EECE488: Analog CMOS Integrated Circuit Design

3. Single-Stage Amplifiers

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Overview

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2. Amplifier Characteristics
3. Amplifier Trade-offs
4. Single-stage Amplifiers
5. Common Source Amplifiers
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   2. Diode-connected Load
   3. Current Source Load
   4. Triode Load
   5. Source Degeneration
Overview

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   1. Resistive Load
   2. Current Source Load
   3. Voltage Division in Source Followers

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Reading Assignments

• Reading:
  Chapter 3 of Razavi’s book

• In this set of slides we will study low-frequency small-signal behavior of single-stage CMOS amplifiers. Although, we assume long-channel MOS models (not a good assumption for deep submicron technologies) the techniques discussed here help us to develop basic circuit intuition and to better understand and predict the behavior of circuits.

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Why Amplifiers?

- Amplifiers are essential building blocks of both analog and digital systems.

- Amplifiers are needed for variety of reasons including:
  - To amplify a weak analog signal for further processing
  - To reduce the effects of noise of the next stage
  - To provide a proper logical levels (in digital circuits)

- Amplifiers also play a crucial role in feedback systems

- We first look at the low-frequency performance of amplifiers. Therefore, all capacitors in the small-signal model are ignored!

Amplifier Characteristics - 1

- Ideally we would like that the output of an amplifier be a linear function of the input, i.e., the input times a constant gain:

\[ y = \alpha x \]

- In real world the input-output characteristics is typically a nonlinear function:
Amplifier Characteristics - 2

- It is more convenient to use a linear approximation of a nonlinear function.
- Use the tangent line to the curve at the given (operating) point.

![Graph showing tangent line approximation](image)

- The larger the signal changes about the operating point, the worse the approximation of the curve by its tangent line.
- This is why small-signal analysis is so popular!

Amplifier Characteristics - 3

- A well-behaved nonlinear function in the vicinity of a given point can be approximated by its corresponding Taylor series:
  \[ y = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n \]

- Let \( \alpha_n = \frac{f^{(n)}(x_0)}{n!} \) to get:
  \[ y \approx \alpha_0 + \alpha_1(x-x_0) + \alpha_2(x-x_0)^2 + \cdots + \alpha_n(x-x_0)^n \]

- If \( x-x_0 \approx \Delta x \) is small, we can ignore the higher-order terms (hence the name small-signal analysis) to get:
  \[ y \approx \alpha_0 + \alpha_1(x-x_0) \]

- \( \alpha_0 \) is referred to as the operating (bias) point and \( \alpha_1 \) is the small-signal gain.
  \[ \Delta y = y - f(x_0) = y - \alpha_0 \approx \alpha_1 \Delta x \]
In practice, when designing an amplifier, we need to optimize for some performance parameters. Typically, these parameters trade performance with each other, therefore, we need to choose an acceptable compromise.

We will examine the following types of amplifiers:
1. Common Source
2. Common Drain (Source Follower )
3. Common Gate
4. Cascode and Folded Cascode

Each of these amplifiers have some advantages and some disadvantages. Often, designers have to utilize a cascade combination of these amplifiers to meet the design requirements.
Common Source Basics - 1

• In common-source amplifiers, the input is (somehow!) connected to the
gate and the output is (somehow!) taken from the drain.

\[ V_{in} \rightarrow V_{gs} \rightarrow V_{ds} \rightarrow V_{out} \]

• We can divide common source amplifiers into two groups:
  1. Without source degeneration (no body
effect for the main transistor):

  2. With source degeneration (have to take
body effect into account for the main
transistor):

Common Source Basics - 2

In a simple common source amplifier:

\[ V_{DD} \]

\[ R_D \]

\[ V_{in} \]

\[ M_1 \]

\[ V_{out} \]

• gate voltage variations times \( g_m \) gives the drain current
variations,
• drain current variations times the load gives the output voltage
variations.
• Therefore, one can expect the small-signal gain to be:

\[ |A_v| = g_m \cdot R_D \]
Common Source Basics - 3

- Different types of loads can be used in an amplifier:
  1. Resistive Load
  2. Diode-connected Load
  3. Current Source Load
  4. Triode Load

- The following parameters of amplifiers are very important:
  1. Small-signal gain
  2. Voltage swing

Resistive Load - 1

- Let’s use a resistor as the load.
- The region of operation of $M_1$ depends on its size and the values of $V_{in}$ and $R$.
- We are interested in the small-signal gain and the headroom (which determines the maximum voltage swing).
- We will calculate the gain using two different methods
  1. Small-signal model
  2. Large-signal analysis
Resistive Load - 2

Gain – Method 1: Small-Signal Model

- This is assuming that the transistor is in saturation, and channel length modulation is ignored.
- The current through $R_D$:
  \[ i_D = g_m \cdot v_{IN} \]
- Output Voltage:
  \[ v_{OUT} = -i_D \cdot R_D = -g_m \cdot v_{IN} \cdot R_D \]
- Small-signal Gain:
  \[ A_v = \frac{v_{OUT}}{v_{IN}} = -g_m \cdot R_D \]

