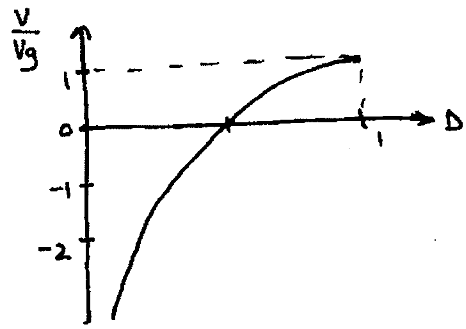
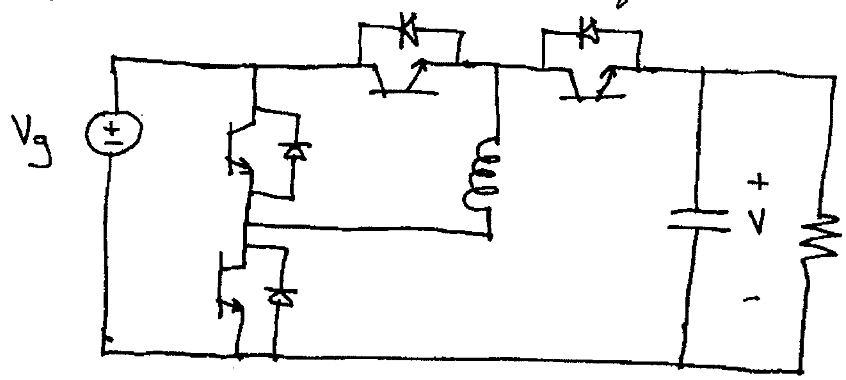
Problem 4.1 (Watkins-Johnson converter)

$$V = \frac{D - D'}{D} V_g$$

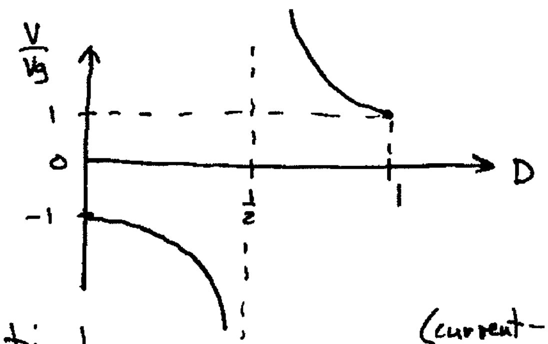


Requires current-bidirectional two-quadrant switches



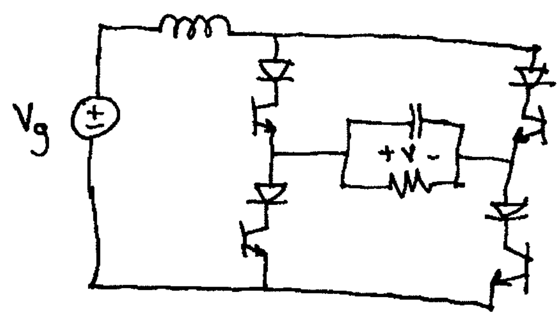
Problem 4.2

$$V = \frac{1}{D - D'} V_g$$



(current-fed bridge)

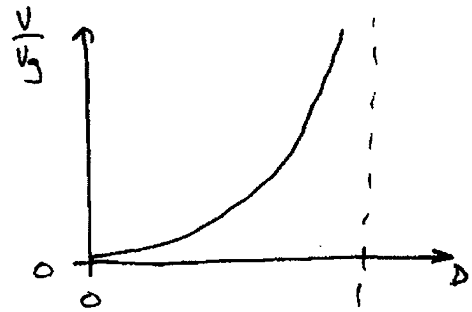
Requires voltage-bidirectional two-quadrant switches

