
EECE488: Analog CMOS Integrated Circuit Design

Set 6

Frequency Response of Amplifiers

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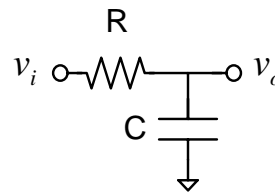
Simple Pole

$$v_o / v_i = \frac{1/sC}{R + 1/sC}$$

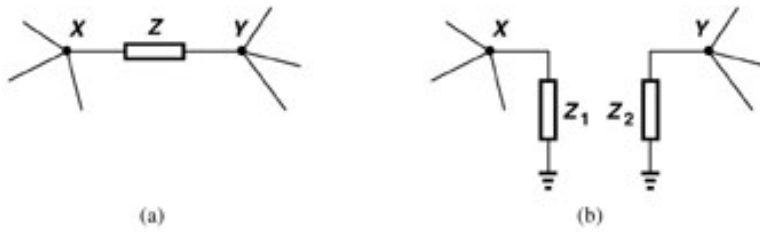
$$v_o / v_i = \frac{1}{sRC + 1}$$

$$\frac{v_o}{v_i}(f) = \frac{1}{1 + j2\pi fRC}$$

$$\frac{v_o}{v_i}(f) = \frac{1}{1 + j\left(\frac{f}{f_p}\right)}, \quad f_p = \frac{1}{2\pi RC}$$



Miller Effect

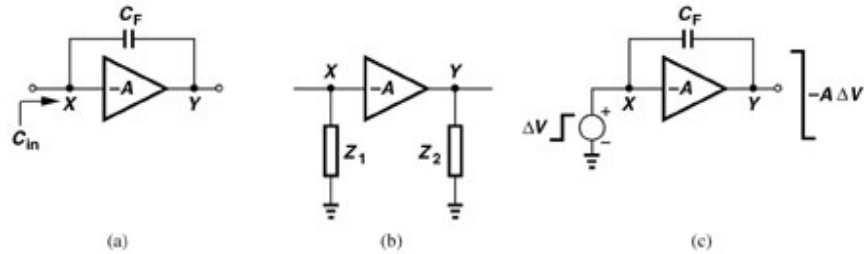


$$Z_1 = \frac{Z}{(1 - A_v)}$$

$$Z_2 = \frac{Z}{(1 - A_v^{-1})}$$

Board Notes

Miller Capacitive Multiplication



$$C_1 = C_F(1 - A_v)$$

$$C_2 = C_F(1 - A_v^{-1}) \approx C_F$$

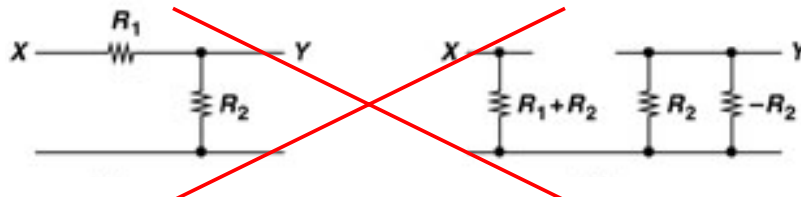
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Applicability of Miller's Theorem

If the only signal path between X and Y is through impedance Z then Miller's theorem is typically not applicable.



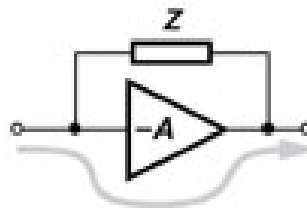
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Applicability of Miller's Theorem

Miller's Theorem is typically useful in the cases where there is impedance in parallel with the main signal path.



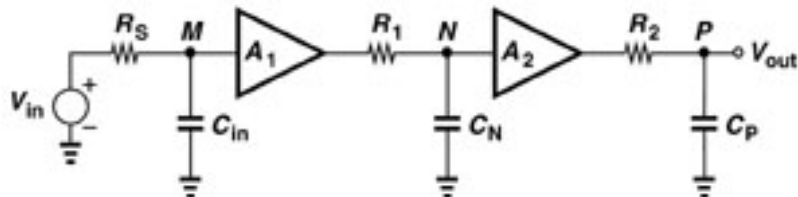
Main Signal Path

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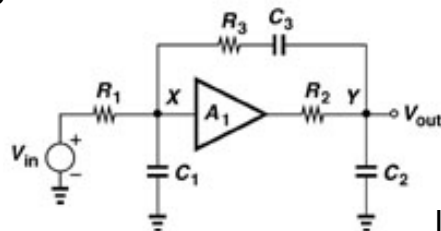
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Poles and Nodes