Resistive Load - 3

Gain – Method 2: Large-Signal Analysis

- If $V_{IN} < V_{TH}$, $M_1$ is off, and $V_{OUT} = V_{DD} = V_{DS}$:
  \[ V_{OUT} = V_{DD} - R_D \cdot i_D = V_{DD} \]
  \[ A_v = \frac{\partial V_{OUT}}{\partial V_{IN}} = 0 \]
- As $V_{IN}$ becomes slightly larger than $V_{TH}$, $M_1$ turns on and goes into saturation ($V_{DS} \approx V_{DD} > V_{GS} - V_{TH} \approx 0$).
  \[ v_{OUT} = V_o - R_o \cdot i_o = V_o - R_o \cdot \frac{1}{2} \mu \cdot C_o \cdot \frac{W}{L} \cdot (V_o - V_m) \]
  \[ A_v = \frac{\partial V_{OUT}}{\partial V_{IN}} = -R_o \cdot \mu \cdot C_o \cdot \frac{W}{L} \cdot (V_o - V_m) = -R_o \cdot g_m \]
- As $V_{DS}$ increases, $V_{DS}$ decreases, and $M_1$ goes into triode when $V_{IN} - V_{TH} = V_{OUT}$. We can find the value of $V_{IN}$ that makes $M_1$ switch its region of operation.
  \[ V_{OUT} = V_o - R_o \cdot i_o = V_o - R_o \cdot \frac{1}{2} \mu \cdot C_o \cdot \frac{W}{L} \cdot (V_o - V_m) = (V_o - V_m) \]
Resistive Load - 4

Gain – Method 2: Large-Signal Analysis (Continued)

- As \( V_{IN} \) increases, \( V_{DS} \) decreases, and \( M_1 \) goes into triode.

\[
V_{OUT} = V_{DD} - R_D \cdot i_D = V_{DD} - R_D \cdot \mu_n \cdot C_w \cdot \frac{W}{L} \left( V_{IN} - V_{TH} \right) \cdot V_{OUT} - \frac{V_{OUT}^2}{2}
\]

\[
\frac{\partial V_{OUT}}{\partial V_{IN}} = -R_D \cdot \mu_n \cdot C_w \cdot \frac{W}{L} \left( V_{IN} - V_{TH} \right) \frac{\partial V_{OUT}}{\partial V_{IN}} + V_{OUT} \frac{\partial V_{OUT}}{\partial V_{IN}}
\]

- We can find \( A_v \) from above. It will depend on both \( V_{IN} \) and \( V_{OUT} \).

- If \( V_{IN} \) increases further, \( M_1 \) goes into deep triode if \( V_{OUT} \ll 2(V_{IN} - V_{TH}) \).

\[
V_{OUT} = V_{DD} - R_D \cdot i_D = V_{DD} - R_D \cdot \mu_n \cdot C_w \cdot \frac{W}{L} (V_{IN} - V_{TH}) \cdot V_{OUT}
\]

\[
\frac{V_{OUT}}{1 + R_D \cdot \mu_n \cdot C_w \cdot \frac{W}{L} (V_{IN} - V_{TH})} = V_{DD} \cdot \frac{1}{R_{ON} + R_D} = V_{DD} \cdot \frac{1}{R_{ON}}
\]

Resistive Load - 5

Example: Sketch the drain current and \( g_m \) of \( M_1 \) as a function of \( V_{IN} \).

- \( g_m \) depends on \( V_{IN} \), so if \( V_{IN} \) changes by a large amount the small-signal approximation will not be valid anymore.

- In order to have a linear amplifier, we don’t want gain to depend on parameters like \( g_m \) which depend on the input signal.
Resistive Load - 6

- Gain of common-source amplifier:

\[ A_v = -g_{m} \cdot R_D = -\mu_n C_{ox} \frac{W}{L}(V_{IN} - V_{TH}) \cdot \frac{V_{RD}}{I_D} = -\sqrt{2\mu_n C_{ox}} \frac{W}{L} \cdot \frac{V_{RD}}{\sqrt{I_D}} = -2 \cdot \frac{V_{RD}}{V_{eff}} \]

- To increase the gain:
  1. Increase \( g_m \) by increasing \( W \) or \( V_{IN} \) (DC portion or bias). Either way, \( I_D \) increases (more power) and \( V_{RD} \) increases, which limits the voltage swing.
  2. Increase \( R_D \) and keep \( I_D \) constant (\( g_m \) and power remain constant). But, \( V_{RD} \) increases which limits the voltage swing.
  3. Increase \( R_D \) and reduce \( I_D \) so \( V_{RD} \) remains constant.
    - If \( I_D \) is reduced by decreasing \( W \), the gain will not change.
    - If \( I_D \) is reduced by decreasing \( V_{IN} \) (bias), the gain will increase. Since \( R_D \) is increased, the bandwidth becomes smaller (why?).
- Notice the trade-offs between gain, bandwidth, and voltage swings.

Resistive Load - 7

- Now let’s consider the simple common-source circuit with channel length modulation taken into account.

- Channel length modulation becomes more important as \( R_D \) increases (in the next slide we will see why!).

- Again, we will calculate the gain in two different methods
  1. Small-signal Model
  2. Large Signal Analysis
Resistive Load - 8

**Gain – Method 1: Small-Signal Model**

- This is assuming that the transistor is in saturation.
- The current through $R_D$:
  \[ i_i = g_v \cdot v_i \]
- Output Voltage:
  \[ v_{os} = -i_i \cdot \left( R_o \parallel \frac{R_D}{2} \right) = -g_v \cdot v_i \cdot \left( R_o \parallel \frac{R_D}{2} \right) \]
- Small-signal Gain:
  \[ A_i = \frac{v_{os}}{v_i} = -g_v \cdot \left( R_o \parallel \frac{R_D}{2} \right) \]

Resistive Load - 9

**Gain – Method 2: Large-Signal Analysis**

- As $V_{IN}$ becomes slightly larger than $V_{TH}$, $M_1$ turns on and goes into saturation ($V_{DS} \approx V_{DD} > V_{GS} - V_{TH} \approx 0$).

