m \cdot r_O + g_{mb} \cdot r_O)\cdot R_S)\cdot R_D}{r_O + (1+g_m \cdot r_O + g_{mb} \cdot r_O)\cdot R_S + R_D}$$

$$= \frac{-g_m \cdot r_O \cdot R_D}{r_O + (1+(g_m + g_{mb})\cdot R_S)\cdot r_O + R_D}$$

Source Degeneration - 6

- If we ignore body effect and channel-length modulation:

Method 1 – Small-signal Model:



$$v_{OUT} = -g_m \cdot v_{GS} \cdot R_D \quad , \quad v_{GS} = v_{IN} - g_m \cdot v_{GS} \cdot R_S$$

$$v_{GS} = v_{IN} \cdot \frac{1}{1 + g_m \cdot R_S} \quad \rightarrow \quad A_v = \frac{v_{OUT}}{v_{IN}} = \frac{-g_m \cdot R_D}{1 + g_m \cdot R_S}$$

Method 2 – Taking limits:

$$G_m = \lim_{\substack{r_O \to \infty \\ g_{mb} \to 0}} \frac{g_m \cdot r_O}{r_O + r_O \cdot (g_m \cdot R_S + g_{mb} \cdot R_S) + R_S} = \frac{g_m}{1 + (g_m + g_{mb}) \cdot R_S} = \frac{g_m}{1 + g_m \cdot R_S}$$

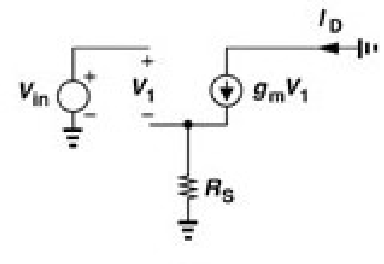
$$R_{OUT} = \lim_{\substack{r_O \to \infty \\ g_{mb} \to 0}} \frac{(r_O + (1 + g_m \cdot r_O + g_{mb} \cdot r_O) \cdot R_S) \cdot R_D}{r_O + (1 + g_m \cdot r_O + g_{mb} \cdot r_O) \cdot R_S + R_D} = \frac{(1 + (g_m + g_{mb}) \cdot R_S) \cdot R_D}{1 + (g_m + g_{mb}) \cdot R_S} = R_D$$

$$A_v = -G_m \cdot R_{OUT} = \frac{-g_m \cdot R_D}{1 + g_m \cdot R_S}$$

Source Degeneration - 7

Obtaining G_m and R_{out} directly assuming $\lambda=\gamma=0$:

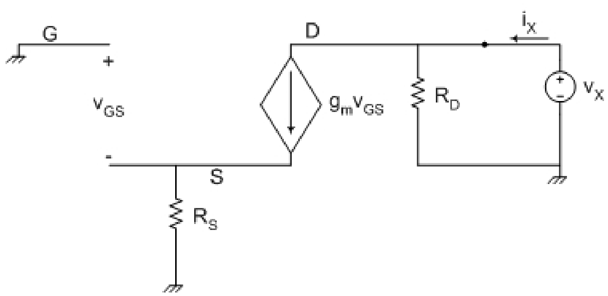
1. G_m:



$$i_D = g_m \cdot v_{GS} \quad , \quad v_{GS} = v_{IN} - g_m \cdot v_{GS} \cdot R_S$$

$$v_{GS} = v_{IN} \cdot \frac{1}{1+g_m \cdot R_S} \quad \rightarrow \quad G_m = \frac{i_D}{v_{IN}} = \frac{g_m}{1+g_m \cdot R_S}$$

2. R_{OUT}:



$$v_{GS} = -g_m \cdot v_{GS} \cdot R_S \quad \rightarrow \quad v_{GS} = 0$$

$$i_X = \frac{v_X}{R_D} + g_m \cdot v_{GS} = \frac{v_X}{R_D}$$

$$R_{OUT} = \frac{v_X}{i_X} = R_D$$

$$A_v = -G_m \cdot R_{OUT} = \frac{-g_m \cdot R_D}{1+g_m \cdot R_S}$$

Source Degeneration - 8

- If we ignore body effect and channel-length modulation:

$$G_m = \frac{g_m}{1 + g_m \cdot R_S} \quad , \quad R_{OUT} = R_D \quad \rightarrow \quad A_v = \frac{-g_m \cdot R_D}{1 + g_m \cdot R_S}$$

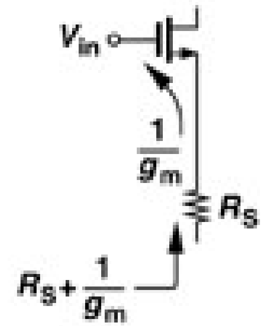
- We Notice that as R_S increases G_m becomes less dependent on g_m:

$$\lim_{R_S \to \infty} G_m = \lim_{R_S \to \infty} \frac{g_m}{1 + g_m \cdot R_S} = \frac{1}{R_S}$$

- That is for large R_S: $G_m = \frac{i_{OUT}}{v_{IN}} \approx \frac{1}{R_S} \quad \rightarrow \quad v_{IN} \approx R_S \cdot i_{OUT}$
- Therefore, the amplifier becomes more linear when R_S is large enough. Intuitively, an increase in v_{IN} tend to increase I_D, however, the voltage drop across R_S also increases. This makes the amplifier less sensitive to input changes, and makes I_D smoother!
- The linearization is achieved at the cost of losing gain and voltage headroom.

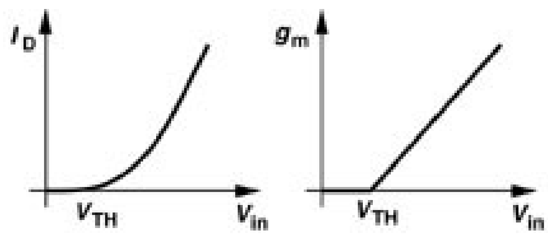
Source Degeneration - 9

- We can manipulate the gain equation so the numerator is the resistance seen at the drain node, and the denominator is the resistance in the source path.



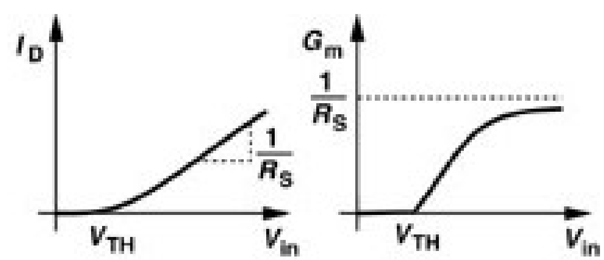
$$A_v = \frac{-g_m \cdot R_D}{1 + g_m \cdot R_S} = \frac{-R_D}{\frac{1}{g_m} + R_S}$$

- The following are I_D and g_m of a transistor without R_S.