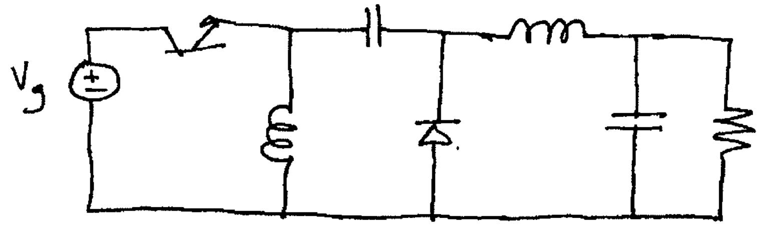


Problem 4.3 (inverse SEPIC)

$$\frac{V}{V_g} = +\frac{D}{D'}$$

single-quadrant switches

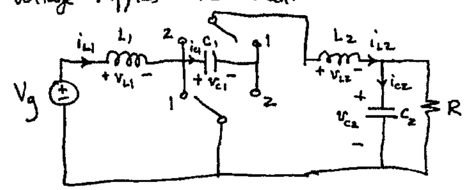


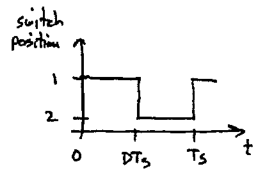


Tutorial

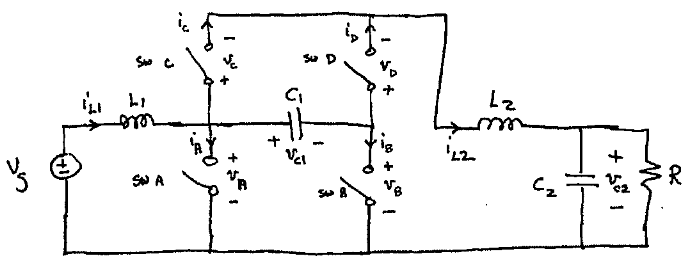
Solution to problem 4.4

Realize switches using transistors and diodes, such that the converter operates in CCM over the entire range $0 \le D \le 1$. The inductor current ripples and capacitor voltage ripples are small.





a) Realize the switches using SPST ideal switches, and explicitly define the voltage and current of each switch



Polarity of switch voltage can be arbitrarily defined. Switch current polarity must then be defined to flow through switch from + terminal to − terminal.

subinterval 1: sw A and sw D are closed

subinterval 2: sw B and sw C are closed

b) Express the on-state current and off-state voltage of each switch in terms of the converter inductor currents, capacitor voltages, and independent sources.

Switch A

on state: $i_A = i_{L1} - i_{L2}$, $v_A = 0$ (subinterval 1)

off state: $v_A = v_{C1}$, $i_A = 0$ (subinterval 2)

Switch B

on state: $i_B = i_{L1} - i_{L2}$, $v_B = 0$ (subinterval 2)

off state: $v_B = -v_{C1}$, $i_B = 0$ (subinterval 1)

Switch C

on state: $i_C = i_{L2}$, $v_C = 0$ (subinterval 2)

off state: $v_C = v_{C1}$, $i_C = 0$ (subinterval 1)

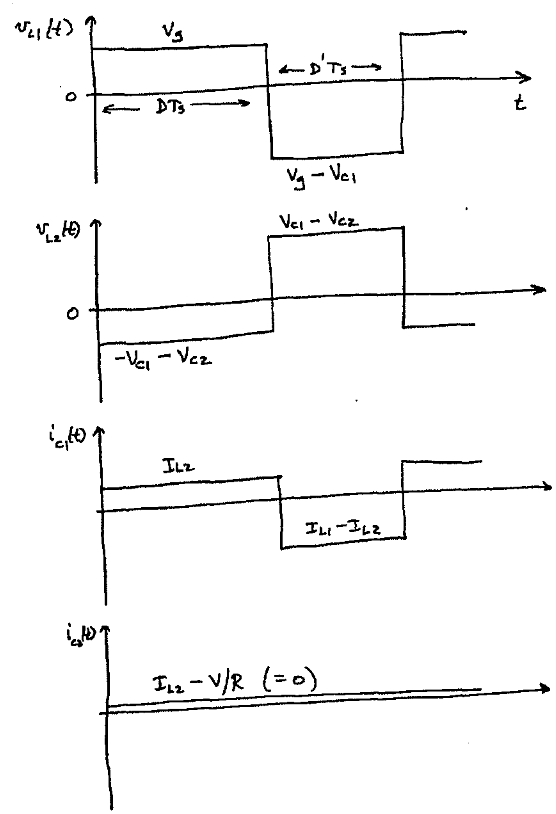
Switch D

on state: $i_D = i_{L2}$, $v_D = 0$ (subinterval 1)

off state: $v_D = -v_{C1}$, $i_D = 0$ (subinterval 2)

c) Solve the converter to determine the dc components of the inductor currents and capacitor voltages, as in chapter 2.

