EEC488: Analog CMOS Integrated Circuit Design

Set 4

Differential Amplifiers

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Overview

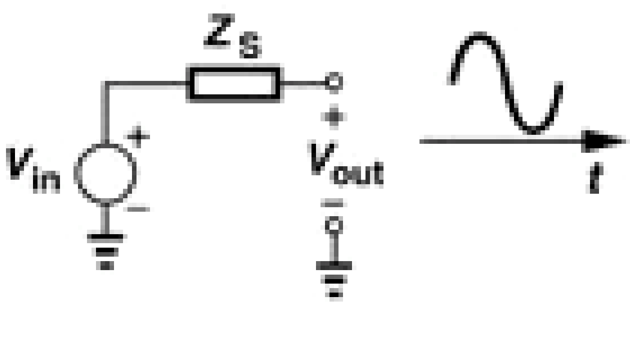
- The “differential amplifier” is one of the most important circuit inventions.
- Their invention dates back to vacuum tube era (1930s).
- Alan Dower Blumlein (a British Electronics Engineer, 1903-1942) is regarded as the inventor of the vacuum-tube version of differential pair.

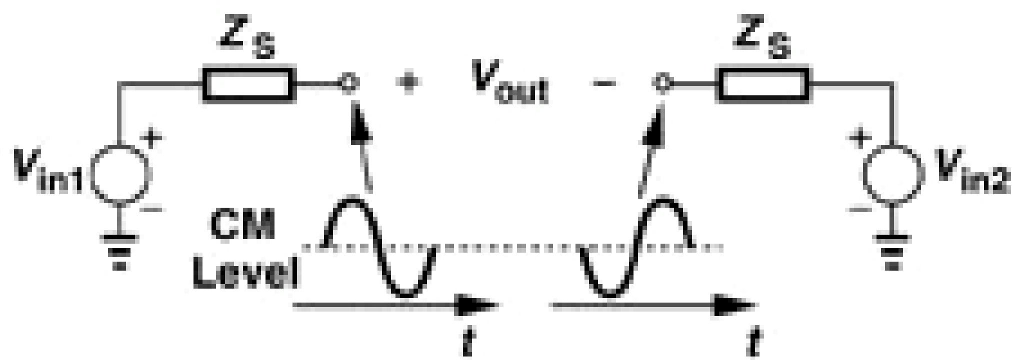
- Differential operation offers many useful properties and is widely used in analog and mixed-signal integrated circuits

Single-ended and Differential Signals

• A "single-ended" signal is a signal that is measured with respect to a fixed potential (typically ground).

• "Differential signal" is generally referred to a signal that is measured as a difference between two nodes that have equal but opposite-phase signal excursions around a fixed potential (the fixed potential is called common-mode (CM) level).





Board Notes (Differential Amplifiers)

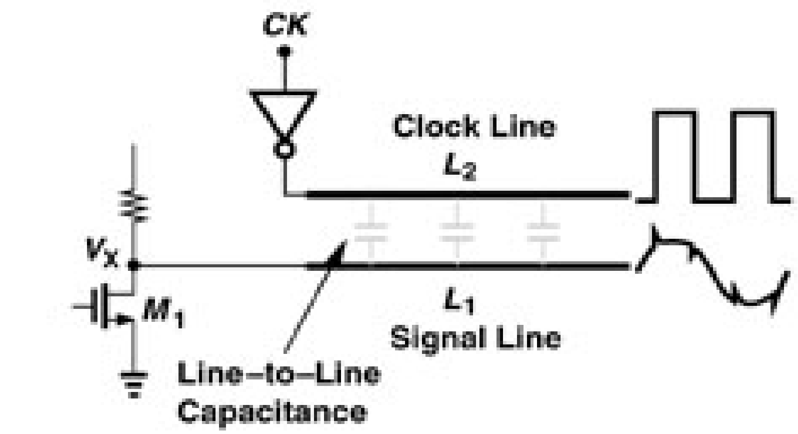
Why Differential?

- Better immunity to environmental noise

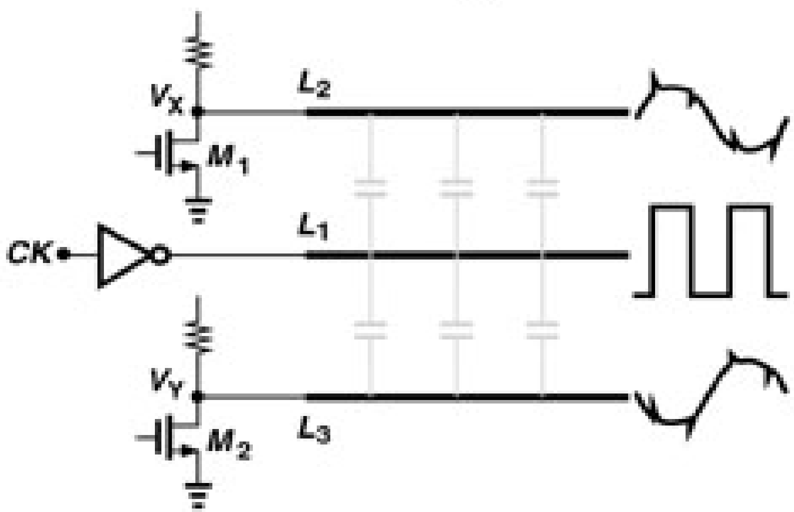
- Improved linearity

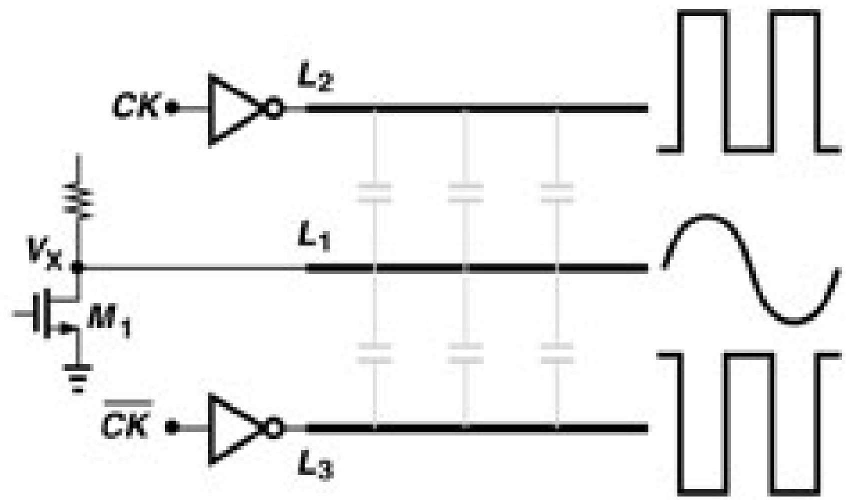
- Higher signal swing compared to single-ended

Higher Immunity to Noise Coupling

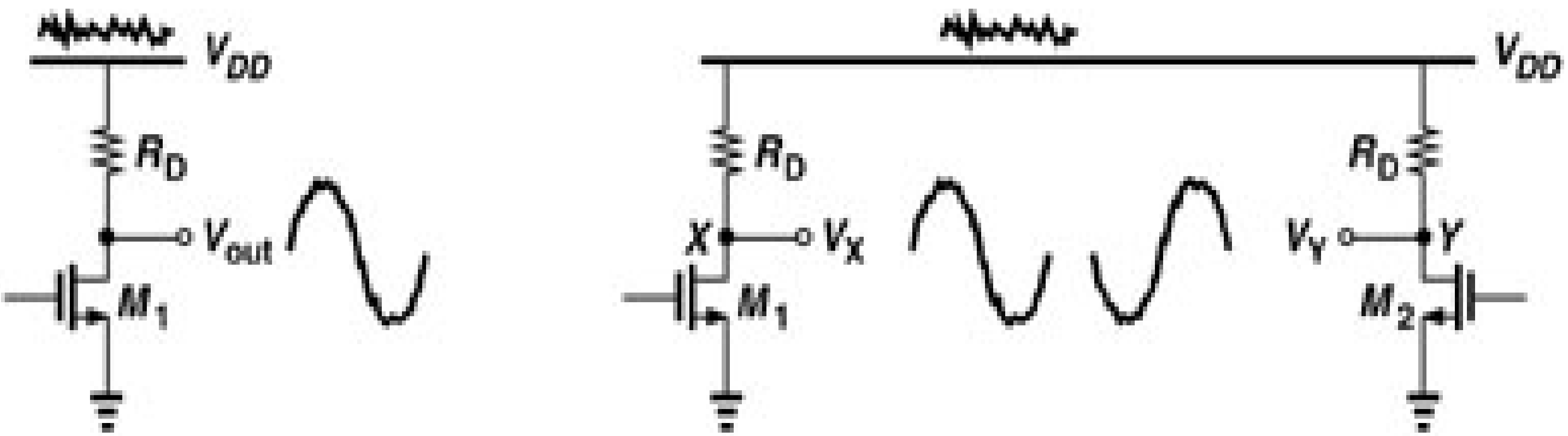


(a)