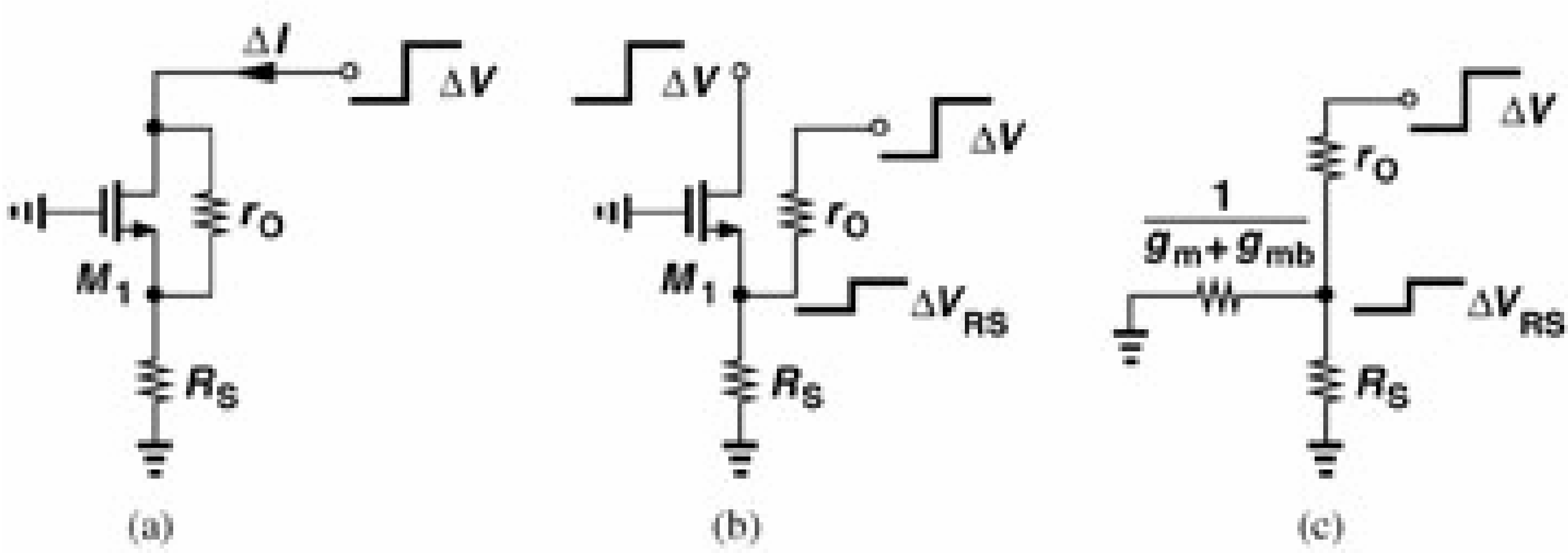
(a)

- I_D and g_m of a transistor considering R_S are:
- When I_D is small such that $R_S\, g_m << 1$, $G_m \approx g_m$.
- When I_D is large such that $R_S\, g_m >> 1$, $G_m \approx 1/R_S$.



(b)

Alternative Method to Find the Output-Resistance of a Degenerated Common-Source Amplifier



Why Buffers?

- Common Source amplifiers needed a large load impedance to provide a large gain.

- If the load is small but we need a large gain (can you think of an example?) a **buffer** is used.

- Source-follower (common-drain) amplifiers can be used as buffers.

$$R_{IN} = \infty \quad , \quad R_{OUT} = 0 \quad , \quad A_V = 1$$

Ideal Buffer:

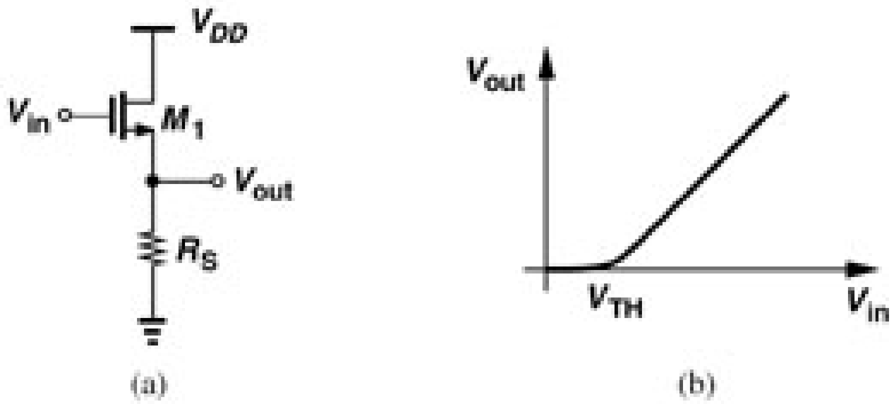
1. $R_{IN}=\infty$: the input current is zero; it doesn't load the previous stage.
2. $R_{OUT}=0$: No voltage drop at the output; behaves like a voltage source.

Resistive Load - 1

- We will examine the Source follower amplifier with two different loads:
  1. Resistive Load
  2. Current Source Load

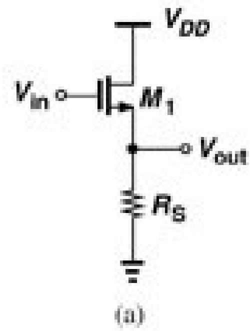
– **Resistive Load:**

- As shown below the output (source voltage) will follow the input (gate voltage). We will analyze the following circuit using large-signal and small-signal analysis.



Resistive Load - 2

Large Signal Analysis:



- The relationship between V_{IN} and V_{OUT} is:

$$V_{OUT} = R_S \cdot I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 \cdot (1 + \lambda \cdot V_{DS}) \cdot R_S$$

$$V_{OUT} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{IN} - V_{OUT} - V_{TH})^2 \cdot (1 + \lambda \cdot V_{DD} - \lambda \cdot V_{OUT}) \cdot R_S$$

- Differentiate with respect to V_{IN}:

$$\frac{\partial V_{OUT}}{\partial V_{IN}} = \mu_n C_{ox}\frac{W}{L}(V_{IN} - V_{OUT} - V_{TH}) \cdot \left(1 - \frac{\partial V_{OUT}}{\partial V_{IN}} - \frac{\partial V_{TH}}{\partial V_{IN}}\right) \cdot (1 + \lambda \cdot V_{DS}) \cdot R_S$$

$$+ \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 \cdot R_S \cdot (-\lambda) \cdot \frac{\partial V_{OUT}}{\partial V_{IN}}$$

- Need to find the derivative of V_{TH} with respect to V_{IN}:

$$V_{TH} = V_{TH0} + \gamma \cdot \left(\sqrt{|2 \cdot \Phi_F + V_{SB}|} - \sqrt{|2 \cdot \Phi_F|}\right) \quad , \quad V_{SB} = V_{OUT}$$

$$\frac{\partial V_{TH}}{\partial V_{IN}} = \frac{\partial V_{TH}}{\partial V_{OUT}} \cdot \frac{\partial V_{OUT}}{\partial V_{IN}} = \frac{\gamma}{2\sqrt{|2 \cdot \Phi_F + V_{SB}|}} \cdot \frac{\partial V_{OUT}}{\partial V_{IN}} = \eta \cdot \frac{\partial V_{OUT}}{\partial V_{IN}}$$

Resistive Load - 3

Large Signal Analysis (Continued):

- The small signal gain can be found:

$$\frac{\partial V_{OUT}}{\partial V_{IN}} \cdot \left(1 + g_m \cdot R_S + g_m \cdot \eta \cdot R_S + I_D \cdot R_S \cdot \lambda\right) = g_m \cdot R_S$$

$$A_V = \frac{\partial V_{OUT}}{\partial V_{IN}} = \frac{g_m \cdot R_S}{1 + g_m \cdot R_S + g_{mb} \cdot R_S + \frac{R_S}{r_o}} = \frac{g_m \cdot R_S}{1 + \left(g_m + g_{mb} + \frac{1}{r_o}\right) \cdot R_S}$$

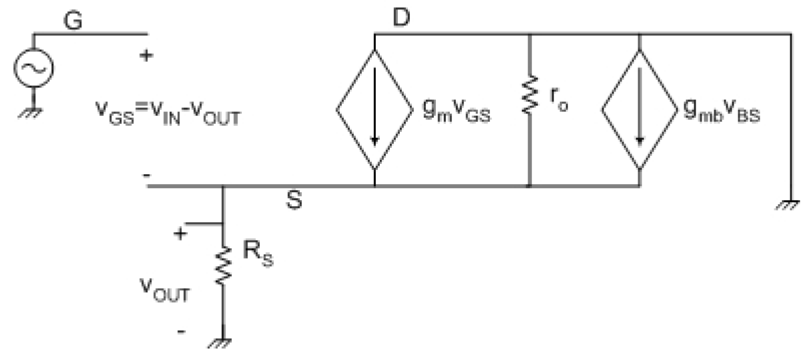
- If channel-length modulation is ignored ($r_o = \infty$) we get:

$$A_V = \frac{\partial V_{OUT}}{\partial V_{IN}} = \frac{g_m \cdot R_S}{1 + \left(g_m + g_{mb}\right) \cdot R_S}$$