Inductor voltage and capacitor current waveforms, using small ripple approximation:



Volt-second balance on L_1: $\langle v_{L1} \rangle = 0 = DV_g + D'(V_g - V_{c1})$

Volt-second balance on L_2: $\langle v_{L2} \rangle = 0 = D(-V_{c1} - V_{c2}) + D'(V_{c1} - V_{c2})$

Charge balance on C_1: $\langle i_{c1} \rangle = 0 = DI_{L2} + D'(I_{L1} - I_{L2})$

Charge balance on C_2: $\langle i_{c2} \rangle = 0 = I_{L2} - V/R$

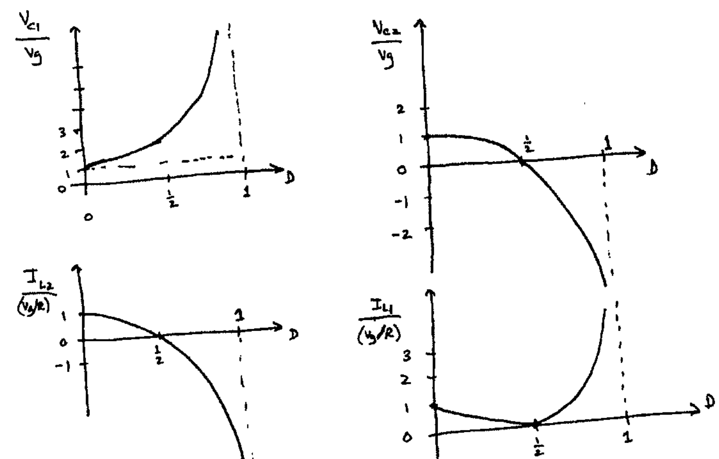
Solution:

$$V_{c1} = \frac{1}{D'} V_g$$

$$V_{c2} = (D' - D) V_{c1} = \frac{D' - D}{D'} V_g = \frac{1-2D}{1-D} V_g$$

$$I_{L2} = \frac{V}{R} = \frac{1-2D}{1-D} \frac{V_g}{R}$$

$$I_{L1} = \frac{D' - D}{D'} I_{L2} = \left(\frac{1-2D}{1-D}\right)^2 \frac{V_g}{R}$$



d) Polarities of switch voltages and current vs. duty cycle D

Switch A

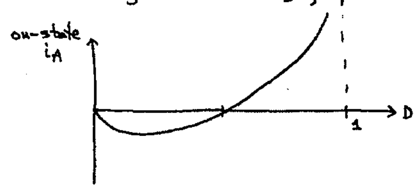
on state: $i_A = i_{L1} - i_{L2} = \left(\frac{1-2D}{1-D}\right)^2 \frac{V_g}{R} - \frac{1-2D}{1-D}\frac{V_g}{R}$

$= \frac{1-2D}{1-D}\frac{V_g}{R}\left(\frac{1-2D}{1-D} - 1\right)$

$= \frac{V_g}{R}\frac{1-2D}{1-D}\frac{1-2D-(1-D)}{1-D}$

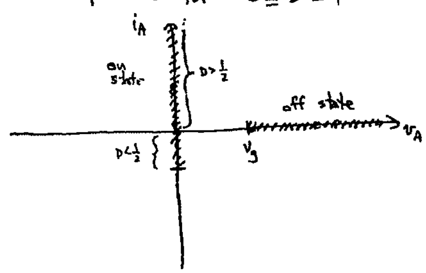
$= \frac{V_g}{R}(1-2D)\frac{(-D)}{(1-D)^2}$

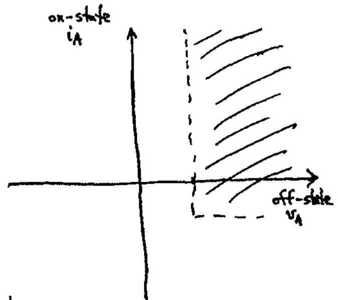
negative for $D < \frac{1}{2}$, positive for $D > \frac{1}{2}$



