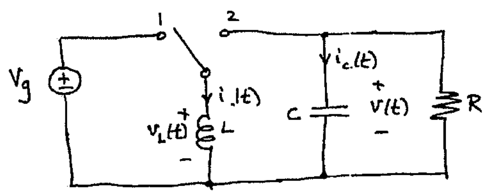
EECE 493 2008W1
ASSIGNMENT #1 SOLUTION

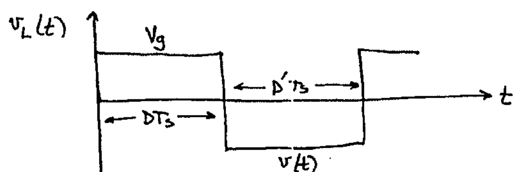
Solution to Problem 2.1

Analysis and Design of a buck-boost converter



a)

Inductor voltage waveform



volt-second balance

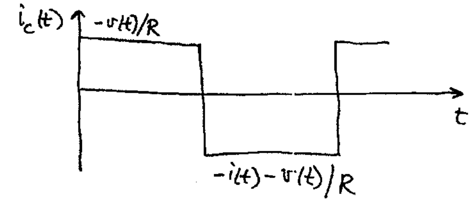
$$\langle v_L(t)\rangle = D(V_g) + D'(v(t)) \approx DV_g + D'V = 0$$

small ripple approximation

solve for V:

$$V = -\frac{D}{D'}V_g$$

Capacitor current waveform



Capacitor charge balance:

$$\langle i_c(t)\rangle = D\left(-\frac{v(t)}{R}\right) + D'\left(-i(t) - \frac{v(t)}{R}\right)$$

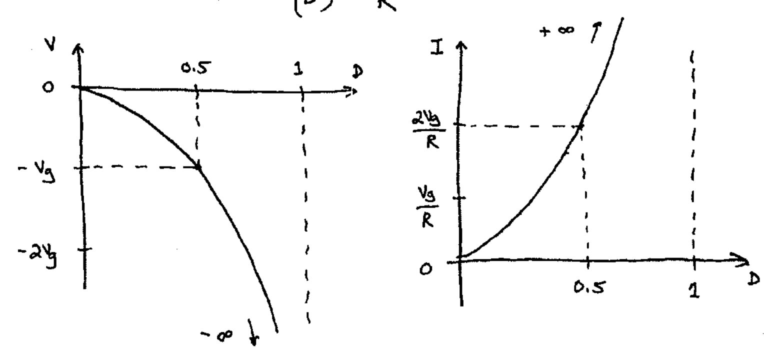
$$\approx D\left(-\frac{V}{R}\right) + D'\left(-I - \frac{V}{R}\right) = 0$$

solve for I:

$$D'I = -\frac{V}{R}\underbrace{(D+D')}_{=1}$$

$$\Rightarrow I = -\frac{V}{D'R} = \frac{D}{(D')^2}\frac{V_g}{R}$$

b)



c) DC design

Given $V_g = 30V$
$V = -20V$
$R = 4\Omega$
$f_s = 40kHz \Rightarrow T_s = \frac{1}{40kHz} = 25\mu sec$

(i) Find D and I

we know that $\frac{V}{V_g} = -\frac{D}{1-D}$

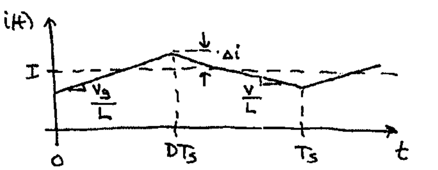
so $V(1-D) = -D V_g$

$\Rightarrow V = D(V - V_g)$

$\Rightarrow D = \frac{V}{V-V_g} = \frac{-20}{-20-30} = 0.4$

$I = \frac{D}{(1-D)^2}\frac{V_g}{R} = \frac{0.4}{(0.6)^2}\frac{30V}{4\Omega} = 8.33A$

(ii) Choose L such that Δi is 10% of $I = 0.833A$

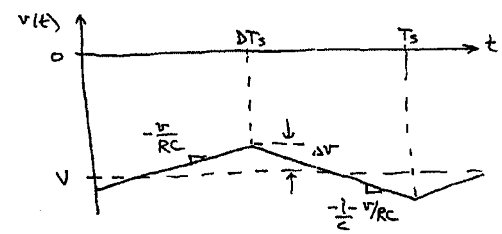


$2\Delta i = \frac{V_g}{L} DT_s \Rightarrow L = \frac{V_g DT_s}{2\Delta i}$

$= \frac{(30V)(0.4)(25\mu sec)}{2(0.833A)}$

$= 180\mu H$

(iii) Choose C such that $\Delta v = 0.1$ Volt



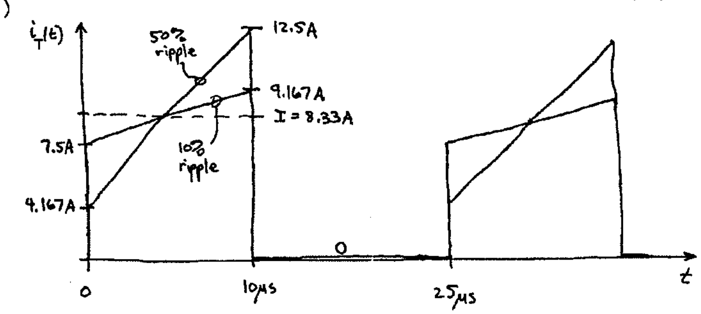
$2\Delta v = \frac{-v}{RC} DT_s \approx \frac{-V}{RC} DT_s$