Resistive Load - 4

Small Signal Analysis:

- We get the following small signal model:



$$v_{OUT} = \left(g_m \cdot v_{GS} + g_{mb} \cdot v_{BS}\right) \cdot R_S \| r_O \quad , \quad v_{GS} = v_{IN} - v_{OUT} \quad , \quad v_{BS} = -v_{OUT}$$

$$v_{OUT} = \left(g_m \cdot \left(v_{IN} - v_{OUT}\right) + g_{mb} \cdot \left(-v_{OUT}\right)\right) \cdot \frac{R_S \cdot r_O}{R_S + r_O}$$

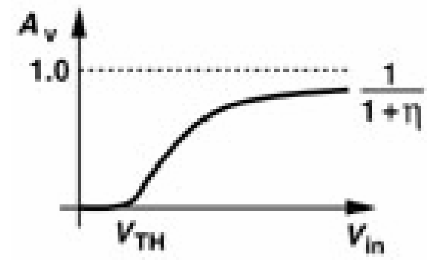
$$v_{OUT} \cdot \left(R_S + r_O + g_m \cdot R_S \cdot r_O + g_{mb} \cdot R_S \cdot r_O\right) = g_m \cdot R_S \cdot r_O \cdot v_{IN}$$

$$A_v = \frac{v_{OUT}}{v_{IN}} = \frac{g_m \cdot R_S \cdot r_O}{R_S + r_O + g_m \cdot R_S \cdot r_O + g_{mb} \cdot R_S \cdot r_O} = \frac{g_m \cdot R_S \cdot r_O}{R_S \cdot \left(1 + g_m \cdot r_O + g_{mb} \cdot r_O\right) + r_O}$$

$$A_v = \frac{g_m \cdot R_S}{R_S \cdot \left(\frac{1}{r_O} + g_m + g_{mb}\right) + 1}$$

Resistive Load - 5

- Graph of the gain of a source-follower amplifier:



1. M_1 never enters the triode region as long as $V_{IN}<V_{DD}$.
2. Gain is zero if V_{IN} is less than V_{TH} (because g_m is 0).
3. As V_{IN} increases, g_m increases and the gain becomes:

$$A_v \approx \frac{g_m}{g_m + g_{mb}} = \frac{1}{1+\eta}$$

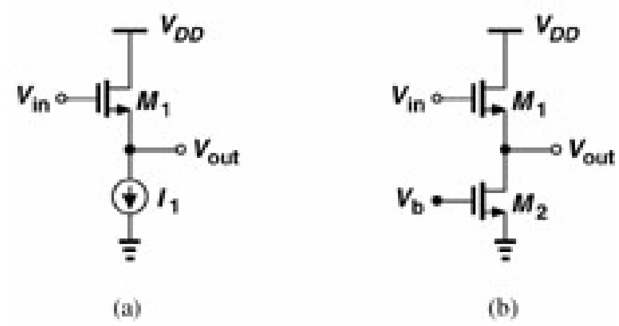
4. As V_{OUT} increases, η decreases, and therefore, the maximum gain increases.
5. Even if $R_S=\infty$, the gain is less than one:

$$A_V \approx \frac{g_m}{g_m + g_{mb} + \frac{1}{r_o}} < 1$$

6. Gain depends heavily on the DC level of the input (nonlinear amplifier).

Current Source Load

- In a source follower with a resistive load, the drain current depends on the DC level of V_{IN}, which makes the amplifier highly nonlinear.



- To avoid this problem, we can use a current source as the load.

- The output resistance is:

$$R_{M1} = r_{o1} \left\| \frac{1}{g_{m1}} \right\| \frac{1}{g_{mb1}} \quad , \quad R_{I1} = r_{o2} \quad \rightarrow \quad R_{OUT} = R_{M1} \| R_{I1} = r_{o1} \left\| \frac{1}{g_{m1}} \right\| \frac{1}{g_{mb1}} \| r_{o2}$$

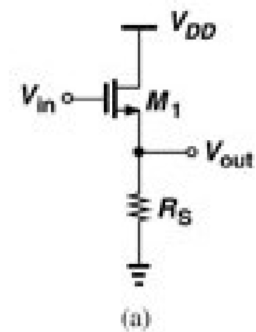
- If channel length modulation is ignored ($r_{o1}=r_{o2}=\infty$) :

$$R_{OUT} = \infty \left\| \frac{1}{g_{m1}} \right\| \frac{1}{g_{mb1}} \| \infty = \frac{1}{g_{m1}} \left\| \frac{1}{g_{mb1}} \right. = \frac{1}{g_{m1} + g_{mb1}}$$

- Note that the body effect reduces the output impedance of the source follower amplifiers.

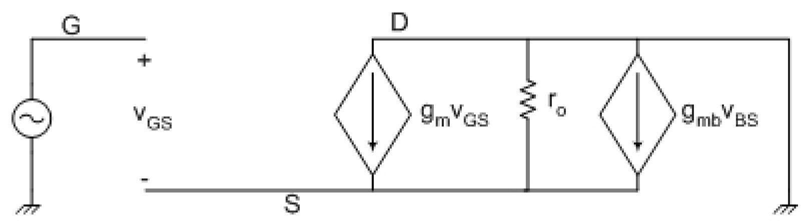
Voltage Division in Source Followers - 1

- When Calculating output resistance seen at the source of M_1, i.e., R_{M1}, we force v_{IN} to zero and find the output impedance:



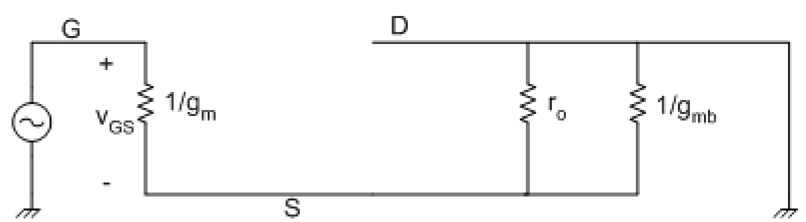
$$R_{M1} = r_{o1} \left\| \frac{1}{g_{m1}} \right\| \frac{1}{g_{mb1}}$$

- However, if we were to find the gain of the amplifier, we would not suppress v_{IN}.
- Here, we would like to find an equivalent circuit of M_1, from which we can find the gain.
- Consider the small-signal model of M_1:

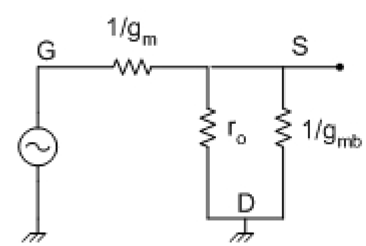


Voltage Division in Source Followers - 2

- For small-signal analysis $v_{BS}= v_{DS}$, so $g_{mb}v_{BS}$ dependant current source can be replaced by a resistor ($1/g_{mb}$) between source and drain.
- Note that, when looking at the circuit from the source terminal, we can replace the $g_m v_{GS}$ dependant current source with a resistor (of value $1/g_m$) between source and gate.