\[
\frac{V_{OUT}}{V_{IN}} = -R_D \cdot \frac{1}{2} \cdot \frac{\mu_n}{C_{ox}} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH})^2 \cdot (1 + \lambda \cdot V_{OUT})
\]

\[
\frac{\partial V_{OUT}}{\partial V_{IN}} = -R_D \cdot \frac{1}{2} \cdot \frac{\mu_n}{C_{ox}} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH}) \cdot \left( 1 + \lambda \cdot V_{OUT} \right) + \frac{1}{1 + R_D \cdot I_D \cdot \lambda} \cdot \frac{\partial V_{OUT}}{V_{IN}}
\]

\[
A_v = \frac{-R_D \cdot \frac{1}{2} \cdot \frac{\mu_n}{C_{ox}} \cdot \frac{W}{L} \cdot (V_{IN} - V_{TH}) \cdot (1 + \lambda \cdot V_{OUT})}{1 + R_D \cdot I_D \cdot \lambda} = \frac{-R_D \cdot g_m}{1 + R_D \cdot \frac{1}{r_o}}
\]

\[
A_v = -\frac{r_o \cdot R_D \cdot 8m}{r_o + R_D} = -g_m \cdot \left( R_D \parallel r_o \right)
\]
Resistive Load - 10

Example:

- Assuming $M_1$ is biased in active region, what is the small-signal gain of the following circuit?

$$A = \frac{v_{\text{out}}}{v_{\text{in}}} = -g_{m} \cdot (\alpha \cdot r) = -g_{m} \cdot r$$

- $I_1$ is a current source and ideally has an infinite impedance.

This is the maximum gain of this amplifier (why?), and is known as the intrinsic gain.

- How can $V_{\text{in}}$ change if $I_1$ is constant?

$$I_1 = \frac{1}{2} \cdot \mu \cdot C \cdot \frac{W}{L} \cdot (V_{\text{in}} - V_t) \cdot (1 + \lambda \cdot V_{\text{in}})$$

Here we have to take channel-length modulation into account. As $V_{\text{in}}$ changes, $V_{\text{OUT}}$ also changes to keep $I_1$ constant.

Diode Connected Load - 1

- Often, it is difficult to fabricate tightly controlled or reasonable size resistors on chip. So, it is desirable to replace the load resistor with a MOS device.

- Recall the diode connected devices:

<table>
<thead>
<tr>
<th>Body Effect</th>
<th>$R_X$ (when $\lambda \neq 0$)</th>
<th>$R_X$ (when $\lambda = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO</td>
<td>$r \frac{1}{g_m}$</td>
<td>$\frac{1}{g_m}$</td>
</tr>
<tr>
<td>YES</td>
<td>$r \frac{1}{g_m + g_{so}}$</td>
<td>$\frac{1}{g_m + g_{so}}$</td>
</tr>
</tbody>
</table>
Diode Connected Load - 2

- Now consider the common-source amplifier with two types of diode connected loads:
  1. PMOS diode connected load:
     (No body effect)
     ![PMOS diode connected load diagram]
     \[ V_{gs} \]
     \[ L, R_x \]
     \[ V_{ds} \]
   
   2. NMOS diode connected load:
     (Body effect has to be taken into account)
     

Diode Connected Load - 3

PMOS Diode Connected Load:
- Note that this is a common source configuration with \( M_2 \) being the load. We have:
  \[ A_c = \frac{V_{oc}}{V_{in}} = -g_{m1} \cdot (R_s) = -g_{m1} \cdot \left( \frac{1}{g_{m2}} \right) \]

- Ignoring the channel length modulation (\( r_{o1} = r_{o2} = \infty \)), we can write:
  \[ A_c = -g_{m1} \cdot \left( \frac{1}{g_{m2}} \right) = \frac{g_{m1}}{g_{m2}} - \frac{2 \mu_p C_{ox} \cdot \frac{W}{L} \cdot I_{D1}}{2 \mu_n C_{ox} \cdot \frac{W}{L} \cdot I_{D2}} = \frac{\mu_p \left( \frac{W}{L} \right)_1}{\mu_n \left( \frac{W}{L} \right)_2} \]
  \[ A_v = -g_{m1} \cdot \left( \frac{1}{g_{m2}} \right) = -g_{m1} \cdot \left( \frac{1}{g_{m2}} \right) \left( \frac{V_{GS1} - V_{TH1}}{V_{GS2} - V_{TH2}} \right) = \frac{V_{GS2} - V_{TH2}}{V_{GS1} - V_{TH1}} \]
Diode Connected Load - 4

NMOS Diode Connected Load:

- Again, note that this is a common source configuration with $M_2$ being the load. We have:

$$A = \frac{v_{\text{out}}}{v_{\text{in}}} = -g_m \cdot \left( R_{r1} \parallel R_{r2} \right) = -g_m \cdot \left( \frac{1}{g_{sa} + g_{mb}} \right)$$

- Ignoring the channel length modulation ($r_{o1}=r_{o2}=\infty$), we can write:

$$A_v = -g_{m1} \cdot \left( \frac{1}{g_{m2} + g_{mb2}} \right) = -g_{m1} \cdot \frac{g_{m1}}{g_{m2} + g_{mb2}} = g_{m1} \cdot \frac{1}{g_{m2} + g_{mb2} \cdot (1 + \eta)}$$

$$A_v = -\frac{W_1}{1+\eta} \cdot \left( \frac{W}{L} \right)_{1} = -\frac{W_1}{1+\eta} \cdot \left( \frac{W}{L} \right)_{2} = -\frac{W_1}{1+\eta} \cdot \frac{V_{GS2} - V_{TH}}{V_{GS1} - V_{TH}}$$

Diode Connected Load - 5

- For a diode connected load we observe that (to the first order approximation):

1. The amplifier gain is not a function of the bias current. So, the change in the input and output levels does not affect the gain, and the amplifier becomes more linear.

2. The amplifier gain is not a function of the input signal (amplifier becomes more linear).

3. The amplifier gain is a weak function (square root) of the transistor sizes. So, we have to change the dimensions by a considerable amount so as to increase the gain.
4. The gain of the amplifier is reduced when body effect should be considered.