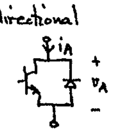
off-state: $v_A = v_{C1} = \frac{1}{1-D} V_g$

positive for $0 \le D \le 1$





Requires a current-bidirectional two-quadrant switch:



Switch B

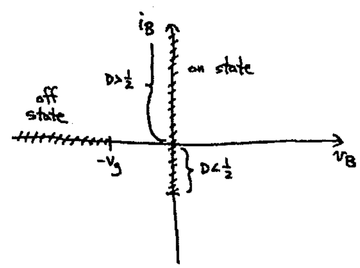
on state: $i_B = i_{L1} - i_{L2} = \frac{V_g}{R}(1-2D)\frac{(-D)}{(1-D)^2}$

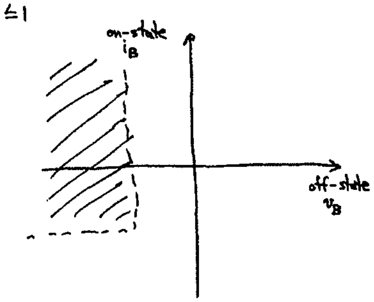
same as switch A.

negative for $D < \frac{1}{2}$, positive for $D > \frac{1}{2}$

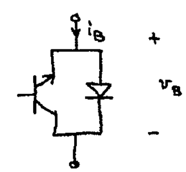
off state: $v_B = -v_{C1} = -\frac{1}{1-D} V_g$

negative for $0 \le D \le 1$





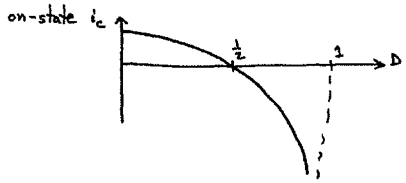
Requires a current-bidirectional two-quadrant switch that blocks negative v_B:



(Note: if we had originally defined v_B and i_B with the opposite polarity, then we would have obtained a current-bidirectional two-quadrant switch having the same polarity and realization as switch A)

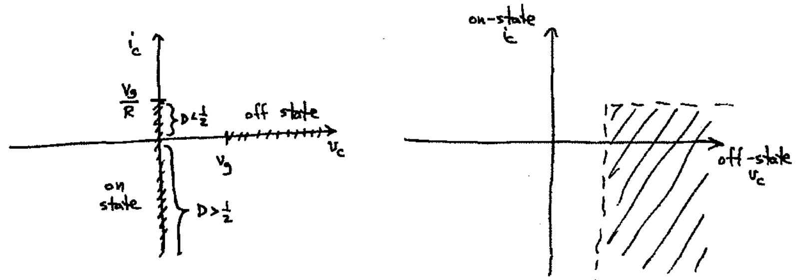
Switch C

on state: $i_C = i_{L2} = \frac{1-2D}{1-D}\frac{V_g}{R}$

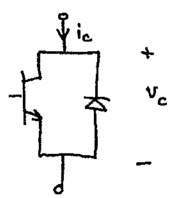


positive for $D < \frac{1}{2}$
negative for $D > \frac{1}{2}$

off state: $v_C = V_{C1} = \frac{1}{1-D} V_g$ positive for $0 \le D \le 1$



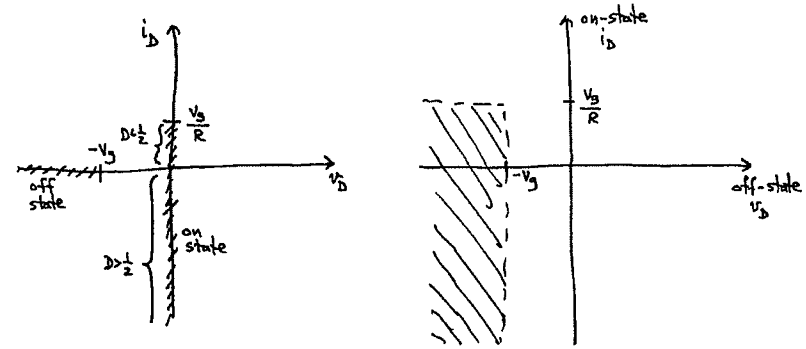
Requires a current-bidirectional two-quadrant switch:



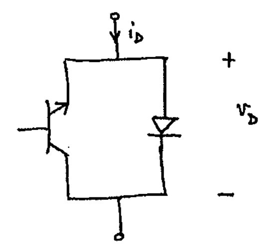
switch D

on state: $i_D = i_{L2} = \frac{1-2D}{1-D}\frac{V_g}{R}$ positive for $D < \frac{1}{2}$
negative for $D > \frac{1}{2}$
(same as switch C)

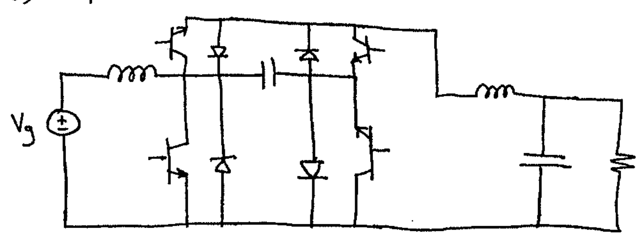
off state: $v_D = -V_{C1} = -\frac{1}{1-D} V_g$ negative for $0 \le D \le 1$



Requires a current-bidirectional two-quadrant switch that blocks negative voltage:



e) Implement switches in converter:

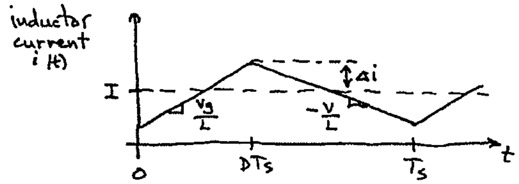


Of course, instead of BJT's, other devices such as MOSFETs or IGBTs could be used.

<u>End of problem 4.4</u>

Problem 5.1 Buck-boost in DCM

a) $I > \Delta i \Rightarrow CCM$, $I < \Delta i \Rightarrow DCM$



from CCM solution: $I = \frac{-V}{D'R} = \frac{D V_g}{D'^2 R}$

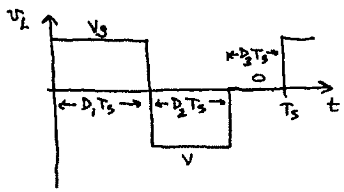
$$\Delta i = \frac{D T_s V_g}{2L}$$

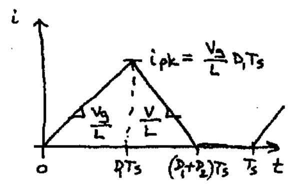
so converter operates in DCM when

$$\frac{D V_g}{D'^2 R} < \frac{D T_s V_g}{2L} \Rightarrow \frac{2L}{R T_s} < (D')^2$$

or, $K < K_{crit}(D)$ where $K = \frac{2L}{RT_s}$, $K_{crit} = (D')^2$

b) Inductor waveforms in DCM:





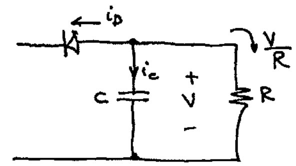
inductor volt-second balance

$$\langle v_L \rangle = 0 = D_1 V_g + D_2 V + D_3 \cdot 0 = D_1 V_g + D_2 V$$

$\Rightarrow V = -\frac{D_1}{D_2} V_g$ problem – don't know D_2

(ok to use small ripple approximation here since $|\Delta v| \ll |V|$)

capacitor charge balance



node equation:

$$i_D + i_C + \frac{V}{R} = 0$$

charge balance:

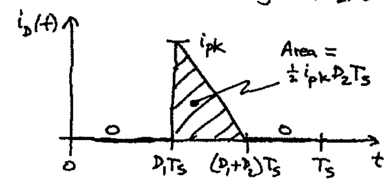
$$\langle i_C \rangle = 0$$

$\Rightarrow \langle i_D \rangle = -\frac{V}{R}$ dc load current is supplied by dc component of diode current

so sketch diode current, and find its average value:

$$i_D(t) = \begin{cases} 0 & \text{during } 0 < t < D_1 T_s \\ i(t) & \text{during } D_1 T_s < t < (D_1+D_2)T_s \\ 0 & \text{during } (D_1+D_2)T_s < t < T_s \end{cases}$$

can't use small ripple approximation here: $\Delta i > I$!