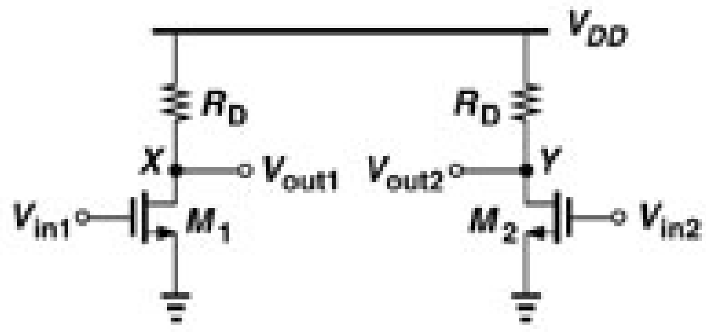


Supply Noise Reduction

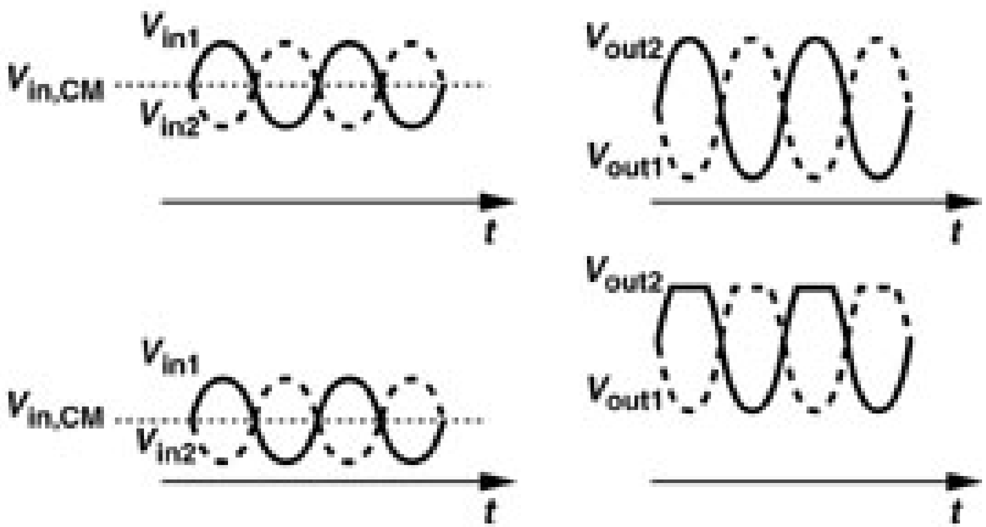


Board Notes (Improved Linearity)

Basic Differential Pair

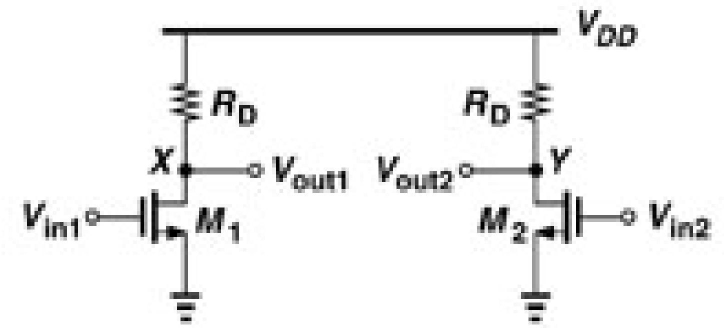


(a)



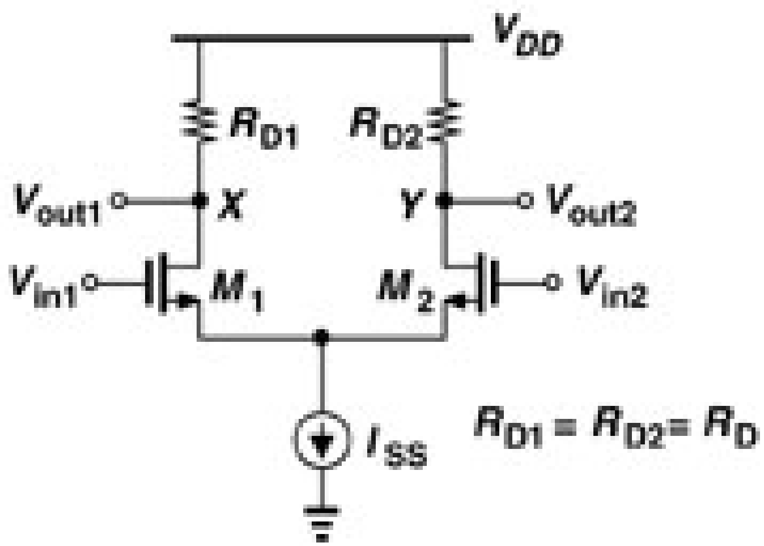
(b)

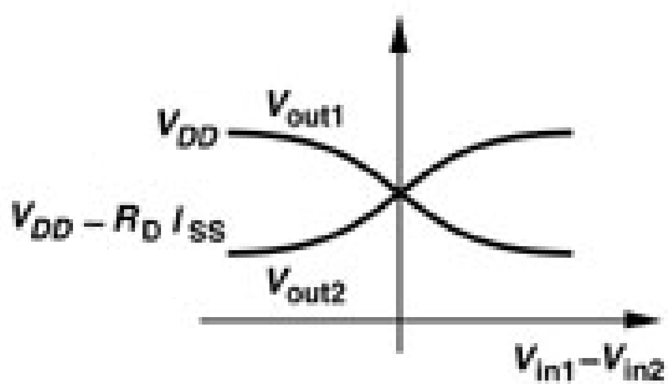
Basic Differential Pair

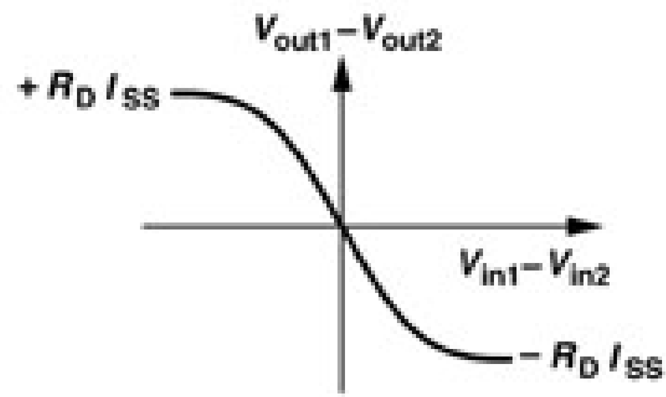


- Problem: Sensitive to input common-mode (CM) level
  - Bias current of the transistors M_1 and M_2 changes as the input CM level changes
  - g_m of the devices as well as output CM level change
- Can we think of a solution?