5. We want $M_1$ to be in saturation, and $M_2$ to be on ($M_2$ cannot be in triode (why?).

6. The voltage swing is constrained by both the required overdrive voltages and the threshold voltage of the diode connected device.

\[ M1: V_{OUT} > V_{GS1} - V_{TH1} = V_{eff1}, \quad M2: V_{OUT} < V_{DD} - V_{TH2} \]

7. A high amplifier gain leads to a high overdrive voltage for the diode connected device which limits the voltage swing.

---

**Diode Connected Load - 6**

**Example:**

- Find the gain of the following circuit if $M1$ is biased in saturation and $I_s = 0.75I_1$.

\[
A = \frac{v_{OUT}}{v_{IN}} = g_{m1} \left( R \right)^{-1} g_{m2} \left( \frac{W}{L} \right) = -g_{m1} \left( \frac{1}{g_{m2}} \right) \left( \frac{W}{L} \right) = -g_{m1} \left( \frac{1}{g_{m2}} \right) \left( \frac{W}{L} \right)
\]

- Ignoring the channel length modulation ($r_{o1} = r_{o2} = \infty$) we get:

\[
A_v = -g_{m1} \left( \frac{1}{g_{m2}} \right) \left( \frac{W}{L} \right) = -g_{m1} \left( \frac{1}{g_{m2}} \right) \left( \frac{W}{L} \right)
\]

\[
A_v = -2 \cdot \left( \frac{\mu_n}{L} \right) \left( \frac{W}{L} \right) \left( \frac{W}{L} \right) = 4 \cdot \frac{V_{SG2} - V_{TH2}}{V_{GS1} - V_{TH1}}
\]
Diode Connected Load - 7

Example (Continued):

- We observe for this example that:

  1. For fixed transistor sizes, using the current source increases the gain by a factor of 2.

  2. For fixed overdrive voltages, using the current source increases the gain by a factor of 4.

  3. For a given gain, using the current source allows us to make the diode connected load 4 times smaller.

  4. For a given gain, using the current source allows us to make the overdrive voltage of the diode connected load 4 times smaller. This increases the headroom for voltage swing.

Current Source Load - 1

- Note that current source $M_2$ is the load.

- Recall that the output impedance of $M_2$ seen from $V_{out}$:

\[
R_o = \frac{V_{out}}{I_o} = r_o
\]

\[
\beta = \frac{V_{sso}}{V_{in}} = -g_{m1} \left( R_s \right) = -g_{m1} \left( r_o \right)
\]

- For large gain at given power, we want large $r_o$ and:

\[
r_o = \frac{1}{\lambda \cdot I_D} \propto \frac{1}{W \cdot L} = \frac{L^2}{W}
\]

Increase $L$ and $W$ keeping the aspect ratio constant (so $r_o$ increases and $I_D$ remains constant). However, this approach increases the capacitance of the output node.

- We want $M_2$ to be in saturation so

\[
V_{SD2} = \frac{V_{DD} - V_{OUT} > V_{SG2} - V_{TH}}{V_{TH}} \Rightarrow V_{eff2} \Rightarrow V_{OUT} < V_{DD} - V_{eff2}
\]
Current Source Load - 2

- We also want $M_1$ to be in saturation:
  \[ V_{\text{sat}} > V_{\text{out}} - V_{\text{in}} > V_{\text{th}} \rightarrow V_{\text{sat}} > V_{\text{th}} \]

- Thus, we want $V_{\text{eff1}}$ and $V_{\text{eff2}}$ to be small, so that there is more headroom for output voltage swing. For a constant $I_D$, we can increase $W_1$ and $W_2$ to reduce $V_{\text{eff1}}$ and $V_{\text{eff2}}$.

- The intrinsic gain of this amplifier is: $A = g_m \cdot r_o$

- In general, we have:
  \[ g_m \propto \frac{W}{L}, \quad r_o \propto \frac{L}{W} \rightarrow A \propto \frac{W}{L} \]

- But since current in this case is roughly constant:
  \[ g_m = \left[ 2\mu \cdot C_D \cdot \frac{W}{L} \cdot I_0 \right] \propto \frac{W}{L}, \quad r_o = \frac{1}{\lambda \cdot I_0} \propto \frac{1}{L} \rightarrow A \propto \sqrt{LW} \]

Triode Load

- We recognize that this is a common source configuration with $M_2$ being the load. Recall that if $M_2$ is in deep triode, i.e., $V_{SD} < 2(V_{SG} - |V_{TH}|)$, it behaves like a resistor.

- $V_b$ should be low enough to make sure that $M_2$ is in deep triode region and usually requires additional complexity to be precisely generated.

- $R_{ON2}$ depends on $\mu$, $C_{ox}$, and $V_{TH}$ which in turn depend on the technology being used.

- In general, this amplifier with triode load is difficult to design and use!

- However, compared to diode-connected load, triode load consumes less headroom: $M_1: V_{\text{OUT}} > V_{GS1} - V_{TH} > V_{\text{eff1}}$, $M_2: V_{\text{OUT}} > V_{DD}$
Source Degeneration - 1

- The following circuit shows a common source configuration with a degeneration resistor in the source.

- We will show that this configuration makes the common source amplifier more linear.

- We will use two methods to derive the gain of this circuit:
  1. Small-signal Model
  2. Using the following Lemma

**Lemma:**

In linear systems, the voltage gain is equal to \(-G_m R_{out}\).