$$\langle i_D \rangle = \frac{1}{T_s} \int_0^{T_s} i_D(t)\,dt$$

$$= \frac{1}{T_s}\left(\frac{1}{2} i_{pk} D_2 T_s\right)$$

$$= \frac{1}{2} D_2 i_{pk} = \frac{V_g D_1 D_2 T_s}{2L}$$

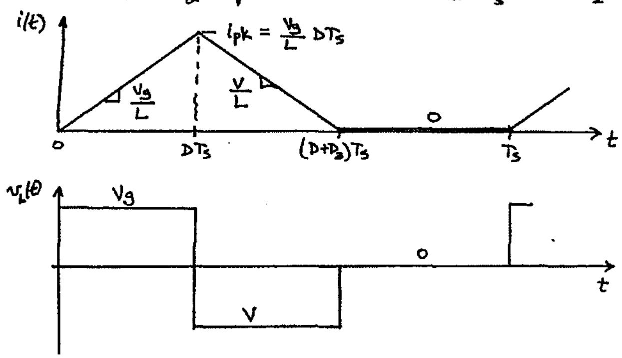
so $\dfrac{-V}{R} = \dfrac{V_g D_1 D_2 T_s}{2L} = \dfrac{D_1}{D_2}\dfrac{V_g}{R}$

$\Rightarrow D_2^2 = \dfrac{2L}{RT_s} = K \Rightarrow D_2 = \sqrt{K}$ (take positive root since D_2 cannot be negative)

and hence $V = -V_g \dfrac{D_1}{\sqrt{K}}$

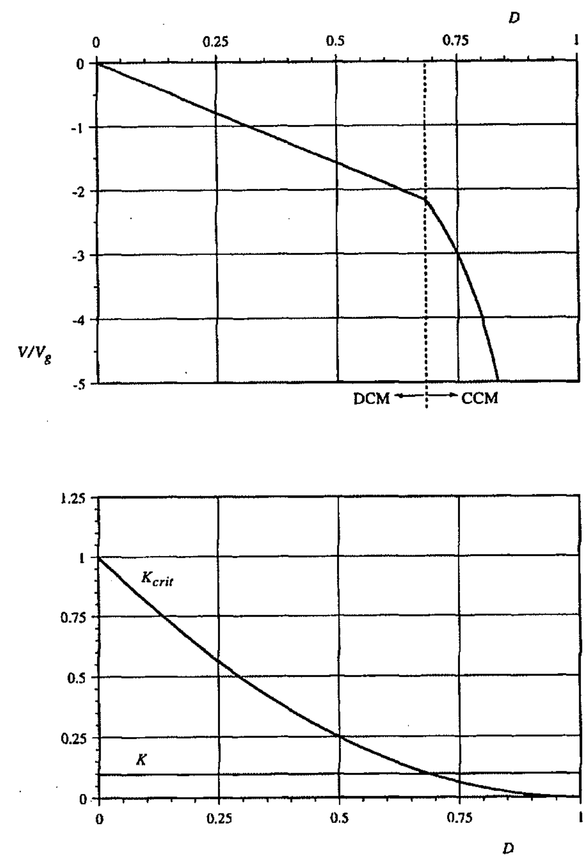
c) see next page

d) For $D=0.3$, $K_{crit} = (1-0.3)^2 = 0.49 \Rightarrow K < K_{crit} \Rightarrow$ DCM
$K = 0.1 \Rightarrow D_2 = \sqrt{0.1} = 0.32$ and $D_3 = 1 - D - D_2 = 0.38$



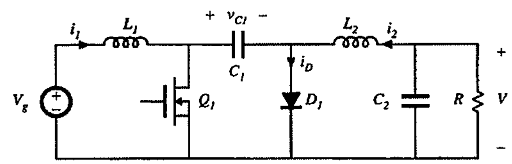
e) At no load, $R \to \infty$ so $K = \frac{2L}{RT_s} \to 0$. DCM, $\frac{V}{V_g} = -\frac{D}{\sqrt{K}} \to -\infty$
In ideal case, $V \to -\infty$. Inductor keeps transferring energy to output capacitor, but there is no load to consume energy.
In practice, output voltage may become very large when load is disconnected.
To avoid exceeding device ratings, could (1) connect minimum load to output, or (2) reduce D to zero.