Solve for C:

$C = \frac{-VDT_s}{2\Delta v R} = \frac{-(-20V)(0.4)(25\mu s)}{2(0.1V)(4\Omega)} = 250\mu F$

Note: in practice, capacitor equivalent series resistance (esr) would contribute to Δv, and would require use of a larger capacitance to achieve $\Delta v = 0.1V$.

d)

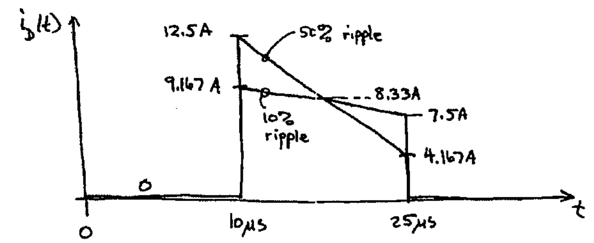


$i_T = i_L$ during $0 < t < DT_s$, and $i_T = 0$ during $DT_s < t < T_s$

Increasing Δi to 50% of I increases the peak transistor current from 9.167 A to 12.5 A.

e) Diode current $i_D(t)$

$$i_D(t) = \begin{cases} 0 & \text{during } 0 < t < DT_s \\ i_L(t) & \text{during } DT_s < t < T_s \end{cases}$$



Increasing Δi also increases the peak diode current.

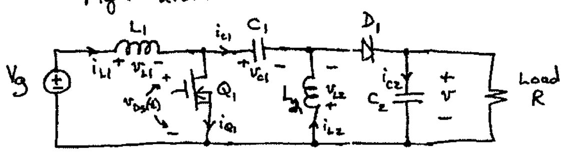
End of Problem 2.1

Tutorial

Solution to Problem 2.2

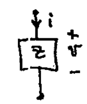
A dc voltage regulator based on the SEPIC Converter analysis

Fig. 2.29



A caveat

Label voltage and current for each inductor and capacitor. Defined directions of current and voltage must be consistent:

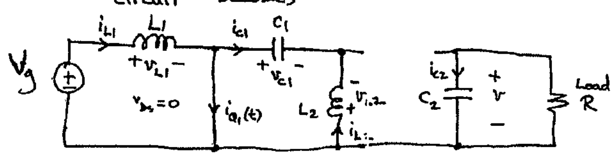


i flows from + to −

First subinterval: $0 \le t \le DT_s$

transistor is on, diode is off

circuit becomes



A caveat

Note: be careful to be consistent in your defined polarities of voltage and current. The same polarities must be followed during all subintervals!

Inductor voltages and capacitor currents for this subinterval:

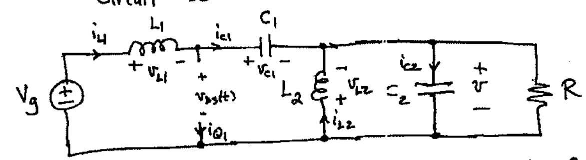
$v_{L1}(t) = V_g, \quad v_{L2}(t) = v_{c1}(t), \quad i_{c1}(t) = -i_{L2}(t),$

$i_{c2}(t) = -v(t)/R$

Second subinterval: $DT_s \le t \le T_s$

transistor is off, diode is on

Circuit becomes



Inductor voltages and capacitor currents for this subinterval:

$v_{L1}(t) = V_g - v_{c1}(t) - v(t)$

$v_{L2}(t) = -v(t)$

$i_{c1}(t) = i_{L1}(t)$

$i_{c2}(t) = i_{L1}(t) + i_{L2}(t) - v(t)/R$

Again, note that v_{L1}, v_{L2}, i_{c1} and i_{c2} are expressed in terms of quantities that have small ripple (i.e., $i_{L1}(t)$, $i_{L2}(t)$, $v_{c1}(t)$, $v(t)$, and V_g) and not as functions of quantities whose ripples are not small (such as $v_{L1}(t)$, $v_{L2}(t)$, $i_{c1}(t)$, $i_{c2}(t)$, and $i_Q(t)$). Use of the small ripple approximation now leads to

$v_{L1}(t) \approx V_g - V_{c1} - V$

$v_{L2}(t) \approx -V$

$i_{c1}(t) \approx I_{L1}$

$i_{c2}(t) \approx I_{L1} + I_{L2} - V/R$

Note that each inductor voltage and capacitor current has been expressed in terms of quantities that have small ripple when L_1, L_2, C_1, and C_2 are sufficiently large. We can therefore make the small ripple approximation and write

$$i_{L1}(t) \approx I_{L1}$$
$$v_{C1}(t) \approx V_{C1}$$
$$i_{L2}(t) \approx I_{L2}$$
$$v(t) \approx V$$

So we obtain

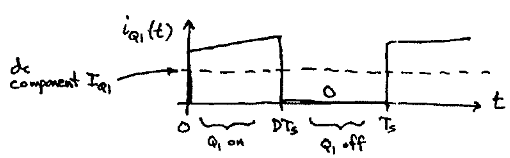
$$v_{L1}(t) = V_g$$
$$v_{L2}(t) \approx V_{C1}$$
$$i_{C1}(t) \approx -I_{L2}$$
$$i_{C2}(t) \approx -V/R$$