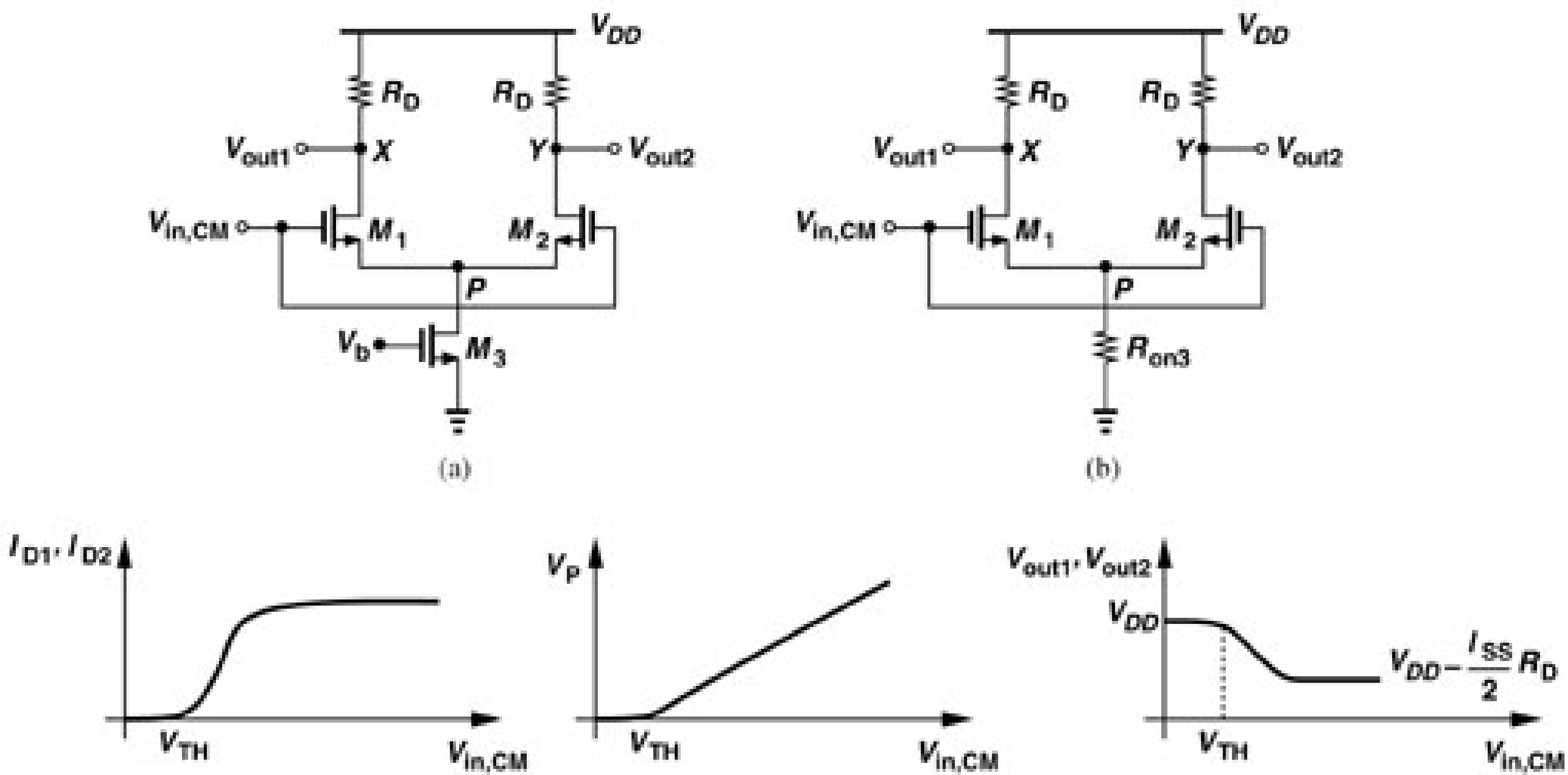
Differential Pair



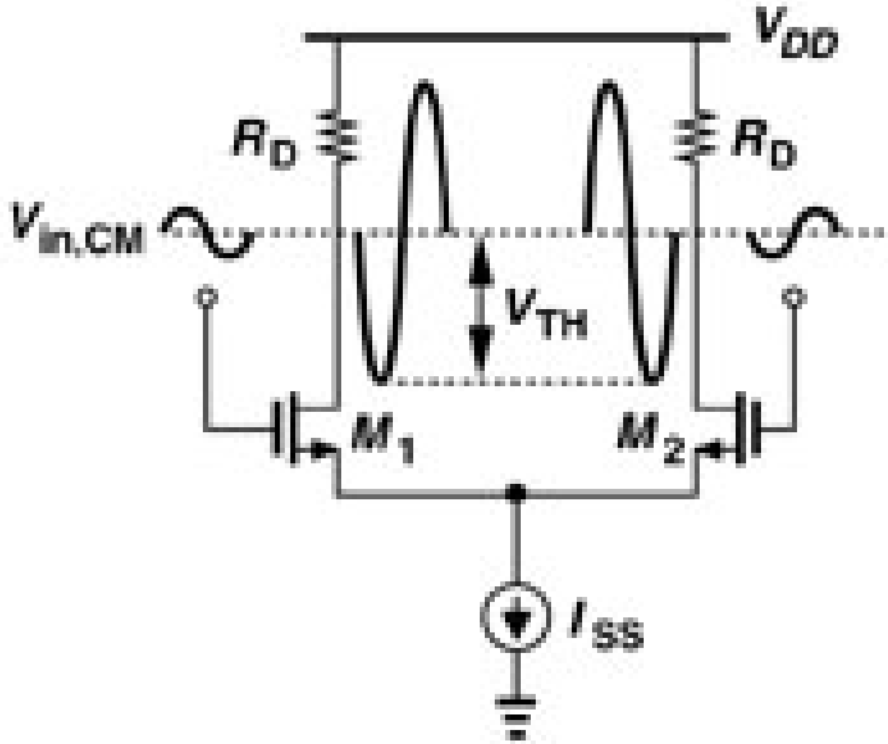




Common-Mode Response



Common-Mode Input versus Output Swing



Board Notes ("Half-Circuit" Concept)

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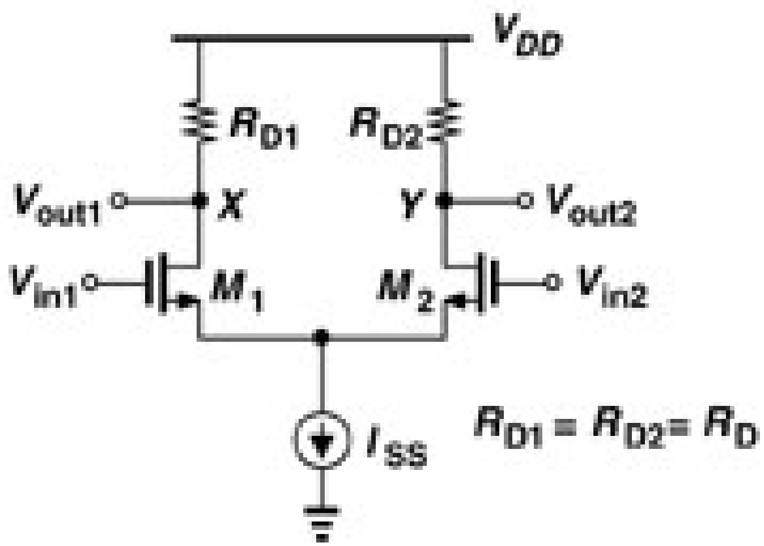
Example

- Using the half-circuit concept, calculate the small-signal differential gain of the following circuit (for two cases of λ=0 and λ≠0).



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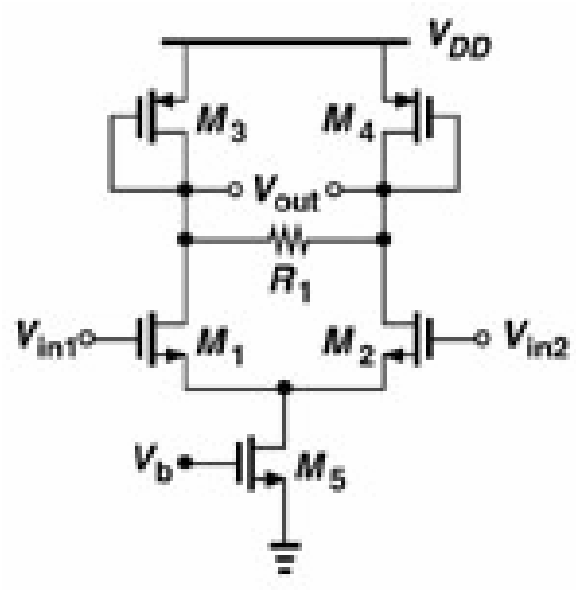
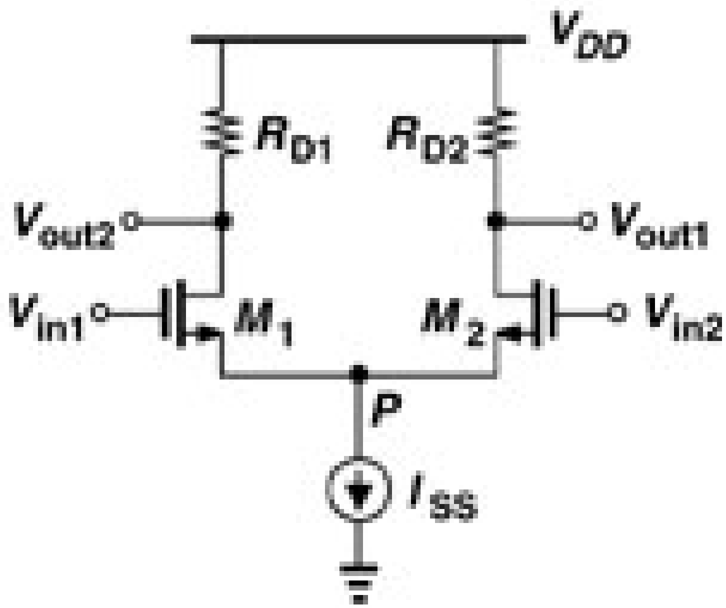


Example

- Sketch the small-signal gain of a differential pair as a function of its input common-mode level.