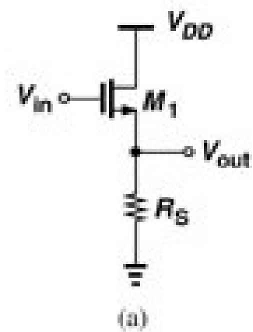
- Simplified circuit:



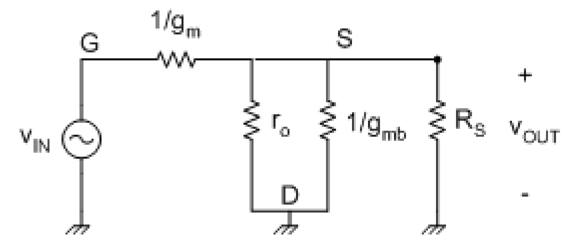
Voltage Division in Source Followers - 3

Example:

- Find the gain of a source follower amplifier with a resistive load.
- We draw the small signal model of this amplifier as shown below to get:



$$v_{OUT} = \frac{R_S \| r_O \| \frac{1}{g_{mb}}}{R_S \| r_O \| \frac{1}{g_{mb}} + \frac{1}{g_m}} \cdot v_{IN} \rightarrow A_v = \frac{v_{OUT}}{v_{IN}} = \frac{R_S \| r_O \| \frac{1}{g_{mb}}}{R_S \| r_O \| \frac{1}{g_{mb}} + \frac{1}{g_m}}$$



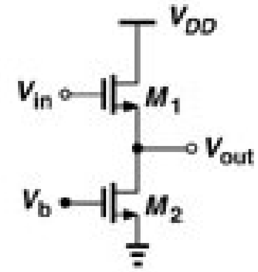
- We can show that this is equal to what we obtained before:

$$A_v = \frac{\frac{1}{\frac{1}{R_S} + \frac{1}{r_O} + g_{mb}}}{\frac{1}{\frac{1}{R_S} + \frac{1}{r_O} + g_{mb}} + \frac{1}{g_m}} = \frac{\frac{R_S \cdot r_O}{R_S + r_O + R_S \cdot r_O \cdot g_{mb}}}{\frac{R_S \cdot r_O}{R_S + r_O + R_S \cdot r_O \cdot g_{mb}} + \frac{1}{g_m}} = \frac{R_S \cdot r_O \cdot g_m}{R_S + r_O + R_S \cdot r_O \cdot g_{mb} + R_S \cdot r_O \cdot g_m}$$

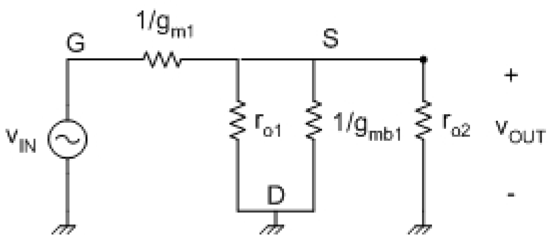
Voltage Division in Source Followers - 4

Example:

- Find the gain of a source follower amplifier with a current source load.
- Small-signal model of this amplifier is:



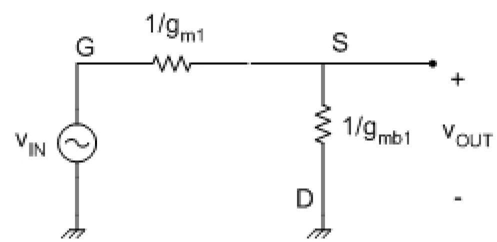
(b)



$$v_{OUT} = \frac{r_{O2} \| r_{O1} \| \frac{1}{g_{mb1}}}{r_{O2} \| r_{O1} \| \frac{1}{g_{mb1}} + \frac{1}{g_{m1}}} \cdot v_{IN} \rightarrow A_v = \frac{v_{OUT}}{v_{IN}} = \frac{r_{O2} \| r_{O1} \| \frac{1}{g_{mb1}}}{r_{O2} \| r_{O1} \| \frac{1}{g_{mb1}} + \frac{1}{g_{m1}}}$$

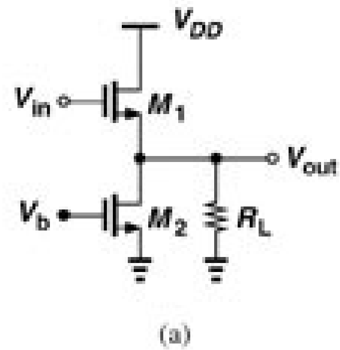
- If we ignore channel length modulation:

$$v_{OUT} = \frac{\frac{1}{g_{mb1}}}{\frac{1}{g_{mb1}} + \frac{1}{g_{m1}}} \cdot v_{IN} \rightarrow A_v = \frac{v_{OUT}}{v_{IN}} = \frac{\frac{1}{g_{mb1}}}{\frac{1}{g_{mb1}} + \frac{1}{g_{m1}}}$$



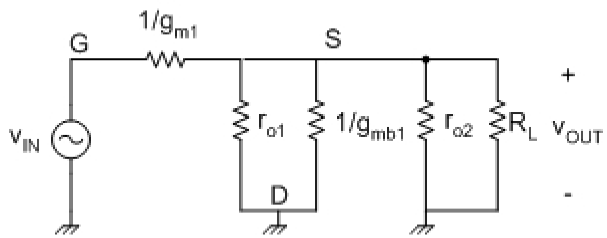
Voltage Division in Source Followers - 5

Example:



- Find the gain of a source follower amplifier with a resistive load and biased with a current source.

- Small-signal model of this amplifier is:

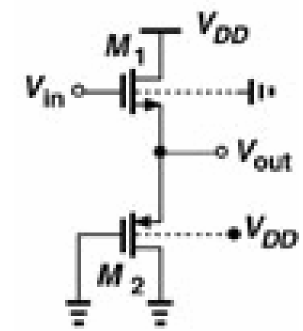


$$v_{OUT} = \frac{r_{O2} \| R_L \| r_{O1} \| \frac{1}{g_{mb1}}}{r_{O2} \| R_L \| r_{O1} \| \frac{1}{g_{mb1}} + \frac{1}{g_{m1}}} \cdot v_{IN} \rightarrow A_v = \frac{v_{OUT}}{v_{IN}} = \frac{r_{O2} \| R_L \| r_{O1} \| \frac{1}{g_{mb1}}}{r_{O2} \| R_L \| r_{O1} \| \frac{1}{g_{mb1}} + \frac{1}{g_{m1}}}$$

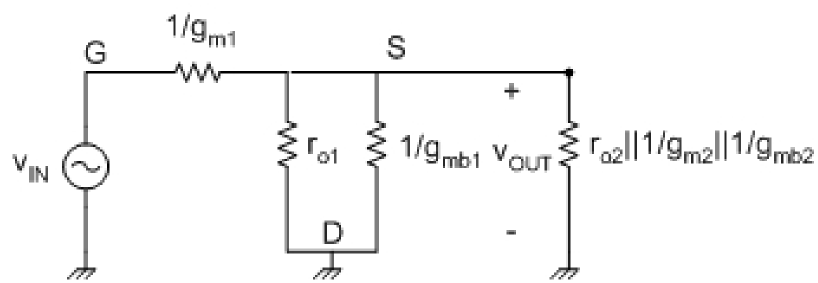
Voltage Division in Source Followers - 6

Example:

- Find the gain of a source follower amplifier with a resistive load.