A caveat

We could have written the equations for subinterval 1 in other ways. For example, we could have expressed $i_{C1}(t)$ as

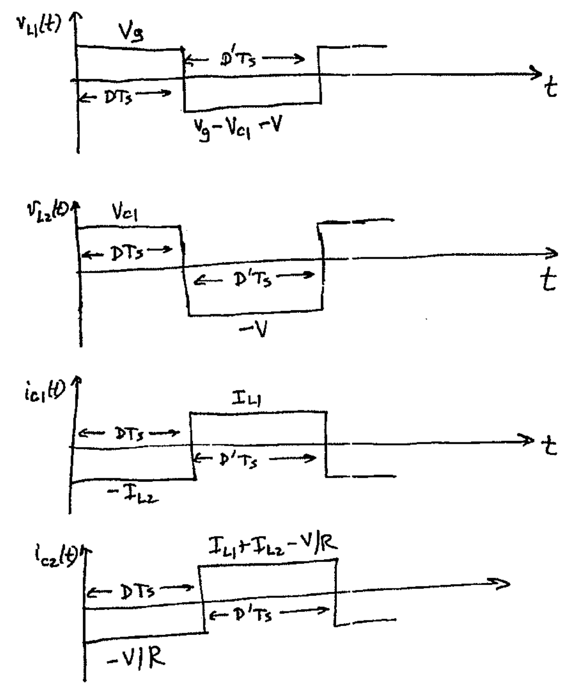
$$i_{C1}(t) = i_{L1}(t) - i_{Q1}(t)$$

While this equation is true, it is not as useful because the switching ripple in the transistor current $i_{Q1}(t)$ is not small:



So we cannot write $i_{Q1}(t) \approx I_{Q1}$ for subinterval 1

Waveforms:



<u>Volt-second balance</u>

$$\langle v_{L1} \rangle_{T_s} = D(V_g) + D'(V_g - V_{C1} - V) = 0$$

$$\langle v_{L2} \rangle_{T_s} = D(V_{C1}) + D'(-V) = 0$$

with $D' = 1 - D$

<u>charge balance</u>

$$\langle i_{C1} \rangle_{T_s} = D(-I_{L2}) + D'(I_{L1}) = 0$$

$$\langle i_{C2} \rangle_{T_s} = D\left(-\frac{V}{R}\right) + D'\left(I_{L1} + I_{L2} - \frac{V}{R}\right) = 0$$

Four equations and four unknowns $(V, V_{C1}, I_{L1}, I_{L2})$.

Solution:

$$V_{C1} = V_g$$

$$V = \frac{D}{1-D} V_g$$

$$I_{L1} = \frac{D}{D'} \frac{V}{R} = \left(\frac{D}{1-D}\right)^2 \frac{V_g}{R}$$

$$I_{L2} = \frac{V}{R} = \frac{D}{(1-D)} \frac{V_g}{R}$$

Part (b): Voltage regulator behavior

Controller automatically adjusts D, to maintain $V = 28$ volts.
Input voltage varies over the range $18 \le V_g \le 36$.
Load current $= 2A = \frac{V}{R}$, so $R = \frac{(28 \text{ volts})}{(2 \text{ amps})} = 14\,\Omega$.
Over what ranges do D and I_{L1} vary?

We know that $V = \frac{D}{1-D} V_g$. Solve for D:

$$V - DV = DV_g$$

$$V = D(V + V_g)$$

$$\Rightarrow D = \frac{V}{V + V_g}$$

and from above,

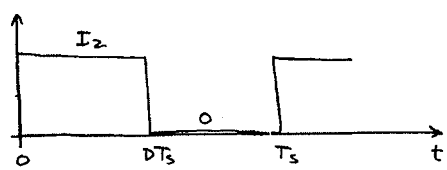
$$I_{L1} = \frac{D}{1-D} \frac{V}{R} = \frac{V^2}{V_g R}$$

V	V_g	D	I_{L1}
28	18	$\frac{28}{46} = 0.609$	$\frac{28^2}{(18)(14)} = 3.11\,A$
28	36	$\frac{28}{64} = 0.438$	$\frac{28^2}{(36)(14)} = 1.56\,A$

End of problem 2.2

Problem 2.9

a)



b)

$V_{c1} = V_g$

$V = DV_{c1} = DV_g$

$I_2 = \frac{V}{R} = \frac{DV_g}{R}$

$I_1 = DI_2 = \frac{D^2 V_g}{R}$

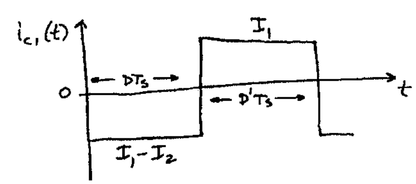
c)

$\Delta i_2 = \frac{V_g D T_s}{2L_2}$ Ripple on L_2

$\Delta v = \frac{\Delta i_2 T_s}{8C_2}$ (see Eq. (2.60))

$= \frac{V_g D T_s^2}{16 L_2 C_2}$ Ripple on C_2

Ripple on C_1:

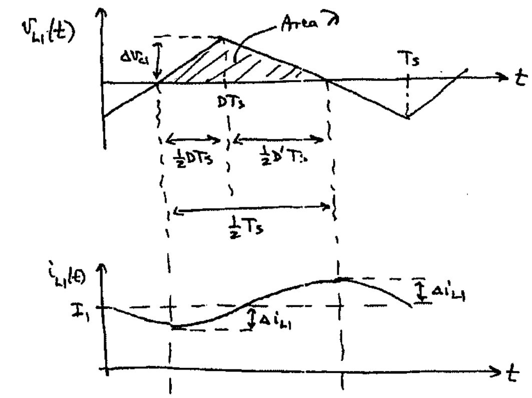


$2\Delta v_{c1} = \frac{I_1 D' T_s}{C_1}$

$\Rightarrow \Delta v_{c1} = \frac{I_1 D' T_s}{2C_1} = \frac{D^2 D' V_g T_s}{2RC_1}$

Ripple on L_1: Voltage waveform applied to L_1 is nonpulsating
$\Rightarrow$ must use arguments as in Fig. 2.27

$v_{L1}(t) = L_1 \frac{di_{L1}(t)}{dt} = V_g - v_{c1}(t)$



Area $\lambda = \frac{1}{2}\left(\frac{1}{2}T_s\right)(\Delta v_{c1}) = L_1 \cdot 2\Delta i_{L1}$

so $\Delta i_{L1} = \frac{T_s \Delta v_{c1}}{8 L_1} = \frac{T_s}{8L_1} \frac{D^2 D' V_g T_s}{2RC_1}$

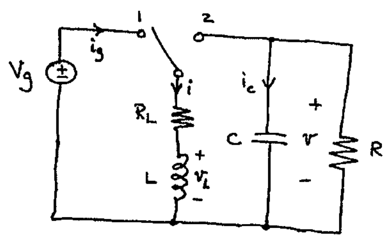
$\Delta i_{L1} = \frac{D^2 D' V_g T_s^2}{16 L_1 C_1 R}$

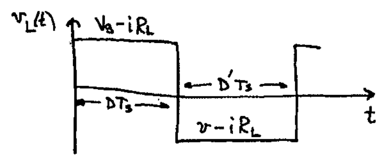
d) $D = \frac{36}{48} = 0.75$

$C_1 = \frac{D^2 D' V_g T_s}{2R(2\% \text{ of } V_g)} = \frac{(0.75)^2(0.25)(10\mu s)}{2(6\Omega)(0.02)} = 5.86\mu F$

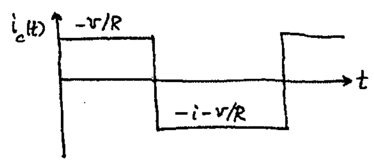
$L_1 = \frac{D^2 D' V_g T_s^2}{16 C_1 R \Delta i_{L1}} = \frac{(0.75)^2(0.25)(48V)(10\mu s)^2}{16(5.86\mu F)(6\Omega)(20mA)} = 60\mu H$

Solution to Problem 3.1





$$\langle v_L \rangle = 0 = D(V_g - iR_L) + D'(v - iR_L)$$
$$\approx D(V_g - IR_L) + D'(V - IR_L)$$
$$\Rightarrow DV_g + D'V - IR_L = 0 \qquad (1)$$



$$\langle i_c \rangle = 0 = D\left(-\frac{V}{R}\right) + D'\left(-I - \frac{V}{R}\right)$$
$$\Rightarrow -D'I - \frac{V}{R} = 0$$
$$\Rightarrow I = -\frac{V}{D'R} \qquad (2)$$

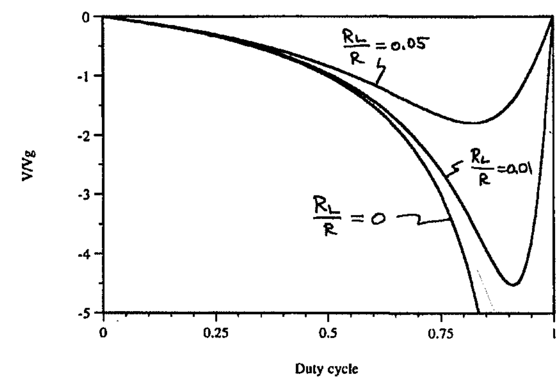
Now insert (2) into (1) and solve for V:

$$DV_g + D'V + \frac{VR_L}{D'R} = 0$$

$$DV_g = D'V\left(1 + \frac{R_L}{(D')^2R}\right)$$

$$\Rightarrow V = \frac{D}{D'} \frac{1}{\left(1 + \frac{R_L}{(D')^2R}\right)} V_g$$

3.1 part (b)



Part (c)

Average powers: $P_{in} = V_g \langle i_g \rangle = DIV_g$

$P_{out} = VD'I$

so $\eta = \frac{P_{out}}{P_{in}} = \frac{VD'I}{DIV_g} = \frac{D'}{D}\frac{V}{V_g}$

substitute solution for V from part (a), and simplify:

$$\eta = \frac{1}{\left(1 + \frac{R_L}{(D')^2R}\right)}$$

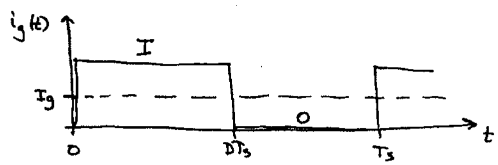
Solution to Problem 3.2

See solution to Problem 3.1 for schematic and for derivation of the following equations:

$$0 = DV_g + D'V - IR_L \quad \text{(from inductor volt-sec balance)}$$

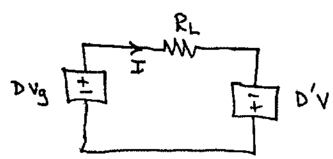
$$0 = -D'I - V/R \quad \text{(from capacitor charge balance)}$$