Analysis of Differential Amplifier



$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}, \qquad V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} + V_{TH}$$

Analysis of Differential Amplifier

$$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$\left(V_{in1} - V_{in2}\right)^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}}\left(I_{D1} + I_{D2} - 2\sqrt{I_{D1} I_{D2}}\right)$$

$$\frac{1}{2}\mu_n C_{ox} \frac{W}{L}\left(V_{in1} - V_{in2}\right)^2 - I_{SS} = -2\sqrt{I_{D1} I_{D2}}$$

$$\frac{1}{4}\left(\mu_n C_{ox} \frac{W}{L}\right)^2 \left(V_{in1} - V_{in2}\right)^4 + I_{SS}{}^2 - I_{SS}\mu_n C_{ox} \frac{W}{L}\left(V_{in1} - V_{in2}\right)^2 = 4 I_{D1} I_{D2}$$

Analysis of Differential Amplifier

Using:

$$4I_{D1}I_{D2} = (I_{D1} + I_{D2})^2 - (I_{D1} - I_{D2})^2 = I_{SS}^2 - (I_{D1} - I_{D2})^2$$

and

$$4I_{D1}I_{D2} = \frac{1}{4}(\mu_n C_{ox}\frac{W}{L})^2 (V_{in1} - V_{in2})^4 + I_{SS}{}^2 - I_{SS}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})^2$$

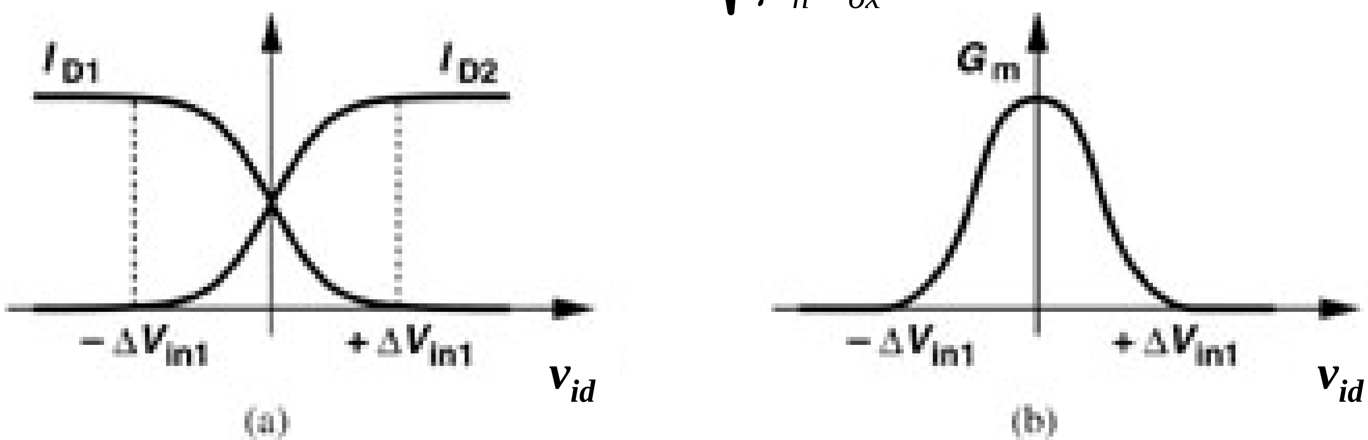
We have:

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

Analysis of Differential Amplifier

$$i_d = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}v_{id}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - v_{id}{}^2}$$

$$\frac{\partial i_d}{\partial v_{id}} = G_m = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}W/L} - v_{id}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}W/L} - v_{id}^2}}$$



Analysis of Differential Amplifier

- For small v_{id}:

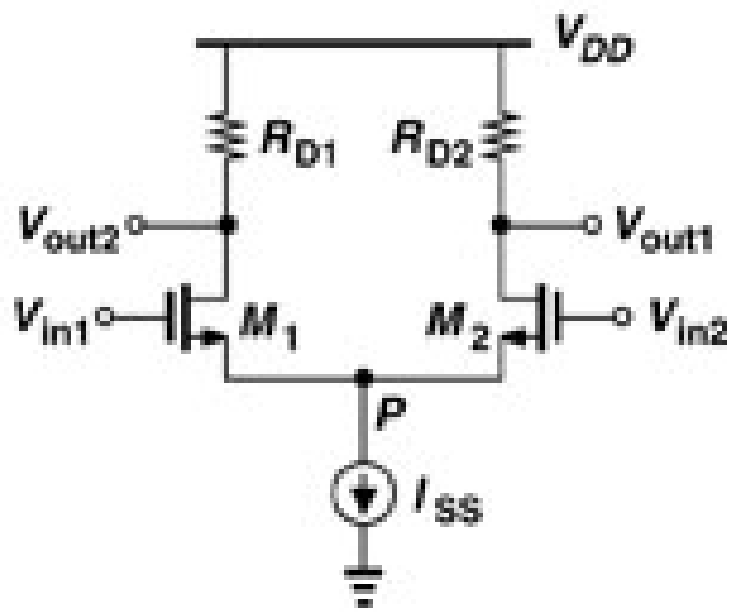
$$G_m = \frac{\partial i_d}{\partial v_{id}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} = g_{m1} = g_{m1}$$

- We have:

$$V_{out1} - V_{out2} = R_D(I_{D1} - I_{D2}) = R_D G_m (V_{in1} - V_{in2})$$

$$\frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = g_{m1} R_D$$

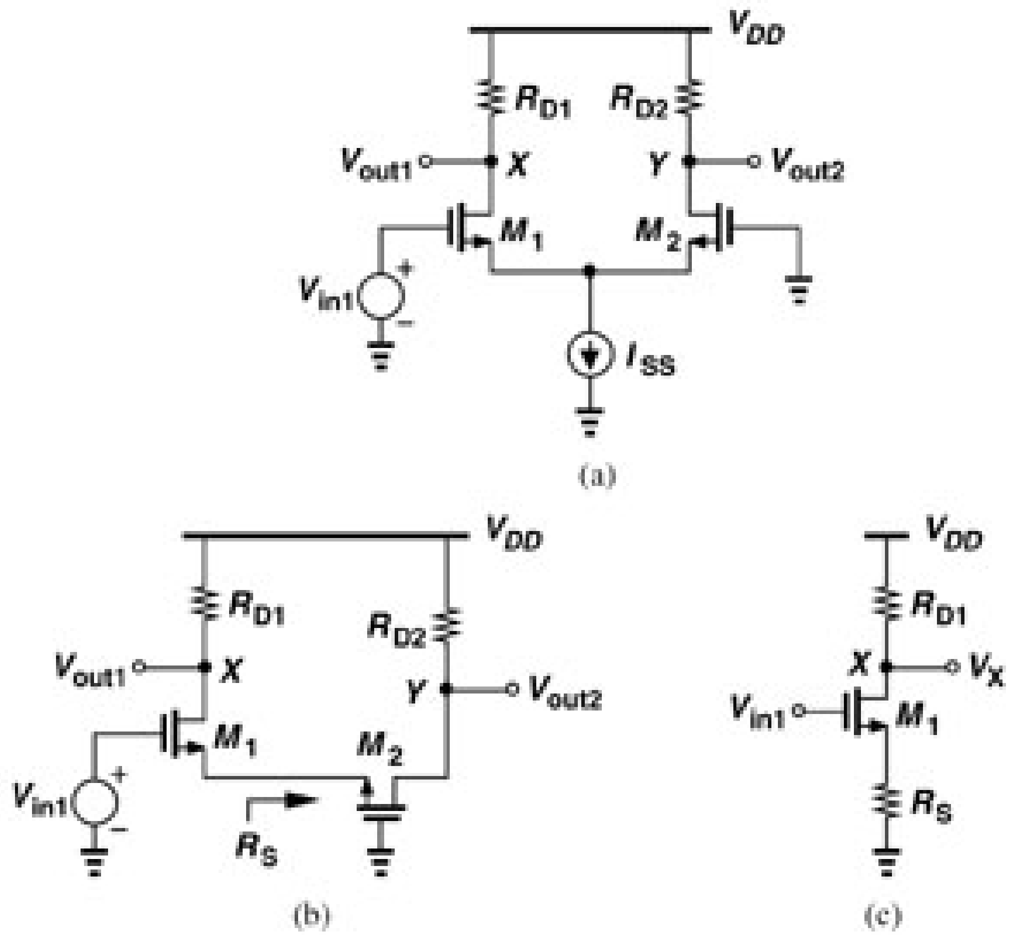
Differential Gain



$$A_v(diff) = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = g_m R_D$$

$$A_v(Single - Ended) = \frac{g_m}{2} R_D$$

Differential Pair as a CS and CD-CG Amplifier

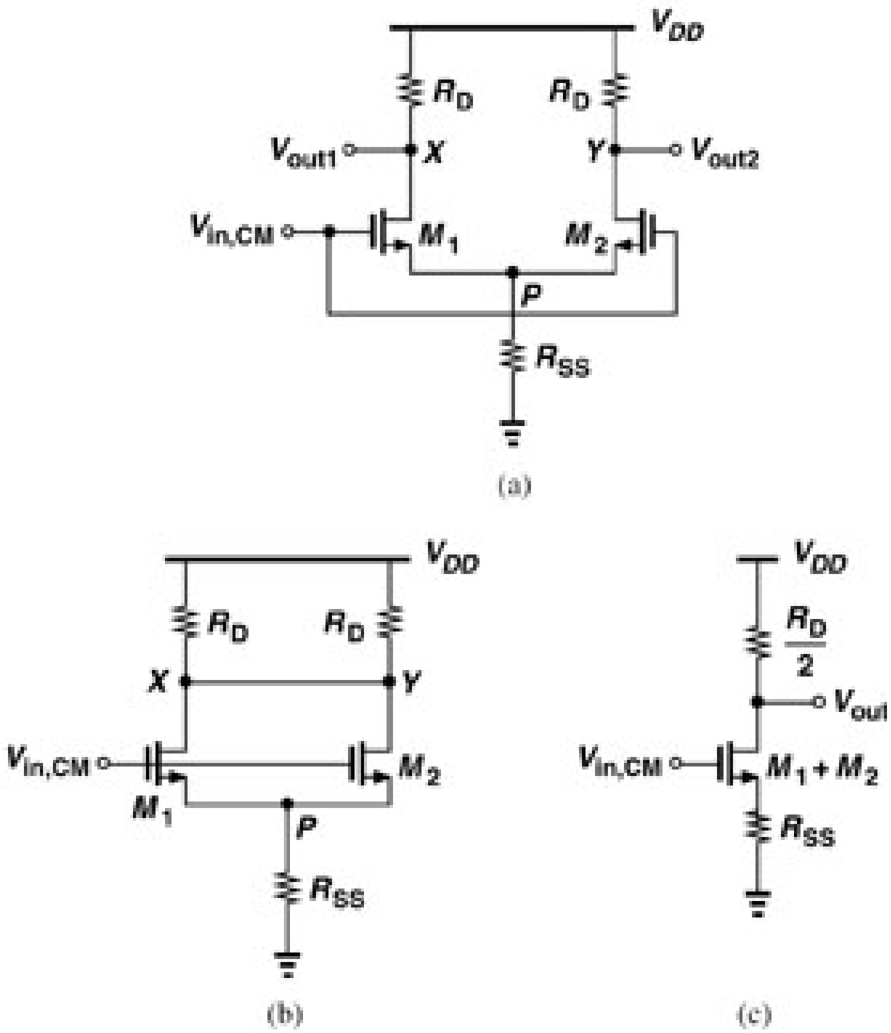


$$A_v = -\frac{g_m R_D}{1 + g_m R_S}$$

$$= -\frac{g_m}{2} R_D \;, \quad (S.E.)$$

$$= g_m R_D \;, \quad (diff)$$

Common-Mode Response



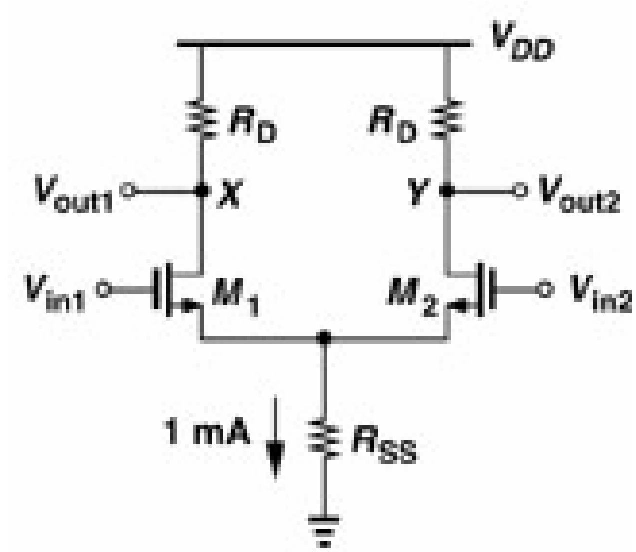
$$A_v = \frac{V_{out}}{V_{in,CM}}$$

$$= -\frac{R_D/2}{1/(2g_m) + R_{SS}}$$

Board Notes

Example

- In the following circuit assume that R_{SS}=500Ω and W/L=25/0.5, $\mu_n C_{ox}$=50μA/V^2, V_{TH} =0.6, λ=γ=0 and V_{DD}=3V.



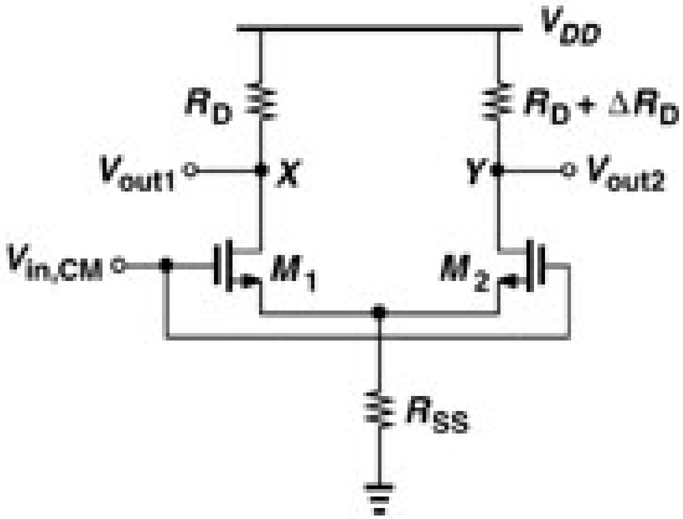
a) What is the required input CM for which R_{SS} sustains 0.5V?

b) Calculate R_D for a differential gain of 5V/V.

c) What happens at the output if the input CM level is 50mV higher than the value calculated in part (a)?