5.1 (c)



Tutorial

Solution to Problem 5.5
DCM mode boundary analysis
of the Cuk converter



CCM analysis of the Cuk converter is given in Section 2.4. Some results:

$$V_{C1} = \frac{V_g}{D'}$$

$$V = -\frac{D}{D'} V_g$$

$$I_1 = \left(\frac{D}{D'}\right)^2 \frac{V_g}{R}$$

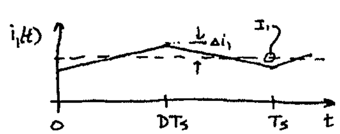
$$I_2 = \frac{D}{D'} \frac{V_g}{R}$$

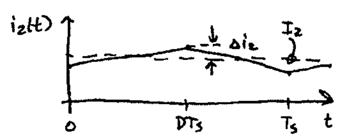
(note that the polarity of i_2 is reversed in Section 2.4, and the quantities v_{C1} and V are called v_1 and v_2 respectively).

Inductor current ripples (from Eq. (2.57)):

$$\Delta i_1 = \frac{V_g D T_s}{2L_1}$$

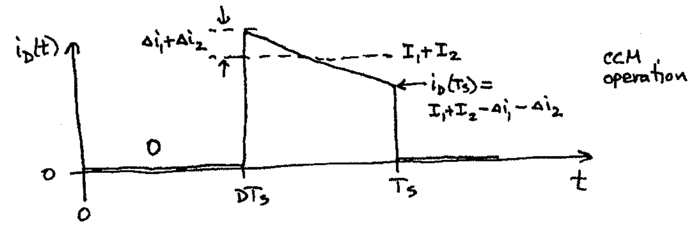
$$\Delta i_2 = \frac{V_g D T_s}{2L_2}$$





a) Sketch diode current waveform

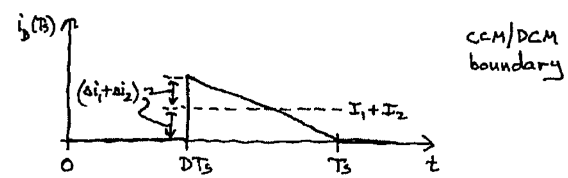
$$i_D(t) = \begin{cases} 0 & \text{during subinterval 1 (diode off)} \\ i_1(t) + i_2(t) & \text{during subinterval 2 (transistor off)} \end{cases}$$



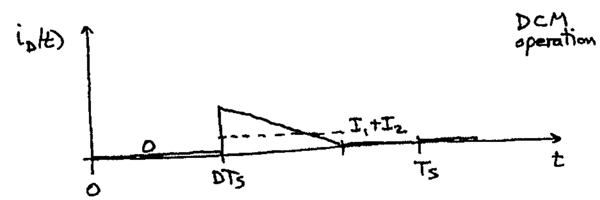
peak $i_D(t) = I_1 + I_2 + \Delta i_1 + \Delta i_2$

b) CCM/DCM mode boundary

The dc components of inductor currents, I_1 and I_2, depend on the load resistance R. The inductor current ripples, Δi_1 and Δi_2, do not depend on the load resistance R. When we increase R, $(I_1 + I_2)$ decreases but $(\Delta i_1 + \Delta i_2)$ does not change. If we increase R such that $(I_1 + I_2) = (\Delta i_1 + \Delta i_2)$, then the diode current will be zero at the end of the switching period: $i_D(T_s) = (I_1 + I_2) - (\Delta i_1 + \Delta i_2) = 0$.