To derive a complete equivalent circuit model, we also need an equation for the average input current I_g

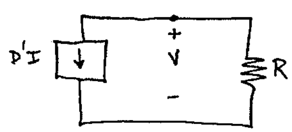


$$I_g = \langle i_g \rangle = DI$$

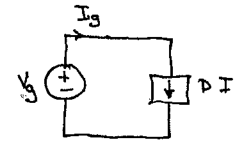
Equivalent circuit corresponding to inductor equation:



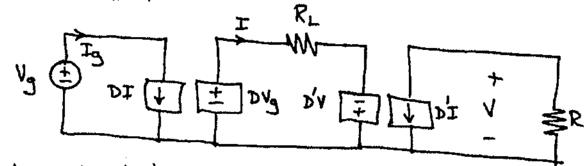
Capacitor equation:



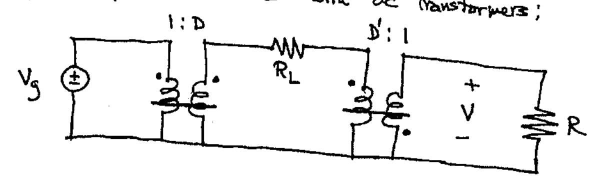
Input current equation:



Combine circuits:

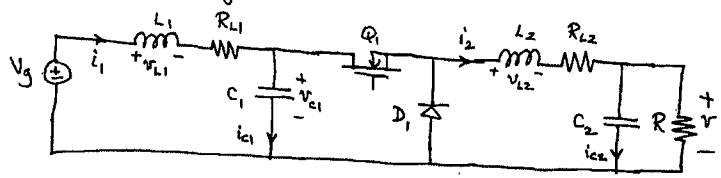


Replace dependent sources with dc transformers:



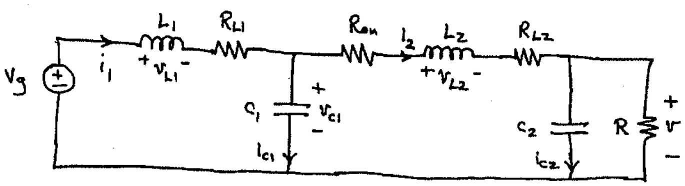
Tutorial
Solution to Problem 3.6

Converter circuit, Fig. 3.33:



Model inductor winding resistances R_{L1} and R_{L2}, MOSFET on-resistance R_{on}, and diode forward voltage drop (constant voltage V_D plus voltage across effective resistance R_D).

Subinterval 1 $\quad 0 < t < DT_s$. $\quad Q_1$ on, D_1 off



Inductor voltages and capacitor currents, expressed as functions of quantities that have small ripple:

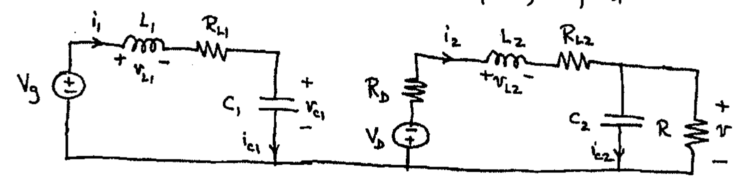
$$v_{L1}(t) = V_g - i_1(t)R_{L1} - v_{c1}(t)$$
$$v_{L2}(t) = v_{c1}(t) - i_2(t)R_{on} - i_2(t)R_{L2} - v(t)$$
$$i_{c1}(t) = i_1(t) - i_2(t)$$
$$i_{c2}(t) = i_2(t) - v(t)/R$$

Small-ripple approximation: $\quad i_1(t) \approx I_1,\ i_2(t) \approx I_2,\ v_{c1}(t) \approx V_{c1},\ v(t) \approx V$

we get

$$v_{L1}(t) \approx V_g - I_1 R_{L1} - V_{c1}$$
$$v_{L2}(t) \approx V_{c1} - I_2 R_{on} - I_2 R_{L2} - V$$
$$i_{c1}(t) \approx I_1 - I_2$$
$$i_{c2}(t) \approx I_2 - V/R$$