Source Degeneration - 2

Gain – Method 1: Small Signal Model

\[
\begin{align*}
    i_{OUT} &= g_m \cdot v_1 + g_{mb} \cdot v_{BS} + \frac{V_{OUT} - i_{OUT} \cdot R_S}{r_D}, \\
    v_1 &= v_{IN} - i_{OUT} \cdot R_S = v_{IN} + \frac{V_{OUT}}{R_D} \cdot R_S, \\
    v_{BS} &= -i_{OUT} \cdot R_S = \frac{V_{OUT}}{R_D} \cdot R_S
\end{align*}
\]

\[
A_v = \frac{V_{OUT}}{v_{IN}} = \frac{\left(-g_m \cdot r_D \cdot R_D\right) + \left(\frac{V_{OUT} \cdot R_S}{r_D}\right)}{r_D \cdot \left(1 + \left(\frac{g_m + g_{mb}}{r_D} \cdot R_S\right) + \frac{R_D + R_S}{r_D}\right)} = -g_m \cdot v_{IN} \cdot R_D
\]
Source Degeneration - 3

Gain – Method 2: Lemma

- The Lemma states that in linear systems, the voltage gain is equal to $-G_m R_{out}$. So we need to find $G_m$ and $R_{out}$.

1. **$G_m$**:
   - Recall that the equivalent transconductance of the above circuit is:
   
   
   ![Circuit Diagram]

   
   
   
   \[
   G_m = \frac{v_{IN}}{v_{OUT}} = \frac{g_m \cdot r_D}{r_D + r_D \cdot (g_m \cdot R_S + g_{mb} \cdot R_S) + R_S} = \frac{R_S}{r_D + (g_m + g_{mb}) \cdot R_S} + R_S
   \]

Source Degeneration - 4

Gain – Method 2: Lemma (Continued)

1. **$R_{OUT}$**:
   - We use the following small signal model to derive the small signal output impedance of this amplifier:

   \[
   v_i = -i_x \cdot R_S \quad , \quad v_{gs} = -i_x \cdot R_S
   \]

   \[
   v_x = i_x \cdot R_S + (i_x \cdot R_S + g_m \cdot v_i - g_m \cdot v_{gs}) \cdot r_D
   \]

   \[
   R_S = \frac{v_x}{i_x} = R_S + (1 + g_m \cdot R_S + g_{mb} \cdot R_S) \cdot r_D = R_S + \left(1 + \frac{g_m + g_{mb}}{R_S} \cdot R_S \right) \cdot r_D
   \]

   \[
   R_{OUT} = R_S \parallel R_D = \frac{R_S + \left(1 + \frac{g_m + g_{mb}}{R_S} \cdot R_S \right) \cdot r_D}{R_S + \left(1 + \frac{g_m + g_{mb}}{R_S} \cdot R_S \right) \cdot r_D} + R_D
   \]

   - Since typically $r_D > > R_S$:

   \[
   R_S = R_S + \left(1 + \frac{g_m + g_{mb}}{R_S} \cdot R_S \right) \cdot r_D = (R_S + (g_m + g_{mb}) \cdot R_S) \cdot r_D = \left(g_m + g_{mb} \right) \cdot R_S \cdot r_D
   \]

\[38\]
Source Degeneration - 5

Gain – Method 2: Lemma (Continued)

\[ G_m = \frac{g_m \cdot r_0}{r_0 (1 + (g_m + g_{mb}) \cdot R_s) + R_S} \]

\[ R_{OUT} = \frac{(R_S + (1 + (g_w + g_{mb}) \cdot R_s) \cdot r_0) \cdot R_D}{R_S + (1 + (g_w + g_{mb}) \cdot R_s) \cdot r_0 + R_D} \]

\[ A_c = -G_m \cdot R_{OUT} = -\frac{g_m \cdot r_0}{r_0 (1 + (g_m + g_{mb}) \cdot R_s) + R_S} \cdot \frac{(r_0 + (1 + g_m \cdot r_0 + g_{mb} \cdot r_0) \cdot R_s) \cdot R_D}{r_0 + (1 + g_m \cdot r_0 + g_{mb} \cdot r_0) \cdot R_s + R_D} \]

\[ = \frac{-g_m \cdot r_0 \cdot R_D}{r_0 + (1 + g_m + g_{mb}) \cdot R_s + R_D} \]

Source Degeneration - 6

- If we ignore body effect and channel-length modulation:

**Method 1 – Small-signal Model:**

\[ v_{out} = -g_m \cdot v_{in} \cdot R_g \quad , \quad v_{in} = v_m - g_m \cdot v_o \cdot R_g \]

\[ A = \frac{v_{out}}{v_{in}} = -\frac{g_m \cdot R_g}{1 + g_m \cdot R_g} \]

**Method 2 – Taking limits:**

\[ G_m = \lim_{v_{out} \to \infty} \frac{g_m \cdot r_0}{r_0 + (g_m \cdot R_s + g_{mb} \cdot R_s) + R_s} = \frac{g_m}{1 + (g_m + g_{mb}) \cdot R_s} = \frac{g_m}{1 + g_m \cdot R_s} \]

\[ R_{out} = \lim_{v_{out} \to \infty} \frac{(r_0 + (1 + g_m \cdot r_0 + g_{mb} \cdot r_0) \cdot R_s) \cdot R_s}{r_0 + (1 + g_m \cdot r_0 + g_{mb} \cdot r_0) \cdot R_s + R_s} = \frac{1 + (g_m + g_{mb}) \cdot R_s}{1 + (g_m + g_{mb}) \cdot R_s} = R_s \]

\[ A_c = -G_m \cdot R_{out} = -\frac{g_m \cdot R_g}{1 + g_m \cdot R_g} \]
Obtaining $G_m$ and $R_{out}$ directly assuming $\lambda = \gamma = 0$:

1. $G_m$:

$$i_n = g_m \cdot v_{in}, \quad v_{os} = v_{in} - g_m \cdot v_{in} \cdot R_s$$

$$v_{os} = v_{in} \cdot \frac{1}{1 + g_m \cdot R_s} \rightarrow G_m = \frac{i_n}{v_{os}} = \frac{g_m}{1 + g_m \cdot R_s}$$