Non-Interacting Poles: One pole associated with each node



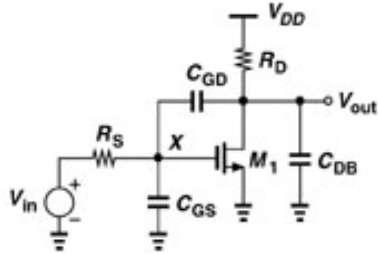
Interacting Poles

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Common Source

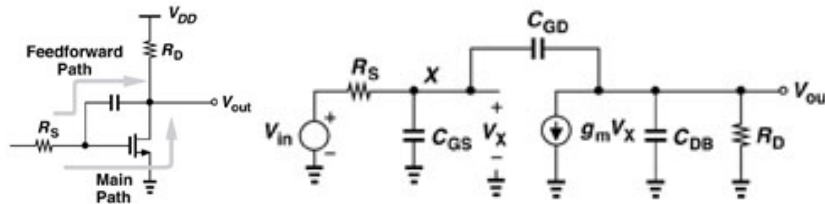


Neglecting input/output interaction,

$$f_{p,in} = \frac{1}{2\pi R_S [C_{GS} + (1 + g_m R_D) C_{GD}]}$$

$$f_{p,out} = \frac{1}{2\pi [(C_{GD} + C_{DB}) R_D]}$$

Common Source



$$\frac{v_o}{v_i} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB}) + s [R_S (1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})] + 1}$$

$$\text{Assume } D = \left(\frac{s}{\omega_{p1}} + 1 \right) \left(\frac{s}{\omega_{p2}} + 1 \right) = \frac{s^2}{\omega_{p1} \omega_{p2}} + \frac{s}{\omega_{p1}} + 1, \quad \omega_{p2} \gg \omega_{p1}$$

$$f_{p,in} = \frac{1}{2\pi (R_S [C_{GS} + (1 + g_m R_D) C_{GD}] + R_D (C_{GD} + C_{DB}))}$$

Common Source

$$f_{p,out} = \frac{R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}{2\pi R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB})}$$

$$f_{p,out} \approx \frac{1}{2\pi R_D (C_{GD} + C_{DB})}, \text{ for large } C_{GS}$$

$$f_{p,out} \approx \frac{g_m R_S R_D C_{GD}}{2\pi R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB})}$$

$$\approx \frac{g_m}{2\pi (C_{GS} + C_{DB})}, \text{ for large } C_{GD}$$

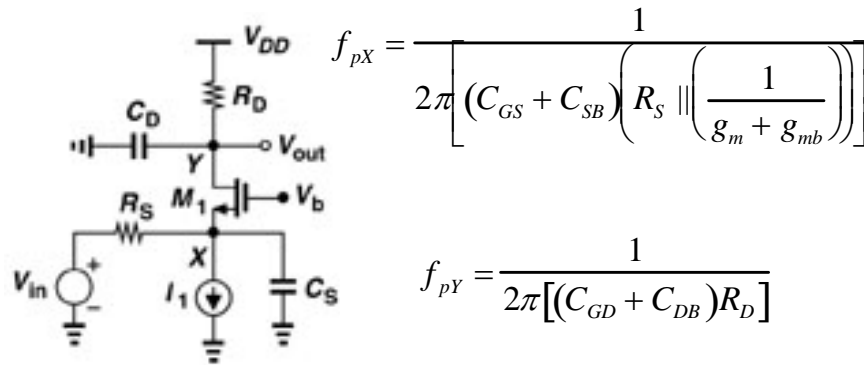
Common Source

Right half plane zero, from the numerator of v_o/v_i

$$\frac{v_o}{v_i} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB}) + s [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})] + 1}$$