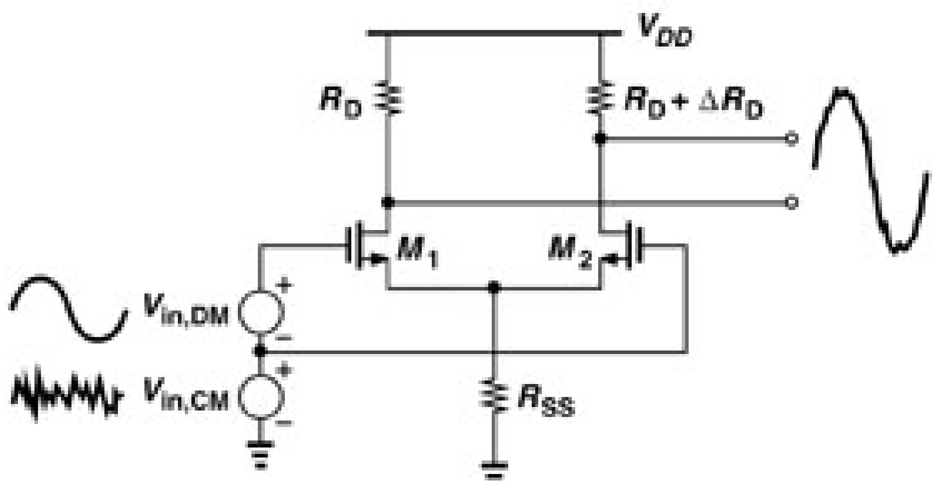
Board Notes

Common-Mode Response

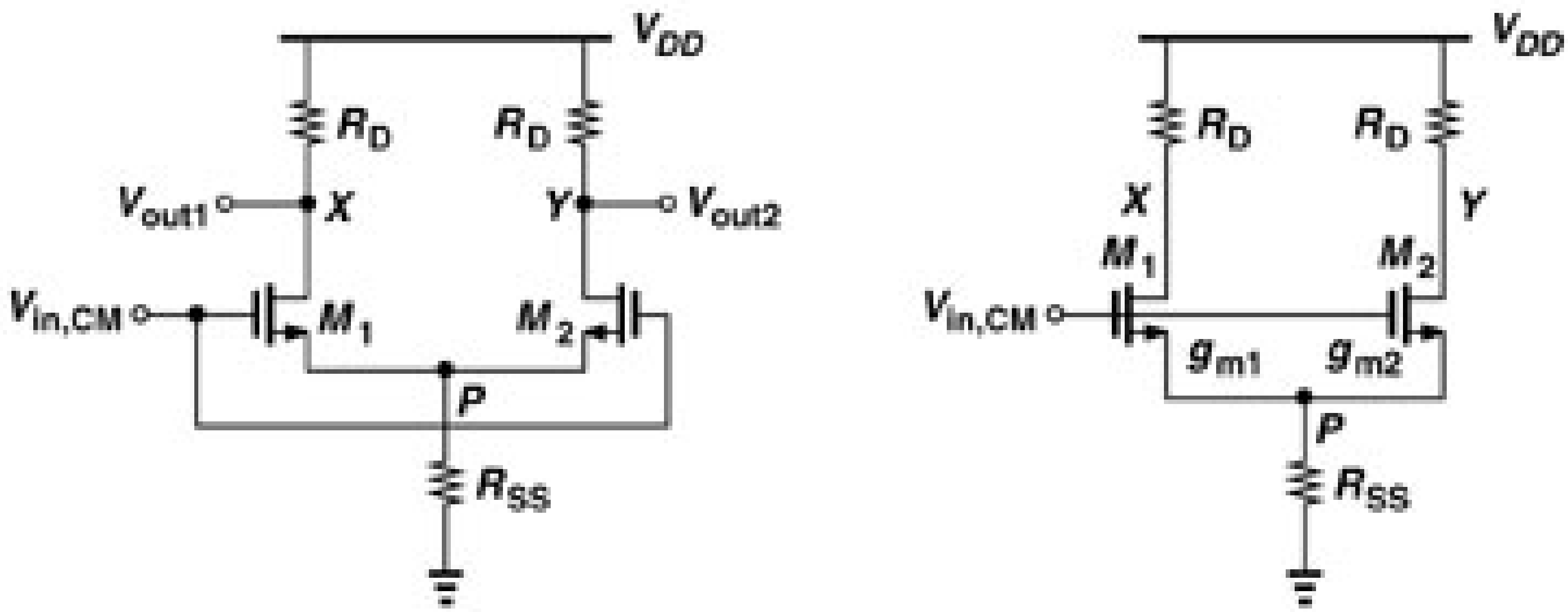


$$\frac{\Delta V_X}{\Delta V_{in,CM}} = -\frac{g_m}{1+2g_m R_{SS}} R_D$$

$$\frac{\Delta V_Y}{\Delta V_{in,CM}} = -\frac{g_m}{1+2g_m R_{SS}} (R_D + \Delta R_D)$$

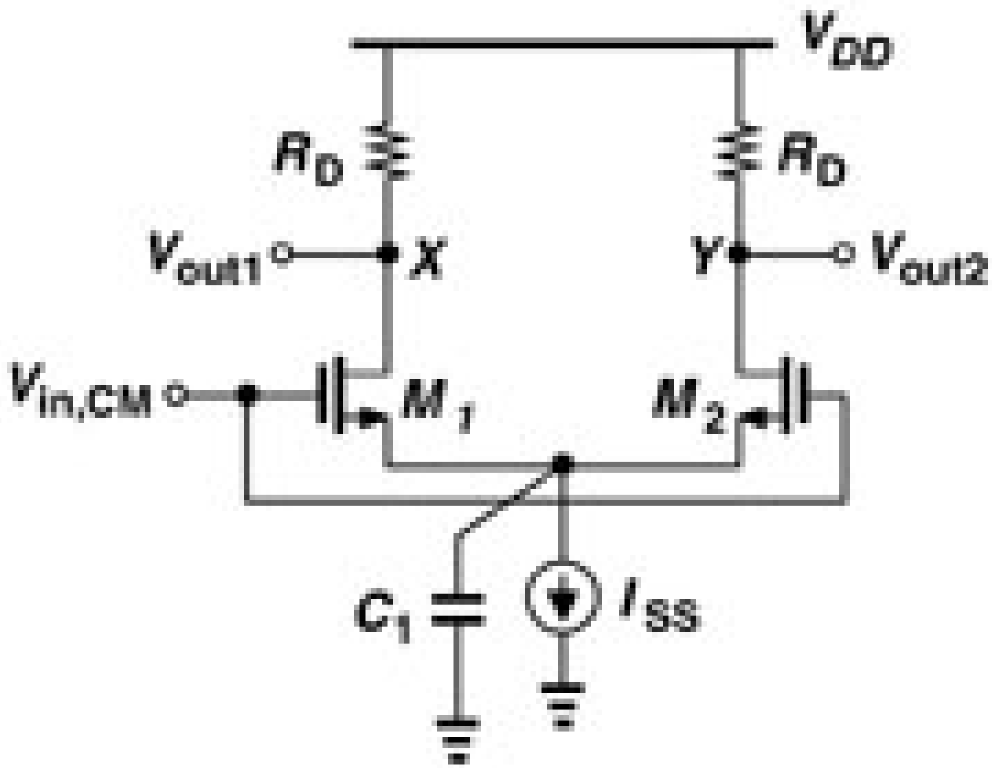


Common-Mode Response

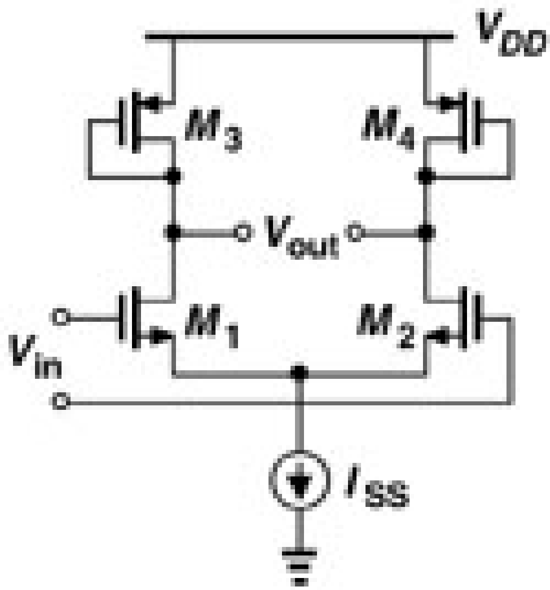


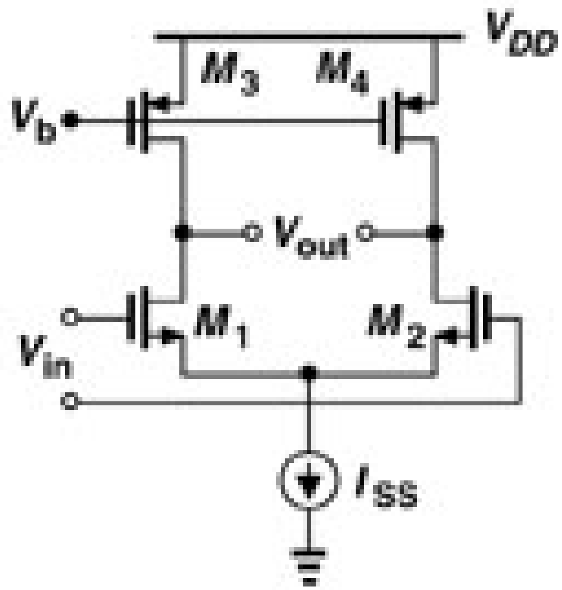
$$\frac{V_X - V_Y}{V_{in,CM}} = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D$$