Subinterval 2 $\quad DT_s < t < T_s \quad Q_1$ off, D_1 on

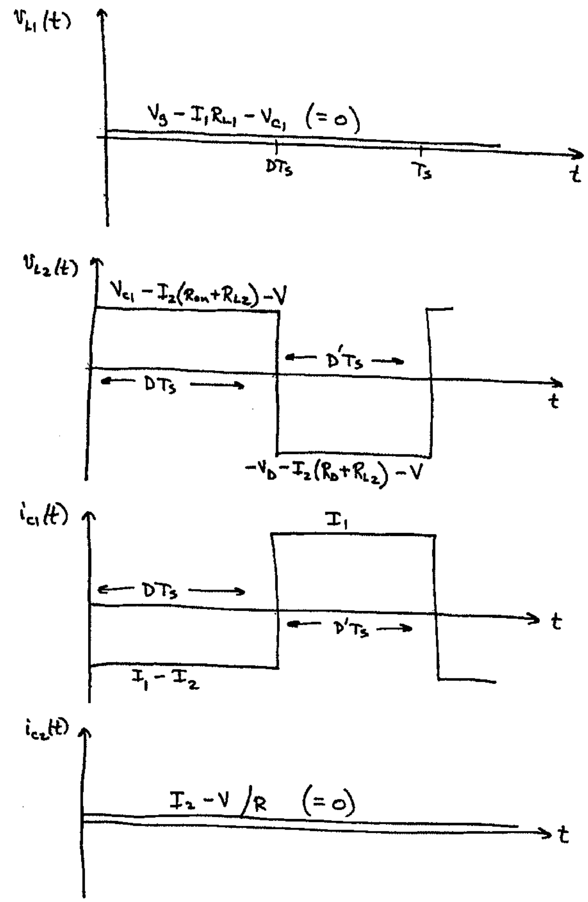


$$v_{L1}(t) = V_g - i_1(t)R_{L1} - v_{c1}(t)$$
$$v_{L2}(t) = -V_D - i_2(t)R_D - i_2(t)R_{L2} - v(t)$$
$$i_{c1}(t) = i_1(t)$$
$$i_{c2}(t) = i_2(t) - v(t)/R$$

Small ripple approximation:

$$v_{L1}(t) \approx V_g - I_1 R_{L1} - V_{c1}$$
$$v_{L2}(t) \approx -V_D - I_2 R_D - I_2 R_{L2} - V$$
$$i_{c1}(t) \approx I_1$$
$$i_{c2}(t) \approx I_2 - V/R$$

Switched waveforms:



Equate average inductor voltages and capacitor currents to zero:

$$\langle v_{L1}(t)\rangle = 0 = V_g - I_1R_{L1} - V_{c1}$$

$$\langle v_{L2}(t)\rangle = 0 = D\left[V_{c1} - I_2(R_{on}+R_{L2}) - V\right] + D'\left[-V_D - I_2(R_D+R_{L2}) - V\right]$$

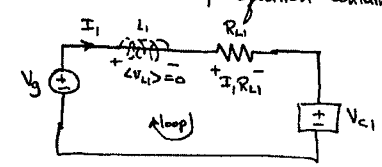
$$\langle i_{c1}(t)\rangle = 0 = D\left[I_1 - I_2\right] + D'\left[I_1\right]$$

$$\langle i_{c2}(t)\rangle = 0 = I_2 - V/R$$

Derive equivalent circuit

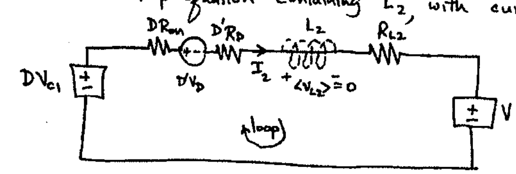
Inductor L_1: $\langle v_{L1}\rangle = 0 = V_g - I_1R_{L1} - V_{c1}$

A loop equation containing L_1, with current I_1



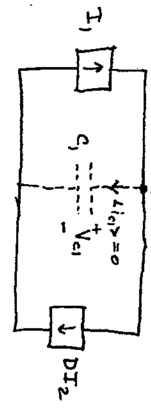
Inductor L_2: $\langle v_{L2}\rangle = 0 = DV_{c1} - DR_{on}I_2 - D'R_DI_2 - R_{L2}I_2 - V - D'V_D$

A loop equation containing L_2, with current I_2



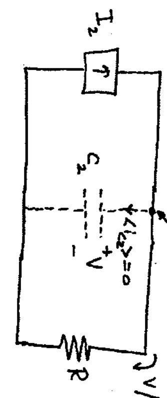
Capacitor C_1: $\langle i_{c1} \rangle = 0 = I_1 - DI_2$

A node equation containing C_1, with voltage V_{c1}

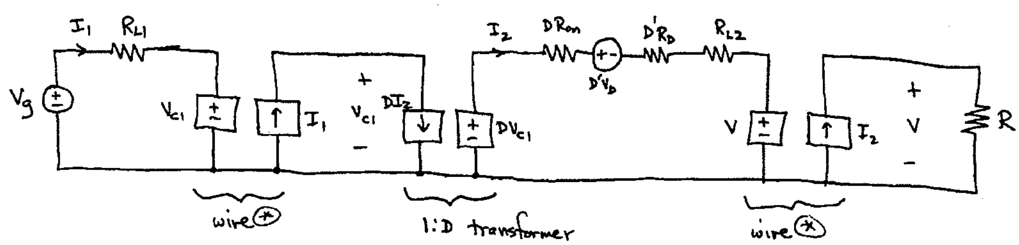


Capacitor C_2: $\langle i_{c2} \rangle = 0 = I_2 - V/R$

A node equation including C_2, with voltage V.

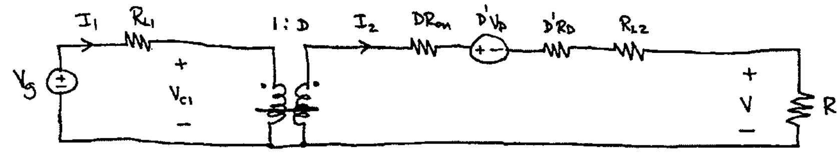


write circuits together:



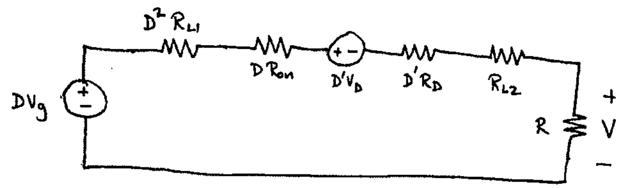
⊛ Note that 1:1 transformers are not needed in these locations – a direct connection will suffice! In fact, in the actual converter there is no switching at these points. Inductor L_1 is directly connected to C_1, and L_2 is always connected to C_2

The DI_2 and DV_{c1} dependent sources form a 1:D effective dc transformer.