$$\frac{sC_{GD} - g_m}{\dots} \rightarrow f_z = \frac{+g_m}{2\pi C_{GD}}$$

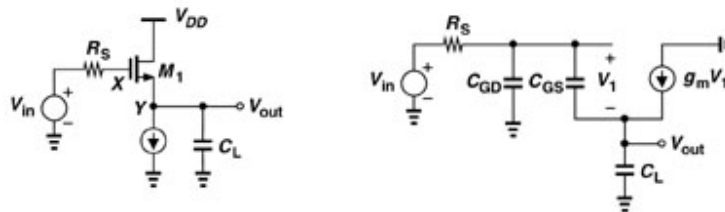
Common Gate



$$f_{pX} = \frac{1}{2\pi \left[(C_{GS} + C_{SB}) \left(R_S \parallel \left(\frac{1}{g_m + g_{mb}} \right) \right) \right]}$$

$$f_{pY} = \frac{1}{2\pi [(C_{GD} + C_{DB})R_D]}$$

Source Follower (Common Drain)

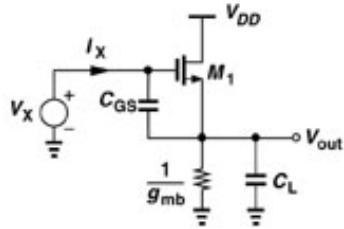


$$\frac{v_o}{v_i} = \frac{g_m + sC_{GS}}{s^2 R_S (C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) + s(g_m R_S C_{GD} + C_L + C_{GS}) + g_m}$$

$$f_{p1} \approx \frac{g_m}{2\pi (g_m R_S C_{GD} + C_L + C_{GS})}, \text{ assuming } f_{p2} \gg f_{p1}$$

$$= \frac{1}{2\pi \left(R_S C_{GD} + \frac{C_L + C_{GS}}{g_m} \right)}$$

Source Follower Input Impedance



Neglecting C_{GD} ,

$$Z_{in} = \frac{1}{sC_{GS}} + \left(1 + \frac{g_m}{sC_{GS}}\right) \frac{1}{g_{mb} + sC_L}$$

At low frequencies, $g_{mb} \gg |sC_L|$

$$Z_{in} \approx \frac{1}{sC_{GS}} \left(1 + \frac{g_m}{g_{mb}}\right) + 1/g_{mb}$$

$$\therefore C_{in} = C_{GS}g_{mb}/(g_m + g_{mb}) + C_{GD} \text{ (same as Miller)}$$

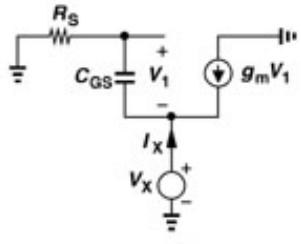
Source Follower

At high frequencies, $g_{mb} \ll |sC_L|$

$$Z_{in} \approx \frac{1}{sC_{GS}} + \frac{1}{sC_L} + \frac{g_m}{s^2C_{GS}C_L}$$

At high frequencies, overall input impedance includes C_{GD} in parallel with series combination of C_{GS} and C_L and a *negative* resistance equal to $-g_m/(C_{GS}C_L\omega^2)$.

Source Follower Output Impedance

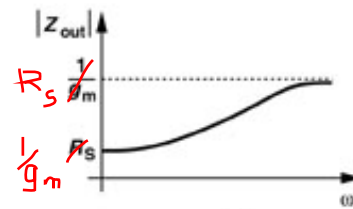
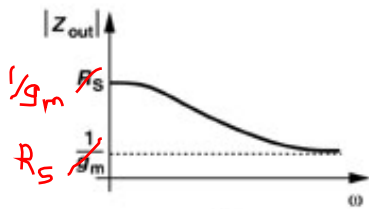


$$Z_{OUT} = V_X / I_X$$

$$= \frac{sR_S C_{GS} + 1}{g_m + sC_{GS}}$$

$\approx 1/g_m$, at low frequencies

$\approx R_S$, at high frequencies

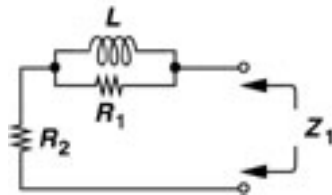
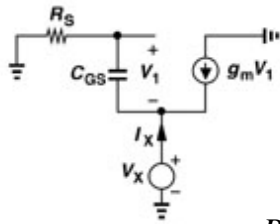