Common-Mode Response



Differential Pair with MOS Loads





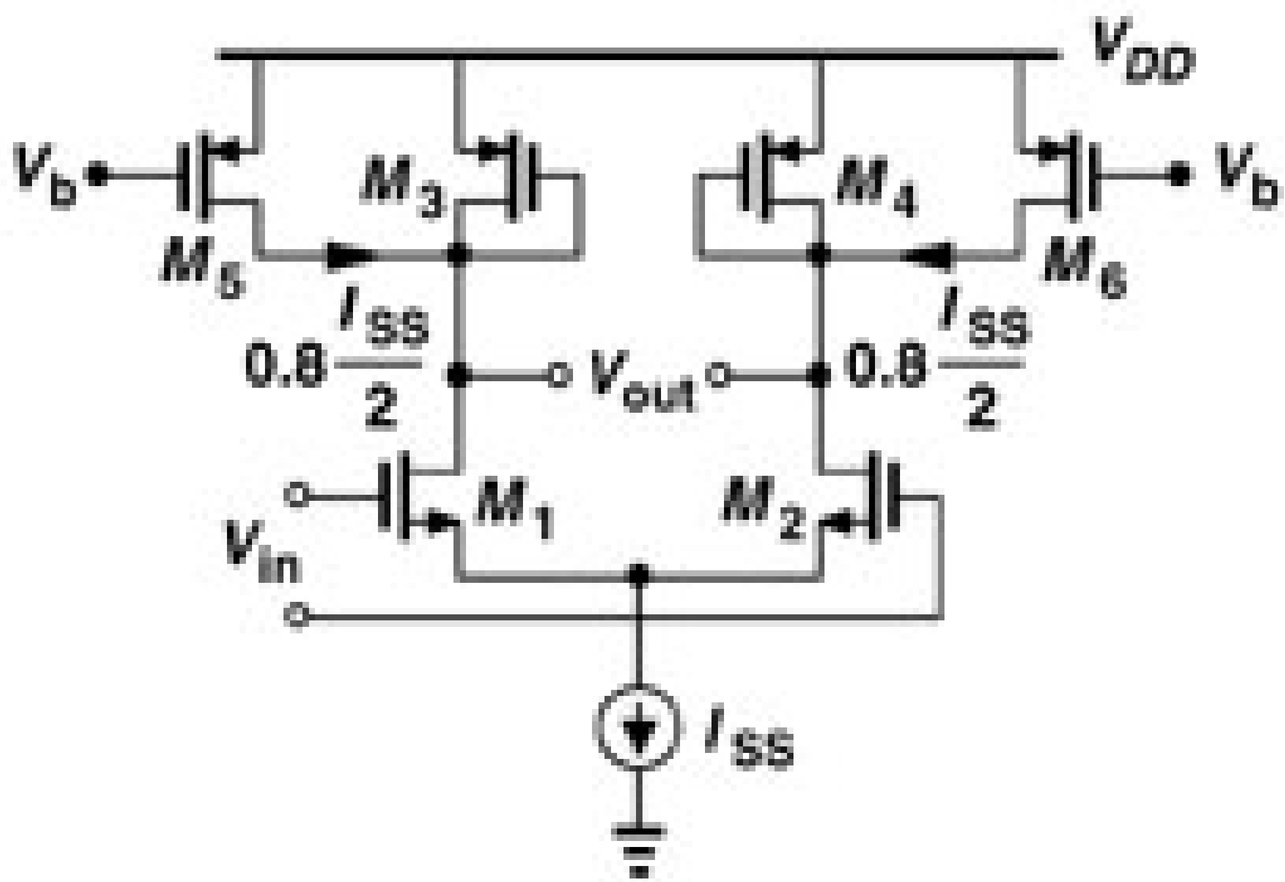
(b)

$$A_v = -g_{mN}(g_{mP}^{-1} \| r_{oN} \| r_{oP})$$

$$\approx -\frac{g_{mN}}{g_{mP}}$$

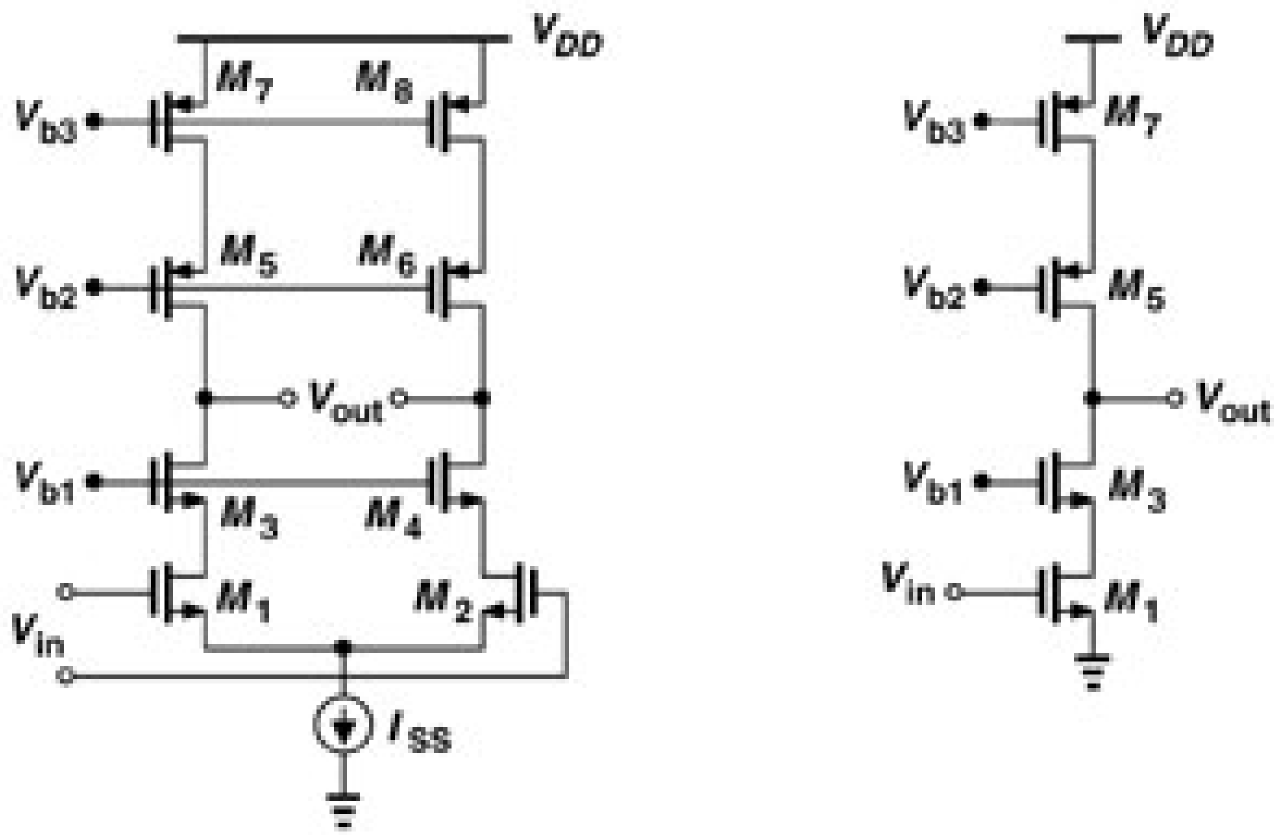
$$A_v = -g_{mN}(r_{oN} \| r_{oP})$$

MOS Loads



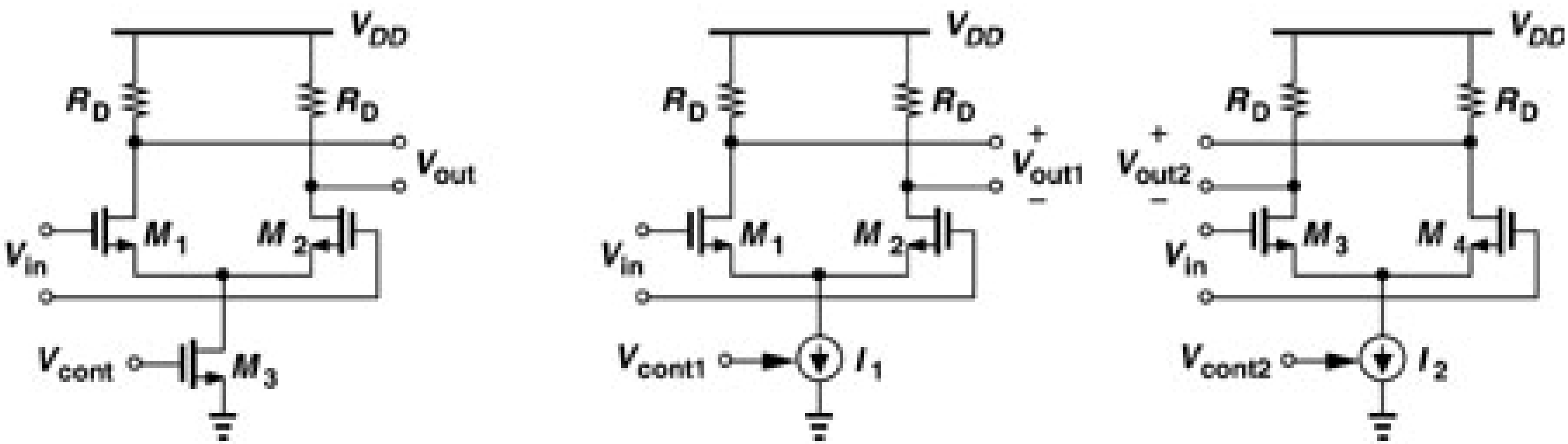
V_{DD}
V_b
V_b
M_3
M_4
M_5
M_6
I_{SS}
$0.8\frac{I_{SS}}{2}$
$0.8\frac{I_{SS}}{2}$
V_{out}
M_1
M_2
V_{in}
I_{SS}