- Small-signal model of this amplifier is:



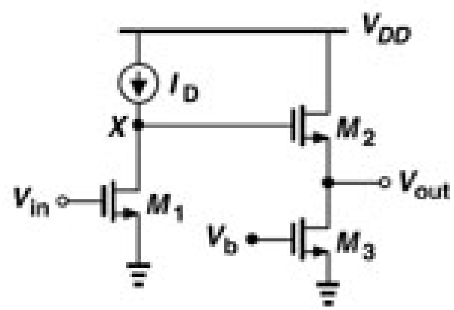
$$v_{OUT} = \frac{r_{O2} \left\| r_{O1} \right\| \frac{1}{g_{mb1}} \left\| \frac{1}{g_{m2}} \right\| \frac{1}{g_{mb2}}}{r_{O2} \left\| r_{O1} \right\| \frac{1}{g_{mb1}} \left\| \frac{1}{g_{m2}} \right\| \frac{1}{g_{mb2}} + \frac{1}{g_{m1}}} \cdot v_{IN} \rightarrow A_v = \frac{v_{OUT}}{v_{IN}} = \frac{r_{O2} \left\| r_{O1} \right\| \frac{1}{g_{mb1}} \left\| \frac{1}{g_{m2}} \right\| \frac{1}{g_{mb2}}}{r_{O2} \left\| r_{O1} \right\| \frac{1}{g_{mb1}} \left\| \frac{1}{g_{m2}} \right\| \frac{1}{g_{mb2}} + \frac{1}{g_{m1}}}$$

Advantages and Disadvantages - 1

1. Source followers have typically small output impedance.
2. Source followers have large input impedance.
3. Source followers have poor driving capabilities..
4. Source followers are nonlinear. This nonlinearity is caused by:
   - Variable bias current which can be resolved if we use a current source to bias the source follower.
   - Body effect; i.e., dependence of V_{TH} on the source (output) voltage. This can be resolved for PMOS devices, because each PMOS transistor can have a separate n-well. However, because of low mobility, PMOS devices have higher output impedance. (In more advanced technologies, NMOS in a separate p-well, can be implemented that potentially has no body effect)
   - Dependence of r_o on V_{DS} in submicron devices.

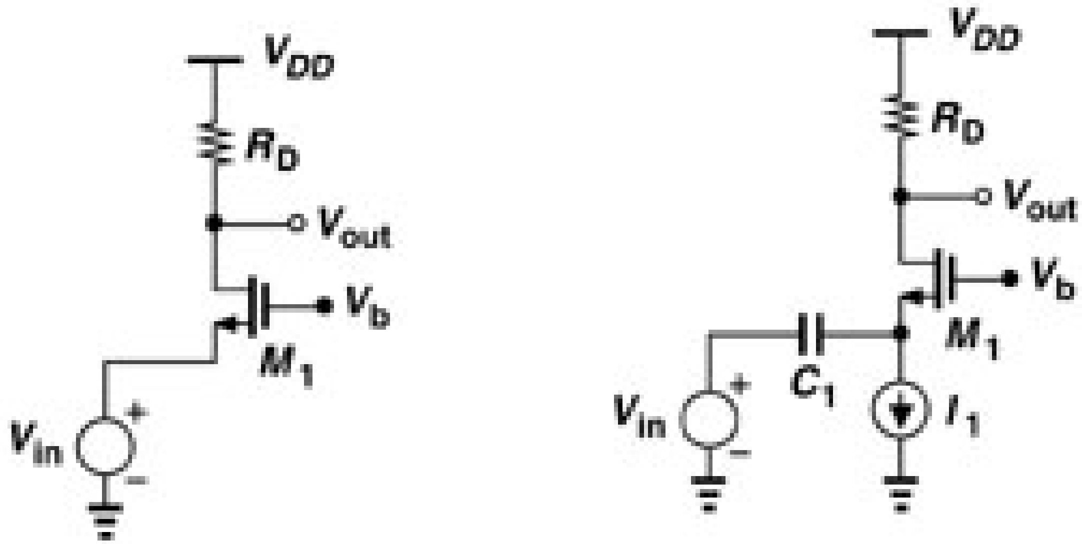
Advantages and Disadvantages - 2

5. Source followers have voltage headroom limitations due to level shift. Consider this circuit (a common source followed by a source follower):



- If we only consider the common source stage, $V_X > V_{GS1} - V_{TH1}$.
- If we only consider the source follower stage, $V_X > V_{GS3} - V_{TH3} + V_{GS2}$.
- Therefore, adding the source follower will reduce the allowable voltage swing at node X.
- The DC value of V_{OUT} is V_{GS2} lower than the DC value of V_X.

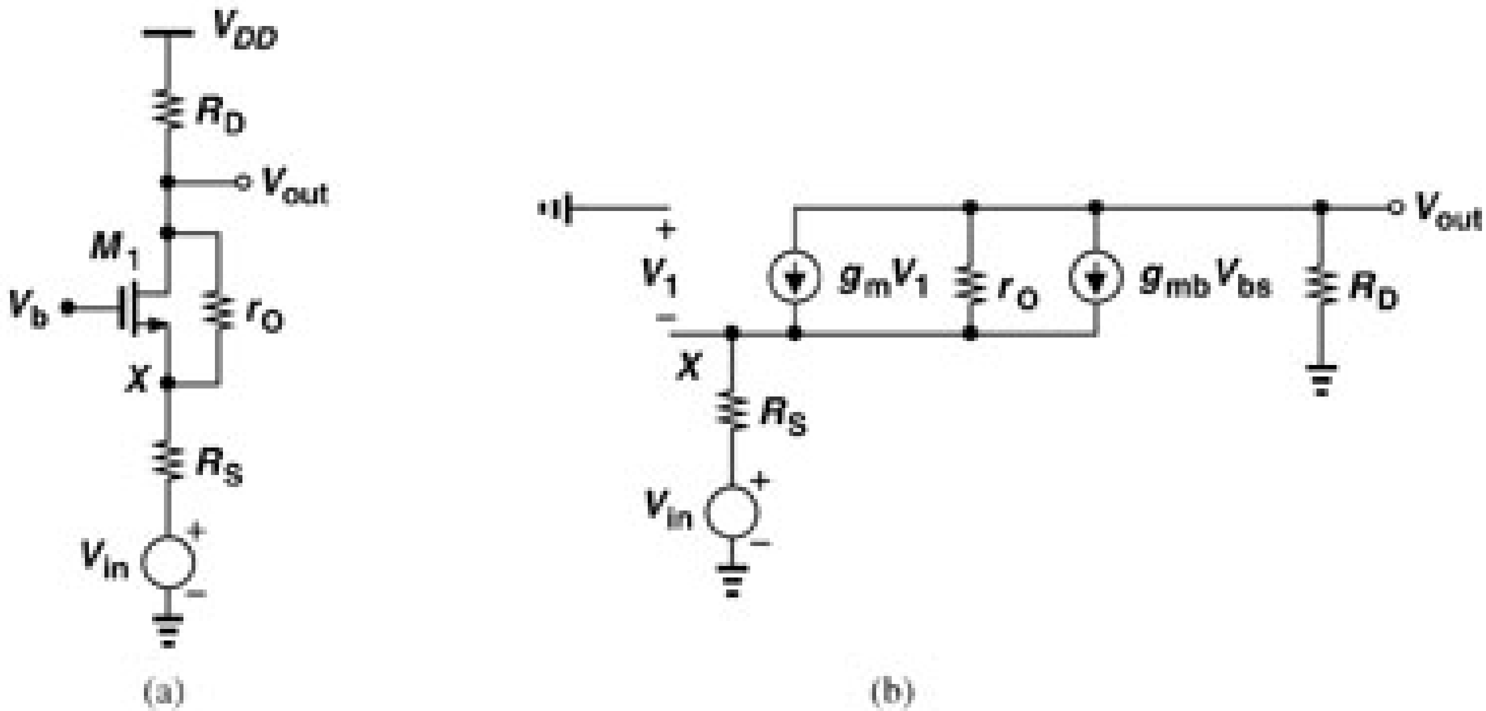
Common-Gate



(a) (b)