2. $R_{out}$:

$$v_{os} = -g_m \cdot v_{in} \cdot R_s \rightarrow v_{os} = 0$$

$$i_n = \frac{v}{R_s} + g_m \cdot v_{in} = \frac{v}{R_s}$$

$$R_{out} = \frac{v}{i_n} = R_s$$

$$A = -G_m \cdot R_{out} = -\frac{g_m \cdot R_s}{1 + g_m \cdot R_s}$$

Source Degeneration - 8

- If we ignore body effect and channel-length modulation:

$$G_m = \frac{R_m}{1 + g_m \cdot R_s}, \quad R_{out} = R_s \rightarrow A = -\frac{g_m \cdot R_s}{1 + g_m \cdot R_s}$$

- We Notice that as $R_s$ increases $G_m$ becomes less dependent on $g_m$:

$$\lim_{R_s \rightarrow \infty} G_m = \lim_{R_s \rightarrow \infty} \frac{g_m \cdot R_s}{1 + g_m \cdot R_s} = \frac{1}{R_s}$$

- That is for large $R_s$:

$$G_m = \frac{i_n}{v_{os}} \approx \frac{1}{R_s} \rightarrow v_{os} \approx R_s \cdot i_{os}$$

- Therefore, the amplifier becomes more linear when $R_s$ is large enough. Intuitively, an increase in $v_{in}$ tend to increase $I_D$, however, the voltage drop across $R_s$ also increases. This makes the amplifier less sensitive to input changes, and makes $I_D$ smoother!

- The linearization is achieved at the cost of losing gain and voltage headroom.
Source Degeneration - 9

- We can manipulate the gain equation so the numerator is the resistance seen at the drain node, and the denominator is the resistance in the source path.

\[ A = \frac{-g_{m} \cdot R_{s}}{1 + g_{m} \cdot R_{s}} = \frac{-R_{s}}{1 + g_{m} \cdot R_{s}} \]

- The following are \( I_{D} \) and \( g_{m} \) of a transistor without \( R_{S} \).

\[
\begin{align*}
\Delta I & = -g_{m} \Delta V \\
\Delta V & = \frac{\Delta V_{DS}}{R_{S}} \\
\end{align*}
\]

- \( I_{D} \) and \( g_{m} \) of a transistor considering \( R_{S} \) are:
  - When \( I_{D} \) is small such that \( R_{S} g_{m} \ll 1 \), \( G_{m} \approx g_{m} \).
  - When \( I_{D} \) is large such that \( R_{S} g_{m} \gg 1 \), \( G_{m} \approx 1/R_{S} \).

Alternative Method to Find the Output-Resistance of a Degenerated Common-Source Amplifier

\[
\begin{align*}
\Delta V & = \frac{\Delta V_{DS}}{R_{S}} \\
\Delta V_{RS} & = \frac{1}{g_{m} + g_{mb}} \\
\end{align*}
\]
Why Buffers?

- Common Source amplifiers needed a large load impedance to provide a large gain.

- If the load is small but we need a large gain (can you think of an example?) a buffer is used.

- Source-follower (common-drain) amplifiers can be used as buffers.

\[ R_{\text{IN}} = \infty \quad , \quad R_{\text{OUT}} = 0 \quad , \quad A_I = 1 \]

Ideal Buffer:
1. \( R_{\text{IN}} = \infty \): the input current is zero; it doesn’t load the previous stage.
2. \( R_{\text{OUT}} = 0 \): No voltage drop at the output; behaves like a voltage source.

Resistive Load - 1

- We will examine the Source follower amplifier with two different loads:
  1. Resistive Load
  2. Current Source Load

- Resistive Load:
  - As shown below the output (source voltage) will follow the input (gate voltage). We will analyze the following circuit using large-signal and small-signal analysis.
Resistive Load - 2

Large Signal Analysis:
- The relationship between $V_{\text{IN}}$ and $V_{\text{OUT}}$ is:

$$V_{\text{out}} = R_s \cdot I_n = \frac{1}{2} \mu C_m \frac{W}{L} (V_{\text{in}} - V_{\text{sat}}) \cdot (1 + \lambda \cdot V_{\text{sat}}) \cdot R_s$$

$$V_{\text{out}} = \frac{1}{2} \mu C_m \frac{W}{L} (V_{\text{in}} - V_{\text{sat}} - V_{\text{th}}) \cdot (1 + \lambda \cdot V_{\text{sat}} - \lambda \cdot V_{\text{th}}) \cdot R_s$$

- Differentiate with respect to $V_{\text{IN}}$:

$$\frac{\partial V_{\text{out}}}{\partial V_{\text{in}}} = \mu C_m \frac{W}{L} (V_{\text{in}} - V_{\text{sat}} - V_{\text{th}}) \left(1 - \frac{\partial V_{\text{in}}}{\partial V_{\text{in}}} - \frac{\partial V_{\text{sat}}}{\partial V_{\text{in}}} \right) \cdot (1 + \lambda \cdot V_{\text{sat}} - \lambda \cdot V_{\text{th}}) \cdot R_s$$

$$+ \frac{1}{2} \mu C_m \frac{W}{L} (V_{\text{in}} - V_{\text{sat}} - V_{\text{th}}) \cdot (1 - \lambda \cdot V_{\text{sat}} + \lambda \cdot V_{\text{th}}) \cdot \frac{\partial V_{\text{in}}}{\partial V_{\text{in}}} \cdot R_s$$