MOS Loads



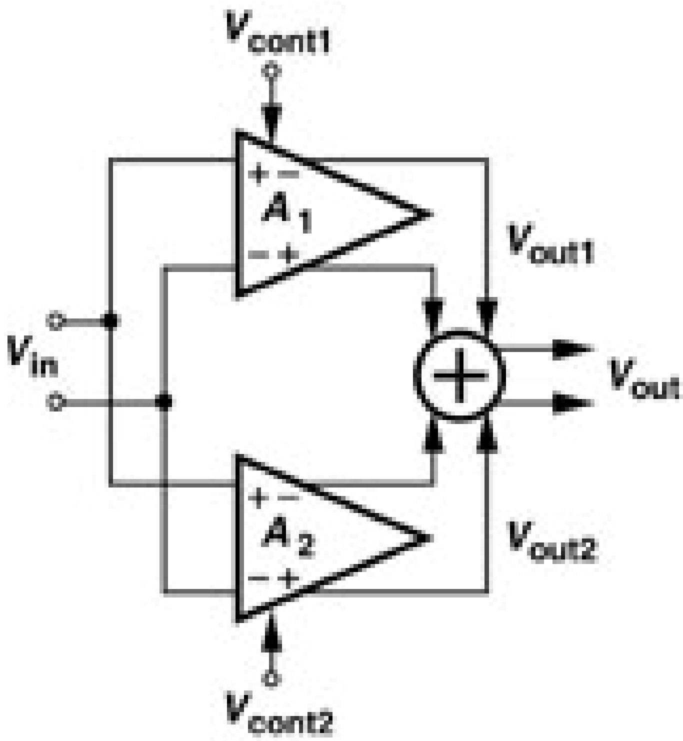
$$A_v \approx g_{m1}[(g_{m3}r_{o3}r_{o1}) \| (g_{m5}r_{o5}r_{o7})]$$

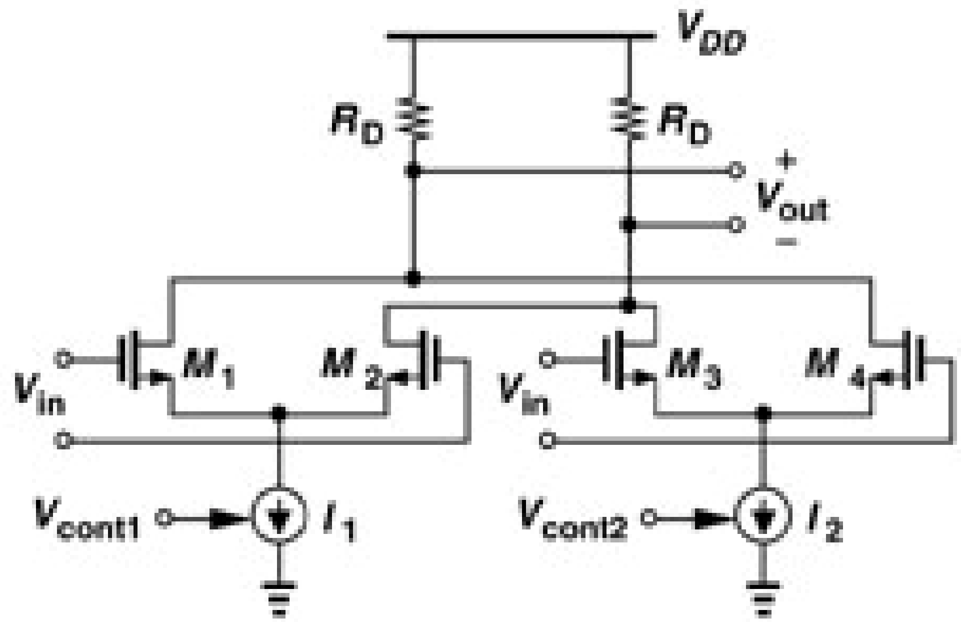
Gilbert Cell



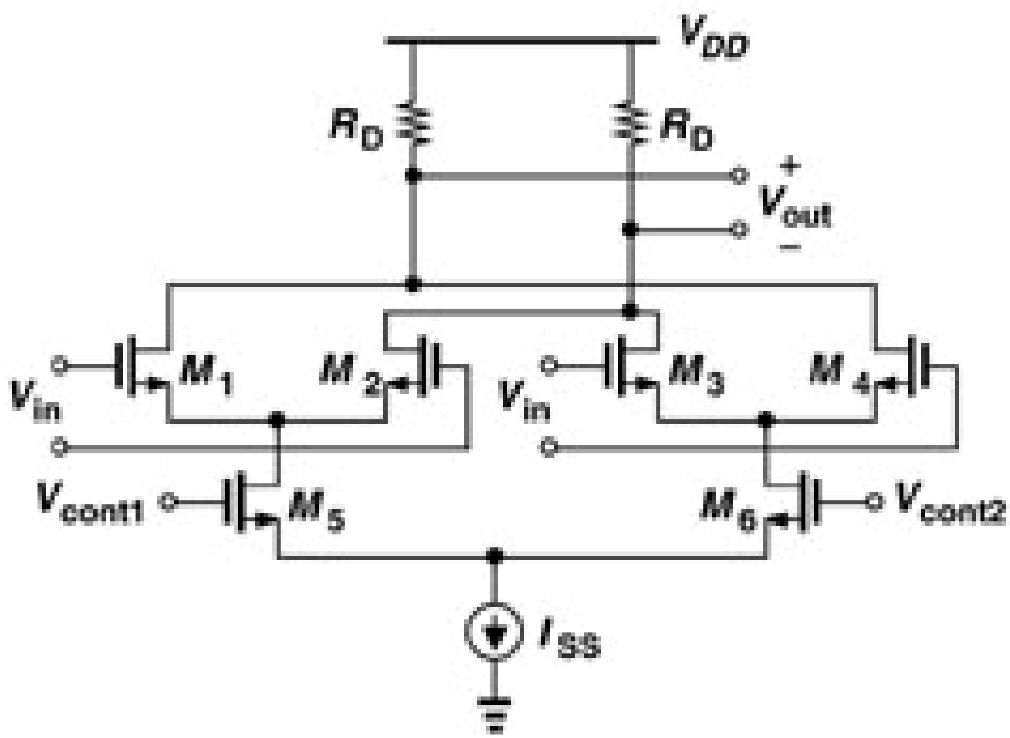
(a) (b)

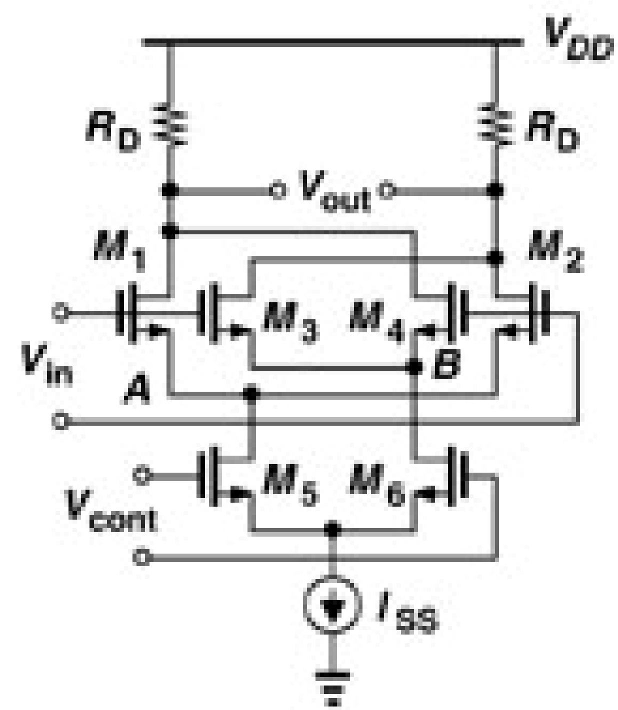
Gilbert Cell





Gilbert Cell





$$V_{OUT} = kV_{in}V_{cont}$$