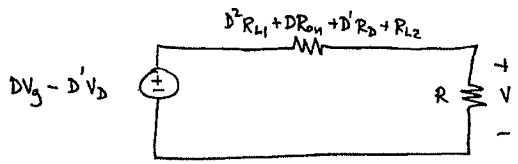


b) Solve model to find V

Push V_g and R_{L1} through dc transformer:



Combine elements:



Voltage divider formula:

$$\boxed{V = \left(DV_g - D'V_D\right) \frac{R}{R + D^2 R_{L1} + D R_{on} + D' R_D + R_{L2}}}$$

c) Derive an expression for efficiency η. Manipulate into form similar to Eq. (3.35)

- From the equivalent circuit on page 6,

$$P_{in} = V_g I_1$$
$$P_{out} = V I_2$$
and $$I_1 = D I_2$$

So

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V I_2}{V_g I_1} = \frac{1}{D} \frac{V}{V_g}$$

From part (b),

$$\frac{V}{V_g} = \left(D - D' \frac{V_D}{V_g}\right) \frac{R}{R + D^2 R_{L1} + D R_{on} + D' R_D + R_{L2}}$$

(note that it is OK for the right side of the equation to depend on V_g).

So

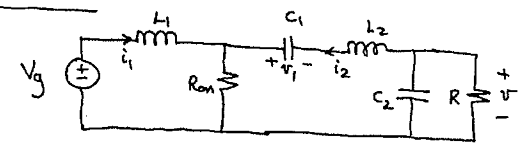
$$\eta = \left(1 - \frac{D'}{D} \frac{V_D}{V_g}\right) \frac{1}{\left(1 + D^2 \frac{R_{L1}}{R} + D \frac{R_{on}}{R} + D' \frac{R_D}{R} + \frac{R_{L2}}{R}\right)}$$

This is a good design-oriented way to express the efficiency, because it exposes how each loss element reduces the efficiency. The effect of the diode voltage drop V_D is expressed in terms of V_g, while the loss resistances are compared to R. The various losses also depend on duty cycle.

<u>End of problem 3.6</u>

Problem 3.11 Ćuk converter

a)



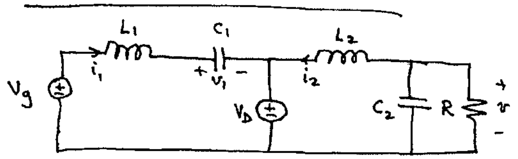
when Q_1 conducts $0<t<DT_s$

$v_{L1} = V_g - (i_1+i_2)R_{on} \approx V_g - (I_1+I_2)R_{on}$

$i_{c1} = -i_2 \approx -I_2$

$v_{L2} = v + v_1 - (i_1+i_2)R_{on} \approx V + V_1 - (I_1+I_2)R_{on}$

$i_{c2} = -i_2 - \frac{v}{R} \approx -I_2 - \frac{V}{R}$



when D_1 conducts $DT_s<t<T_s$

$v_{L1} = V_g - v_1 - V_D \approx V_g - V_1 - V_D$

$i_{c1} = i_1 \approx I_1$

$v_{L2} = v - V_D \approx V - V_D$

$i_{c2} = -i_2 - \frac{v}{R} \approx -I_2 - \frac{V}{R}$

Averaged equations (volt-sec and charge balance)

$0 = V_g - D(I_1+I_2)R_{on} - D'V_1 - D'V_D$

$0 = -DI_2 + D'I_1$

$0 = V + DV_1 - D(I_1+I_2)R_{on} - D'V_D$

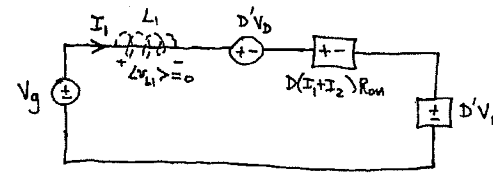
$0 = -I_2 - \frac{V}{R}$

Construct equivalent circuit

$0 = V_g - D(I_1+I_2)R_{on} - D'V_1 - D'V_D$

loop equation for inductor L_1

loop current $= I_1$

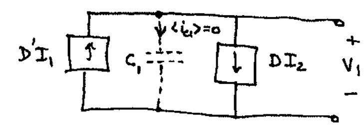


How to handle the $D(I_1+I_2)R_{on}$ term? We would like to model this term using a DR_{on} resistor. But it also depends on I_2. Let's follow the hint, and leave it as a dependent source for now.