- Need to find the derivative of $V_{\text{TH}}$ with respect to $V_{\text{IN}}$:

$$V_{\text{IN}} = V_{\text{sat}} + \gamma \cdot \left( \sqrt{2 \cdot \Phi_m + V_{\text{IN}}} - \sqrt{2 \cdot \Phi_m} \right) \quad V_{\text{sat}} = V_{\text{out}}$$

$$\frac{\partial V_{\text{sat}}}{\partial V_{\text{IN}}} = \frac{\partial V_{\text{sat}}}{\partial V_{\text{sat}}} = \frac{\gamma}{2 \sqrt{2 \cdot \Phi_m + V_{\text{IN}}} + \sqrt{2 \cdot \Phi_m}} \quad \frac{\partial V_{\text{sat}}}{\partial V_{\text{sat}}} = \eta \cdot \frac{\partial V_{\text{sat}}}{\partial V_{\text{sat}}}$$

Resistive Load - 3

Large Signal Analysis (Continued):
- The small signal gain can be found:

$$A_s = \frac{\partial V_{\text{sat}}}{\partial V_{\text{sat}}} \left(1 + g_{\text{m}} \cdot R_s + g_{\text{m}} \cdot \eta \cdot R_s + I_n \cdot R_s \cdot \lambda \right) = g_{\text{m}} \cdot R_s$$

$$A_s = \frac{\partial V_{\text{sat}}}{\partial V_{\text{sat}} \cdot \frac{g_{\text{m}} \cdot R_s}{1 + g_{\text{m}} \cdot R_s + g_{\text{m}} \cdot \eta \cdot R_s + \frac{R_s}{r_c}} = \frac{g_{\text{m}} \cdot R_s}{1 + \left(g_{\text{m}} + g_{\text{m}} + \frac{1}{r_c} \right) \cdot R_s}$$

- If channel-length modulation is ignored ($r_c = \infty$) we get:

$$A_s = \frac{\partial V_{\text{sat}}}{\partial V_{\text{sat}}} = \frac{g_{\text{m}} \cdot R_s}{1 + (g_{\text{m}} + g_{\text{m}}) \cdot R_s}$$
Resistive Load - 4

Small Signal Analysis:
- We get the following small signal model:

\[
\begin{align*}
    v_{os} &= (g_m \cdot v_{os} + g_m \cdot v_{in}) \cdot R_g, \\
    v_{in} &= v_{os} - v_{os}, \\
    v_{os} &= -v_{in}.
\end{align*}
\]

\[
\begin{align*}
    v_{os} &= (g_m \cdot (v_{os} - v_{in}) + g_m \cdot (-v_{os})), \\
    v_{in} &= (R_g + r_g + g_m \cdot R_s \cdot r_s + g_m \cdot R_s \cdot r_s) = g_m \cdot R_s \cdot r_s \cdot v_{os}, \\
    A &= \frac{v_{os}}{v_{in}} = g_m \cdot R_s \cdot r_s = \frac{g_m \cdot R_s \cdot r_s}{R_s \cdot (1 + g_m \cdot r_s + g_m \cdot r_s) + r_s}, \\
    A &= \frac{g_m \cdot R_s}{R_s \cdot \left(\frac{1}{r_s} + g_m + g_m\right) + 1}.
\end{align*}
\]

Resistive Load - 5

- Graph of the gain of a source-follower amplifier:

1. \( M_1 \) never enters the triode region as long as \( V_{in} < V_{dd} \).
2. Gain is zero if \( V_{in} \) is less than \( V_{th} \) (because \( g_m \) is 0).
3. As \( V_{in} \) increases, \( g_m \) increases and the gain becomes:

\[
A \approx \frac{g_m}{g_m + g_m} = \frac{1}{1 + \eta}
\]

4. As \( V_{out} \) increases, \( \eta \) decreases, and therefore, the maximum gain increases.
5. Even if \( R_s = \infty \), the gain is less than one:

\[
A \approx \frac{g_m}{g_m + g_m + \frac{1}{r_s}} < 1
\]

6. Gain depends heavily on the DC level of the input (nonlinear amplifier).
Current Source Load

- In a source follower with a resistive load, the drain current depends on the DC level of $V_{IN}$, which makes the amplifier highly nonlinear.

- To avoid this problem, we can use a current source as the load.

- The output resistance is:

$$R_{out} = r_o \left( \frac{1}{g_{m1}} + \frac{1}{g_{m2}} \right)$$

- If channel length modulation is ignored ($r_o1=r_o2=\infty$):

$$R_{out} = \frac{1}{g_{m1}} + \frac{1}{g_{m2}}$$

- Note that the body effect reduces the output impedance of the source follower amplifiers.

Voltage Division in Source Followers - 1

- When calculating output resistance seen at the source of $M_1$, i.e., $R_{M1}$, we force $V_{IN}$ to zero and find the output impedance:

$$R_{out} = r_o \left( \frac{1}{g_{m1}} + \frac{1}{g_{m2}} \right)$$

- However, if we were to find the gain of the amplifier, we would not suppress $V_{IN}$.

- Here, we would like to find an equivalent circuit of $M_1$, from which we can find the gain.

- Consider the small-signal model of $M_1$:
Voltage Division in Source Followers - 2

- For small-signal analysis $V_{BS} = V_{DS}$, so $g_{mb} V_{BS}$ dependant current source can be replaced by a resistor ($1/g_{mb}$) between source and drain.
- Note that, when looking at the circuit from the source terminal, we can replace the $g_{m} V_{GS}$ dependant current source with a resistor (of value $1/g_{m}$) between source and gate.

