EECE488: Analog CMOS Integrated Circuit Design

Set 6

Frequency Response of Amplifiers

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Simple Pole

\[
\frac{v_o}{v_i} = \frac{1/sC}{R + 1/sC}
\]

\[
\frac{v_o}{v_i} = \frac{1}{sRC + 1}
\]

\[
\frac{v_o}{v_i} (f) = \frac{1}{1 + j2\pi fRC}
\]

\[
\frac{v_o}{v_i} (f) = \frac{1}{1 + j\left(\frac{f}{f_p}\right)} \quad , \quad f_p = \frac{1}{2\pi RC}
\]
Miller Effect

\[ Z_1 = \frac{Z}{(1 - A_v)} \]

\[ Z_2 = \frac{Z}{(1 - \frac{1}{A_v})} \]
Board Notes
Miller Capacitive Multiplication

\[ C_1 = C_F (1 - A_v) \]

\[ C_2 = C_F (1 - A_v^{-1}) \approx C_F \]
Applicability of Miller’s Theorem

If the only signal path between X and Y is through impedance Z then Miller’s theorem is typically not applicable.
Applicability of Miller’s Theorem

Miller’s Theorem is typically useful in the cases where there is impedance in parallel with the main signal path.
Poles and Nodes

Non-Interacting Poles: One pole associated with each node

Interacting Poles
Neglecting input/output interaction,

\[ f_{p,\text{in}} = \frac{1}{2 \pi R_S \left[ C_{GS} + (1 + g_m R_D)C_{GD} \right]} \]

\[ f_{p,\text{out}} = \frac{1}{2 \pi \left[ (C_{GD} + C_{DB})R_D \right]} \]
Common Source

\[
\frac{v_o}{v_i} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB}) + s \left[ R_S (1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB}) \right] + 1}
\]

Assume \( D = \left( \frac{s}{\omega_{p1}} + 1 \right) \left( \frac{s}{\omega_{p2}} + 1 \right) = \frac{s^2}{\omega_{p1} \omega_{p2}} + \frac{s}{\omega_{p1}} + 1 \), \( \omega_{p2} \gg \omega_{p1} \)

\[
f_{p, in} = \frac{1}{2\pi \left( R_S \left[ C_{GS} + (1 + g_m R_D) C_{GD} \right] + R_D (C_{GD} + C_{DB}) \right)}
\]
Common Source

\[ f_{p,\text{out}} = \frac{R_S (1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})}{2 \pi R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB})} \]

\[ f_{p,\text{out}} \approx \frac{1}{2 \pi R_D (C_{GD} + C_{DB})}, \text{ for large } C_{GS} \]

\[ f_{p,\text{out}} \approx \frac{g_m R_S R_D C_{GD}}{2 \pi R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB})} \]

\[ \approx \frac{g_m}{2 \pi (C_{GS} + C_{DB})}, \text{ for large } C_{GD} \]
Common Source

Right half plane zero, from the numerator of $v_o/v_i$

$$\frac{v_o}{v_i} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS} C_{GD} + C_{GS} C_{SB} + C_{GD} C_{DB}) + s\left[R_S (1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})\right] + 1}$$

$$\frac{sC_{GD} - g_m}{\ldots} \rightarrow f_z = \frac{+g_m}{2\pi C_{GD}}$$
Common Gate

\[ f_{pX} = \frac{1}{2\pi \left( C_{GS} + C_{SB} \right) \left( R_S \parallel \left( \frac{1}{g_m + g_{mb}} \right) \right)} \]

\[ f_{pY} = \frac{1}{2\pi \left[ (C_{GD} + C_{DB}) R_D \right]} \]
Source Follower (Common Drain)

\[ \frac{v_o}{v_i} = \frac{g_m + sC_{GS}}{s^2 R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L) + s(g_m R_S C_{GD} + C_L + C_{GS}) + g_m} \]

\[ f_{p1} \approx \frac{g_m}{2\pi(g_m R_S C_{GD} + C_L + C_{GS})}, \text{ assuming } f_{p2} \gg f_{p1} \]

\[ = \frac{1}{2\pi \left( R_S C_{GD} + \frac{C_L + C_{GS}}{g_m} \right)} \]
Source Follower Input Impedance

Neglecting $C_{GD}$,

$$Z_{in} = \frac{1}{sC_{GS}} + \left(1 + \frac{g_m}{sC_{GS}}\right) \frac{1}{g_{mb} + sC_L}$$

At low frequencies, $g_{mb} \gg |sC_L|$

$$Z_{in} \approx \frac{1}{sC_{GS}} \left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}}$$

$\therefore \quad C_{in} = C_{GS}g_{mb} / (g_m + g_{mb}) + C_{GD} \quad (\text{same as Miller})$
Source Follower

At high frequencies, \( g_{mb} \ll |sC_L| \)

\[
Z_{in} \approx \frac{1}{sC_{GS}} + \frac{1}{sC_L} + \frac{g_m}{s^2C_{GS}C_L}
\]

At high frequencies, overall input impedance includes \( C_{GD} \) in parallel with series combination of \( C_{GS} \) and \( C_L \) and a negative resistance equal to \(-g_m/(C_{GS}C_L\omega^2)\).
Source Follower Output Impedance

\[ Z_{OUT} = \frac{V_X}{I_X} \]

\[ = \frac{sR_S C_{GS} + 1}{g_m + sC_{GS}} \]

\[ \approx \frac{1}{g_m}, \text{ at low frequencies} \]

\[ \approx R_S, \text{ at high frequencies} \]
Source Follower Output Impedance

\[ R_2 = \frac{1}{g_m} \]
\[ R_1 = R_S - \frac{1}{g_m} \]
\[ L = \frac{C_{GS}}{g_m} \left( R_S - \frac{1}{g_m} \right) \]

Output impedance inductance dependent on source impedance, \( R_S \)!
Source Follower Ringing

Output ringing due to tuned circuit formed with $C_L$ and inductive component of output impedance.
Cascode Stage
Cascode Stage

\[ f_{pA} = \frac{1}{2\pi R_S \left[ C_{GS1} + C_{GD1} \left( 1 + \frac{g_{m1}}{g_{m2} + g_{mb2}} \right) \right]} \]

\[ f_{pX} = \frac{g_{m2} + g_{mb2}}{2\pi \left( 2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2} \right)} \]

\[ f_{pY} = \frac{1}{2\pi R_D \left( C_{DB2} + C_L + C_{GD2} \right)} \]
Differential Pair
Differential Pair
Differential Pair

\[ f_{p1} \approx \frac{1}{2\pi (r_{oN} \parallel r_{oP}) C_L} \]

\[ f_{p2} = \frac{g_{mP}}{2\pi C_E} \]

\[ f_Z = 2f_{p2} = \frac{2g_{mP}}{2\pi C_E} \]