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Source Follower Output Impedance



$$R_2 = 1/g_m$$

$$R_1 = R_S - 1/g_m$$

$$L = \frac{C_{GS}}{g_m} (R_S - 1/g_m)$$

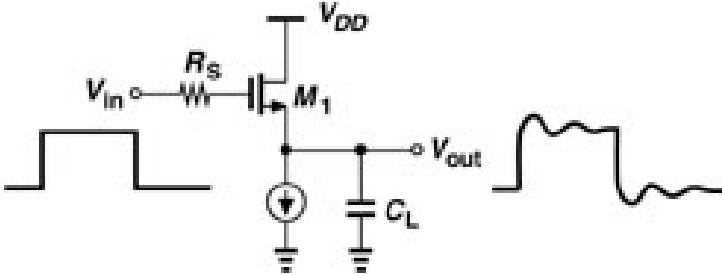
Output impedance inductance dependent on source impedance, R_S !

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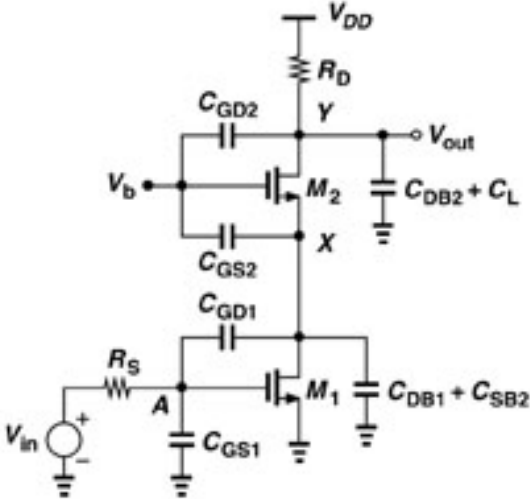
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Source Follower Ringing



Output ringing due to tuned circuit formed with C_L and inductive component of output impedance.

Cascode Stage



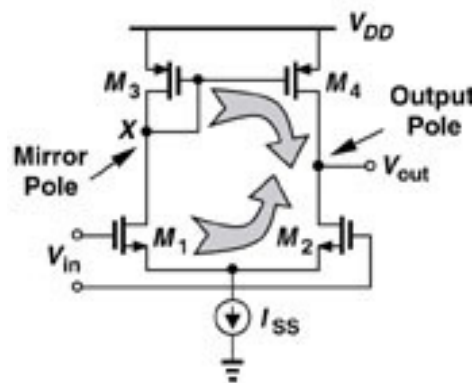
Cascode Stage

$$f_{pA} = \frac{1}{2\pi R_S \left[C_{GS1} + C_{GD1} \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}} \right) \right]}$$

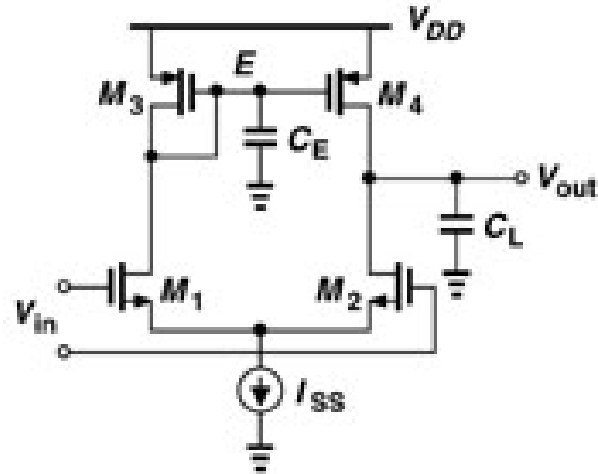
$$f_{pX} = \frac{g_{m2} + g_{mb2}}{2\pi (2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2})}$$

$$f_{pY} = \frac{1}{2\pi R_D (C_{DB2} + C_L + C_{GD2})}$$

Differential Pair



Differential Pair



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Differential Pair

$$f_{p1} \approx \frac{1}{2\pi(r_{oN} \parallel r_{oP})C_L}$$

$$f_{p2} = \frac{g_{mP}}{2\pi C_E}$$

$$f_Z = 2f_{p2} = \frac{2g_{mP}}{2\pi C_E}$$

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