• Simplified circuit:

$$\text{Simplified circuit:}$$

$$\begin{align*}
V_{in} & = \frac{1}{g_{m}} \left( \frac{R}{R_{L}} \right) V_{in} \\
A & = \frac{V_{out}}{V_{in}} = \frac{1}{g_{m}} \left( \frac{1}{R_{L}} + \frac{1}{g_{m}} \right)
\end{align*}$$

Voltage Division in Source Followers - 3

Example:

- Find the gain of a source follower amplifier with a resistive load.
- We draw the small signal model of this amplifier as shown below to get:

$$\begin{align*}
A & = \frac{1}{R_{L} + \frac{1}{g_{m}}} \\
& = \frac{R_{L} \cdot r_{o}}{R_{L} \cdot r_{o} + R_{g} \cdot r_{o} \cdot g_{m} + \frac{1}{g_{m}}}
\end{align*}$$

- We can show that this is equal to what we obtained before:
Example:

- Find the gain of a source follower amplifier with a current source load.
- Small-signal model of this amplifier is:

\[
\begin{align*}
V_{\text{in}} &= \frac{1}{g_{\text{me}}} + \frac{1}{g_{\text{m}}} V_{\text{in}} \\
A &= \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{g_{\text{me}}} + \frac{1}{g_{\text{m}}}
\end{align*}
\]

- If we ignore channel length modulation:

\[
\begin{align*}
V_{\text{out}} &= \frac{1}{g_{\text{me}}} + \frac{1}{g_{\text{m}}} V_{\text{in}} \\
A &= \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{g_{\text{me}}} + \frac{1}{g_{\text{m}}}
\end{align*}
\]
Example:
• Find the gain of a source follower amplifier with a resistive load.

• Small-signal model of this amplifier is:

\[
\begin{align*}
V_{\text{in}} &= \frac{1}{g_{m1}} v_{\text{in}}, \\
V_{\text{out}} &= \frac{1}{g_{m2}} v_{\text{out}} + r_{\text{in}} (1/g_{m2}) (1/g_{\text{ro}})
\end{align*}
\]

\[
V_{\text{in}} = \frac{r_{\text{in}}}{g_{m1} g_{m2} (1 + g_{m2} r_{\text{in}})} \cdot v_{\text{in}} \quad \rightarrow \quad A = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{r_{\text{in}}}{r_{\text{in}} + \frac{1}{g_{m1} g_{m2} (1 + g_{m2} r_{\text{in}})}}
\]

Advantages and Disadvantages - 1

1. Source followers have typically small output impedance.
2. Source followers have large input impedance.
3. Source followers have poor driving capabilities.
4. Source followers are nonlinear. This nonlinearity is caused by:
   - Variable bias current which can be resolved if we use a current source to bias the source follower.
   - Body effect; i.e., dependence of $V_{\text{TH}}$ on the source (output) voltage. This can be resolved for PMOS devices, because each PMOS transistor can have a separate n-well. However, because of low mobility, PMOS devices have higher output impedance. (In more advanced technologies, NMOS in a separate p-well, can be implemented that potentially has no body effect)
   - Dependence of $r_{\text{on}}$ on $V_{\text{DS}}$ in submicron devices.
Advantages and Disadvantages - 2

5. Source followers have voltage headroom limitations due to level shift. Consider this circuit (a common source followed by a source follower):

- If we only consider the common source stage, $V_X > V_{GS1} - V_{TH1}$.
- If we only consider the source follower stage, $V_X > V_{GS3} - V_{TH3} + V_{GS2}$.
- Therefore, adding the source follower will reduce the allowable voltage swing at node X.
- The DC value of $V_{OUT}$ is $V_{GS2}$ lower than the DC value of $V_X$.

Common-Gate

$$A_v = (g_m + g_{mb})R_D = g_m(1 + \eta)R_D$$
Common-Gate

\[
A_v = \frac{(g_m + g_{mb}) r_o + 1}{r_o + (g_m + g_{mb}) r_o R_S + R_S + R_D} \quad R_D
\]

for \( R_S = 0 \): \( A_v \approx (g_m + g_{mb}) (r_o \parallel R_D) \)
Set 3: Single-Stage Amplifiers

Common-Gate Input Impedance

$R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o} = \frac{R_D}{1 + (g_m + g_{mb})r_o} + \frac{r_o}{1 + (g_m + g_{mb})r_o}$

$R_{in} = \frac{R_D}{1 + (g_m + g_{mb})r_o} + (r_o \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}})$

- Input impedance of common-gate stage is relatively low only if $R_D$ is small
- Example: Find the input impedance of the following circuit.
Example

• Calculate the voltage gain of the following circuit:

\[ A_v = 1 + (g_m + g_{mb})r_o \]

Common-Gate Output Impedance

\[ R_{out} = \{[1 + (g_m + g_{mb})R_S]r_o + R_S\} || R_D \]
Example

• Compare the gain of the following two circuits ($\lambda = \gamma = 0$ and 50Ω transmission lines!)

(a) ![Circuit A](image)

(b) ![Circuit B](image)

Cascode Stage

• Cascade of a common-source stage and a common-gate stage is called a “cascode” stage.

$$R_{out} = \left\{ \left[ 1 + (g_{m2} + g_{mb2})r_{o1} \right] r_{o2} + r_{o1} \right\} \parallel R_D$$

$$A_V \approx g_{m1} \left\{ r_{o1} r_{o2} \left( g_{m2} + g_{mb2} \right) \right\} \parallel R_D$$
Cascode Stage

\[ A_V \approx g_{m1} \left( \frac{r_o1}{r_o2} g_{m2} \right) \parallel \left( \frac{r_o3}{r_o4} g_{m3} \right) \]

Output Impedance Comparison
Shielding Property

Board Notes
Triple Cascode

- What is the output resistance of this circuit?

- Problem?

Folded Cascode

- Problem?
Output Impedance of a Folded Cascode

\[ R_{out} = \left[ 1 + (g_{m2} + g_{mb2})r_{o2} \right] (r_{o1} \parallel r_{o3}) + r_{o2} \]