$0 = -DI_2 + D'I_1$

node equation for capacitor C_1

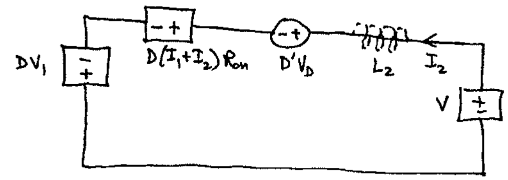
Node voltage $= V_1$



$0 = V + DV_1 - D(I_1+I_2)R_{on} - D'V_D$

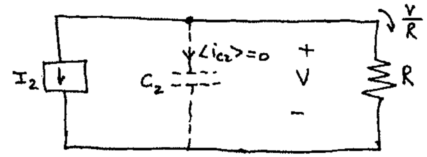
loop equation for inductor L_2

loop current $= I_2$

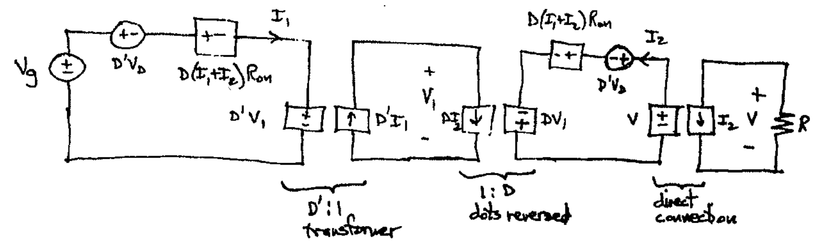


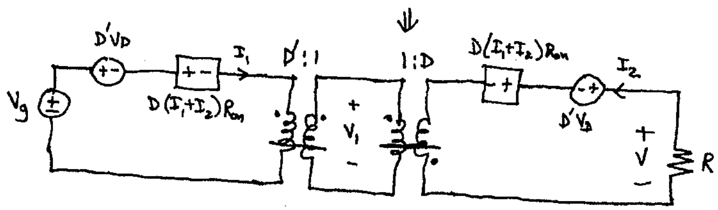
$$0 = -I_2 - \frac{V}{R}$$

node equation for capacitor C_2
Node voltage = V

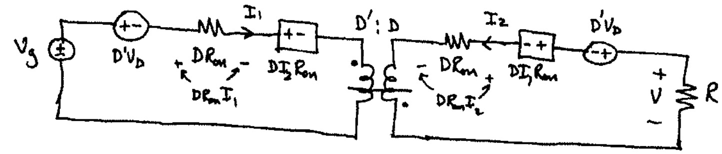


Put the circuits together

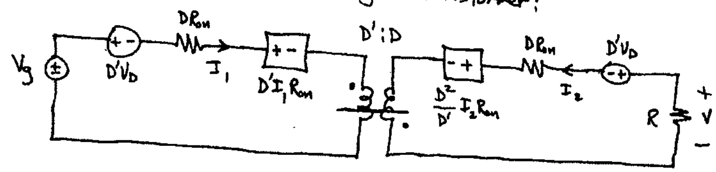




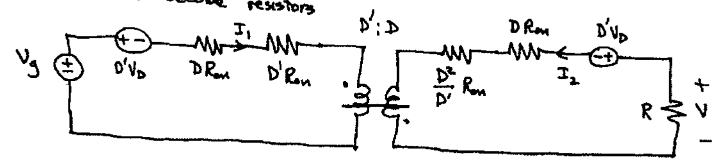
<u>One way to handle the</u> $D(I_1+I_2)R_{on}$ sources;



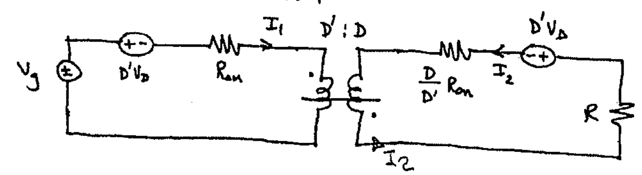
Now push dependent sources through transformer;



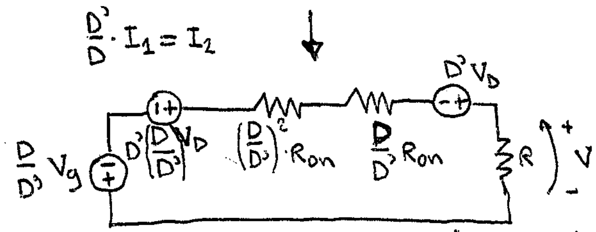
Sources now become resistors



Combine series resistors;



$$\frac{D'}{D} \cdot I_1 = I_2$$



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$$V = V_g\left(-\frac{D}{D'^2} + \frac{V_D}{V_g}\right)\left(\frac{1}{1+\frac{D}{D'^2}\cdot\frac{R_{on}}{R}}\right)$$

b) Solve model for V and η

$$V = \left[-\frac{D}{D'}(V_g - D'V_D) + D'V_D\right] \frac{R}{R + \frac{D}{D'}R_{on} + \left(\frac{D}{D'}\right)^2 R_{on}}$$

$$\frac{V}{V_g} = -\frac{D}{D'} \frac{\left(1 - \frac{D'V_D}{D V_g}\right)}{\left(1 + \frac{D}{(D')^2}\frac{R_{on}}{R}\right)}$$

$$\eta = \frac{\left(1 - \frac{D'V_D}{D V_g}\right)}{\left(1 + \frac{D}{(D')^2}\frac{R_{on}}{R}\right)}$$

$$P_{in} = \frac{D}{D'} V_g \cdot I_2$$
$$P_{out} = V \cdot I_2$$

$$\frac{P_{out}}{P_{in}} = \frac{-V \cdot \cancel{I_2}}{V_g \cdot \cancel{I_2}} \cdot \frac{D'}{D}$$

$$\eta = -\frac{D'}{D} \cdot \frac{V}{V_g}$$

c) and d) see plots on next page

(c) and (d)