$$A_v = (g_m + g_{mb})R_D = g_m(1+\eta)R_D$$

Common-Gate



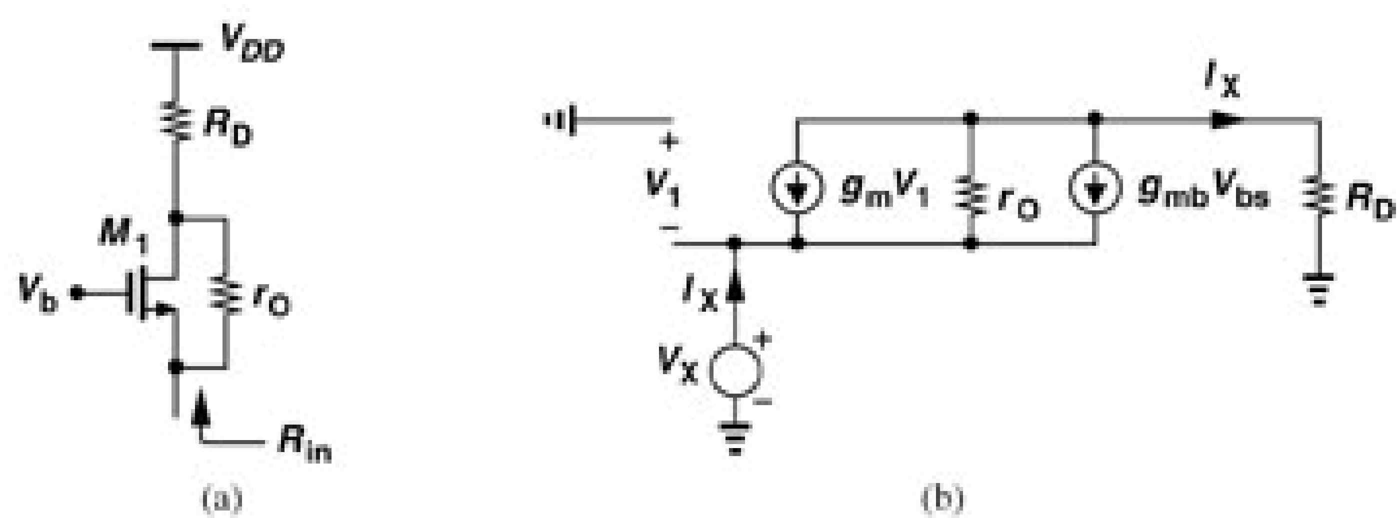
$$A_v = \frac{(g_m + g_{mb})r_o + 1}{r_o + (g_m + g_{mb})r_o R_S + R_S + R_D} R_D$$

Common-Gate

$$A_v = \frac{(g_m + g_{mb})r_o + 1}{r_o + (g_m + g_{mb})r_o R_S + R_S + R_D} R_D$$

for $R_S = 0: A_v \approx (g_m + g_{mb})(r_o \| R_D)$

Common-Gate Input Impedance



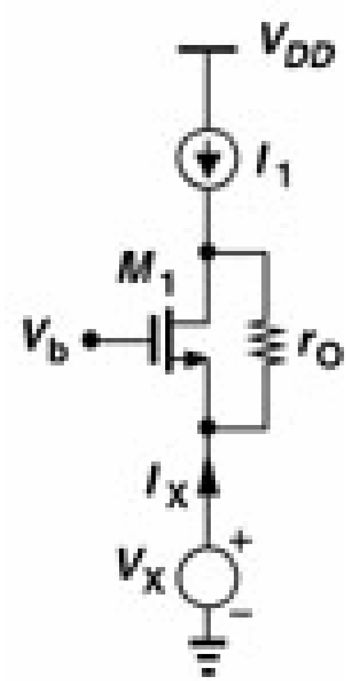
$$R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb}) r_o} = \frac{R_D}{1 + (g_m + g_{mb}) r_o} + \frac{r_o}{1 + (g_m + g_{mb}) r_o}$$

$$R_{in} = \frac{R_D}{1 + (g_m + g_{mb}) r_o} + (r_o \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}})$$

Common-Gate Input Impedance

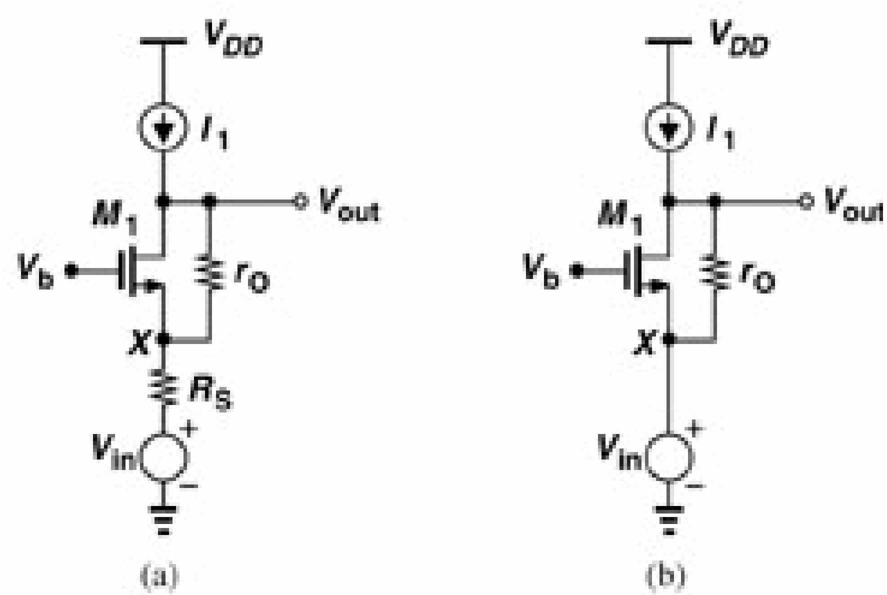
- Input impedance of common-gate stage is relatively low only if R_D is small

- Example: Find the input impedance of the following circuit.



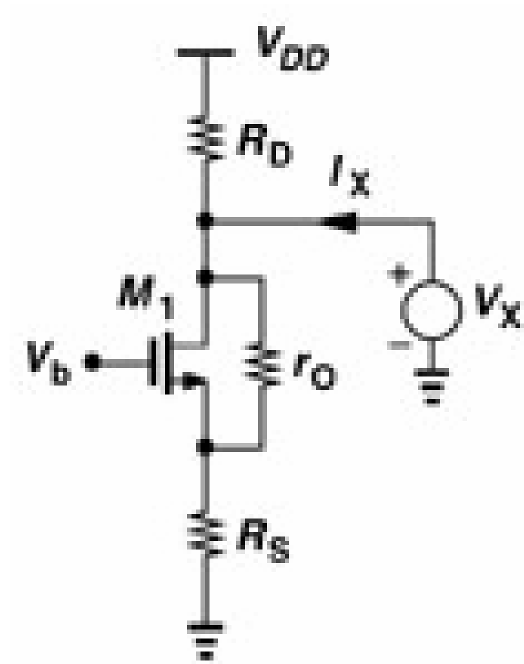
Example

- Calculate the voltage gain of the following circuit:



$$A_v = 1 + (g_m + g_{mb}) r_o$$

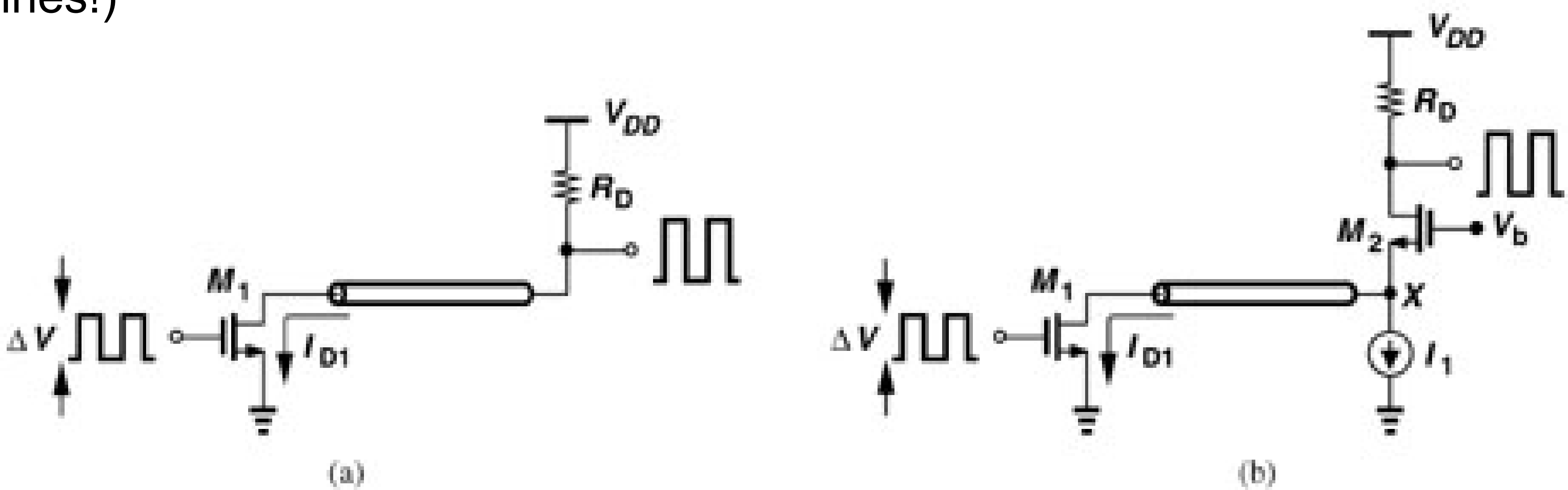
Common-Gate Output Impedance



$$R_{out} = \{[1 + (g_m + g_{mb})R_S]r_o + R_S\} \,||\, R_D$$

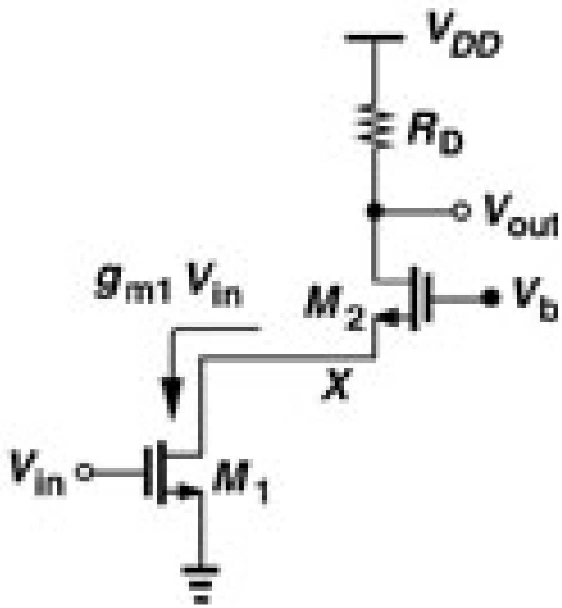
Example

- Compare the gain of the following two circuits ($\lambda=\gamma=0$ and 50Ω transmission lines!)



Cascode Stage

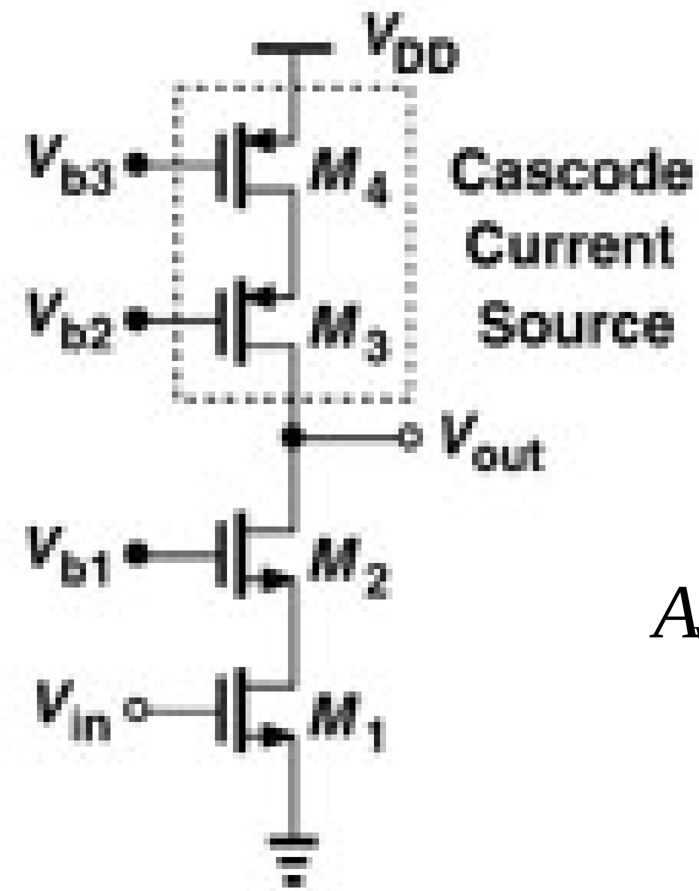
- Cascade of a common-source stage and a common-gate stage is called a “cascode” stage.



$$R_{out} = \{[1+(g_{m2}+g_{mb2})r_{o1}]r_{o2}+r_{o1}\} \,||\, R_D$$
$$\approx [(g_{m2}+g_{mb2})r_{o1}r_{o2}] \,||\, R_D$$

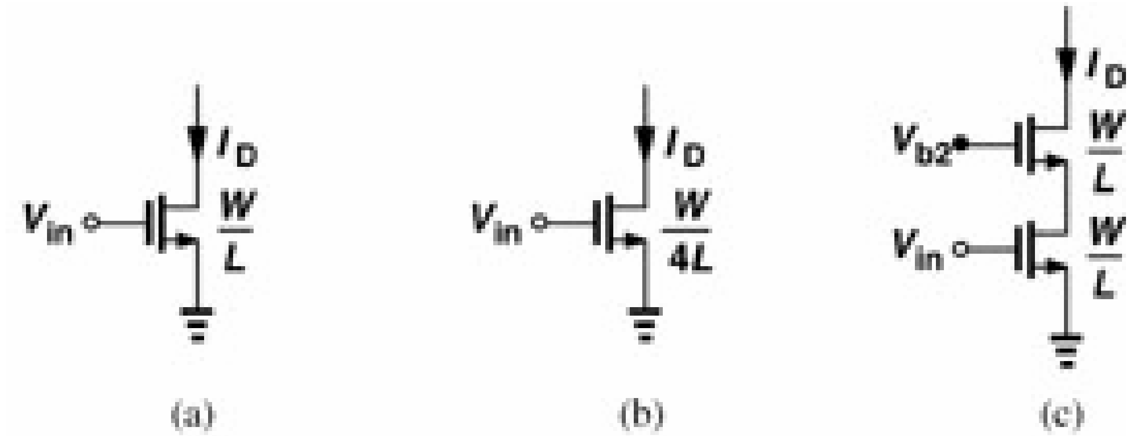
$$A_V \approx g_{m1}\{[r_{o1}r_{o2}(g_{m2}+g_{mb2})] \,||\, R_D\}$$

Cascode Stage

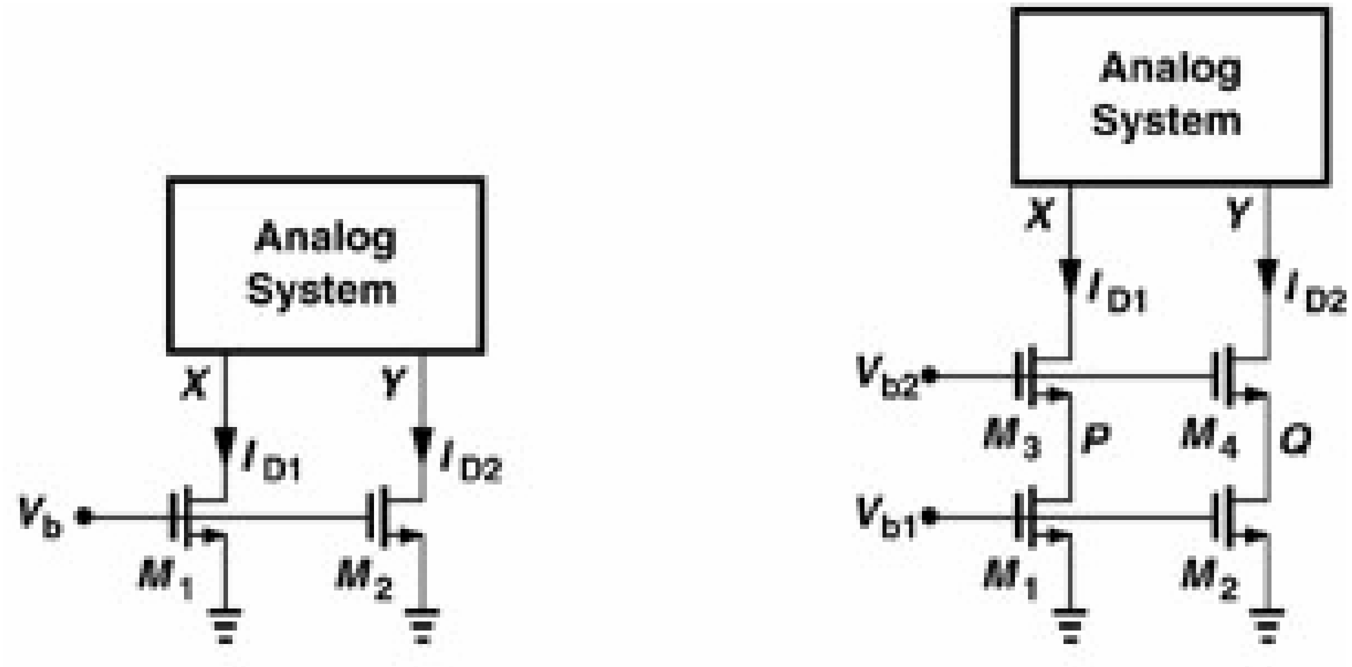


$$A_V \approx g_{m1}[(r_{o1}r_{o2}g_{m2}) \| (r_{o3}r_{o4}g_{m3})]$$

Output Impedance Comparison



Shielding Property

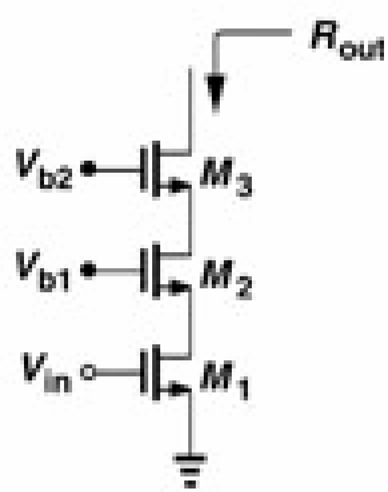


(a) (b)

Board Notes

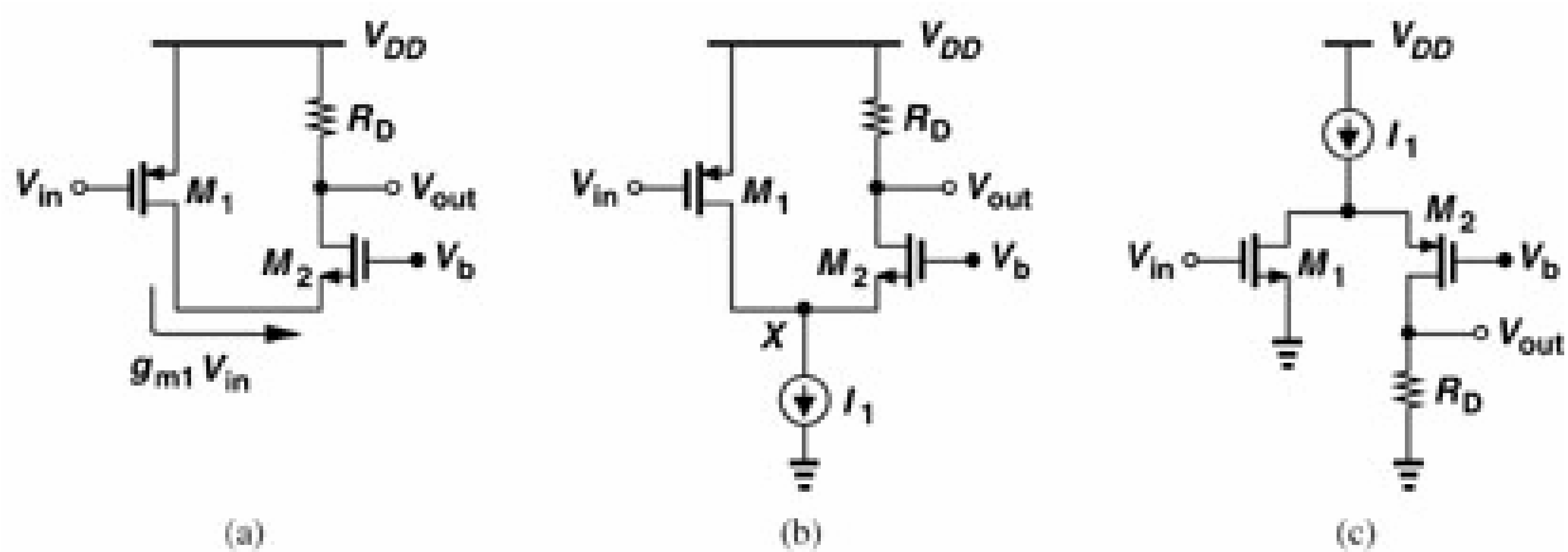
Triple Cascode

- What is the output resistance of this circuit?

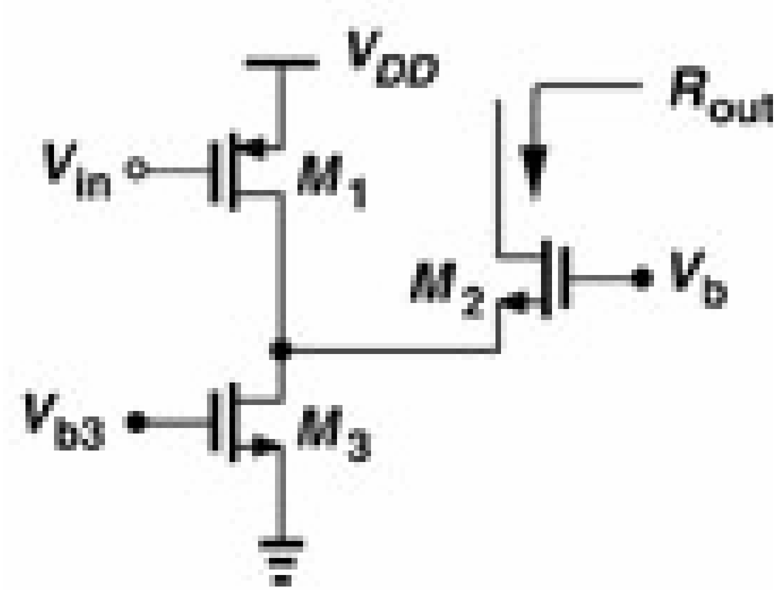


- Problem?

Folded Cascode



Output Impedance of a Folded Cascode



$$R_{out} = [1 + (g_{m2} + g_{mb2})r_{o2}](r_{o1} \| r_{o3}) + r_{o2}$$