If we further increase R, then $i_D(t)$ will reach zero before the end of the switching period. The diode then becomes reverse-biased, and the converter operates in the discontinuous conduction mode:



So the Cuk converter operates in DCM when

$$I_1 + I_2 < \Delta i_1 + \Delta i_2$$

Substitute the CCM expressions for I_1, I_2, Δi_1, Δi_2 (note that the CCM analysis is valid at the CCM/DCM boundary):

$$\left(\frac{D}{D'}\right)^2 \frac{V_g}{R} + \frac{D}{D'}\frac{V_g}{R} < \frac{V_g DT_s}{2L_1} + \frac{V_g DT_s}{2L_2}$$

Rearrange terms:

$$\Rightarrow \quad 2\,\frac{L_1 \| L_2}{R\,T_s} < (D')^2$$

$$K < K_{crit}(D) \quad \text{for DCM}$$

with $K = \dfrac{2\,L_1 \| L_2}{R T_s}$, $K_{crit} = (D')^2$

End of problem 5.5

Tutorial

Solution to Problem 5.6

Conversion ratio analysis of the Cuk converter in DCM

See solution to problem 5.5 for schematic and mode boundary analysis.

a) For the given values, we obtain

$$\left.\begin{aligned} K &= \frac{2\, L_1 \| L_2}{R\, T_s} = \frac{(2)\ (54\mu H \,\|\, 27\mu H)}{(10\Omega)\ (10\mu s)} \\ &= 0.36 \\ K_{crit} &= (D')^2 = (1-0.4)^2 = 0.36 \end{aligned}\right\} \text{these equations derived in Prob. 5.5}$$

so indeed $K = K_{crit}$ and the converter operates at the boundary between CCM and DCM

We can sketch the current waveforms $i_D(t)$, $i_1(t)$, and $i_2(t)$ using the CCM analysis of Section 2.4.

From Eq. (2.53):

$$I_1 = \left(\frac{D}{D'}\right)^2 \frac{V_g}{R} = \left(\frac{0.4}{0.6}\right)^2 \frac{(120)}{(10)} = 5.33\text{ A}$$

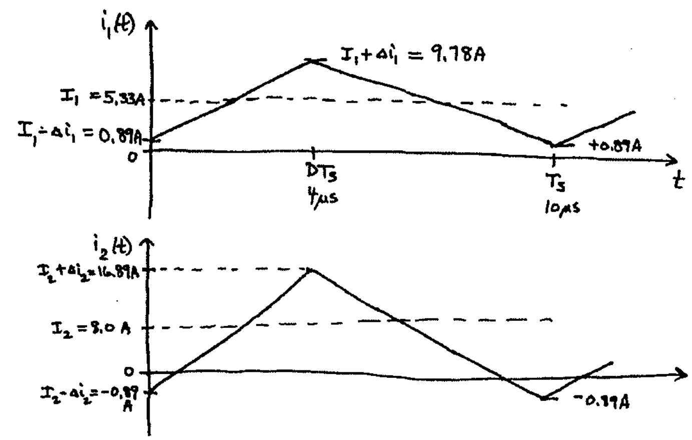
$$I_2 = \left(\frac{D}{D'}\right)\left(\frac{V_g}{R}\right) = \left(\frac{0.4}{0.6}\right)\left(\frac{120}{10}\right) = 8.0\text{ A}$$

(Note: polarity of I_2 in Fig. 5.23 is reversed from polarity used in Section 2.4)

From Eq. (2.57):

$$\Delta i_1 = \frac{V_g D T_s}{2L_1} = \frac{(120V)(0.4)(10\mu s)}{(2)(54\mu H)} = 4.44\text{A}$$

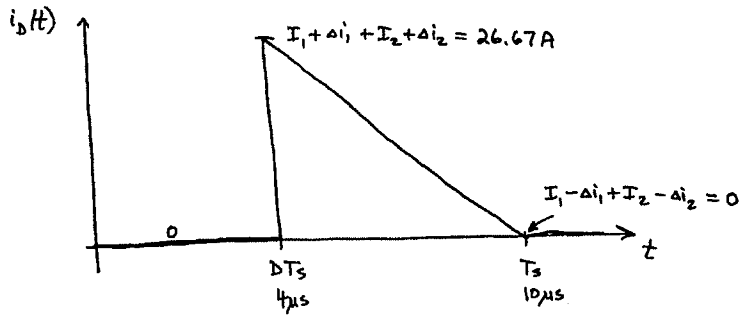
$$\Delta i_2 = \frac{V_g D T_s}{2L_2} = \frac{(120V)(0.4)(10\mu s)}{(2)(27\mu H)} = 8.88\text{A}$$



note $i_2(0) < 0$ and $\Delta i_2 > I_2$

Diode current waveform

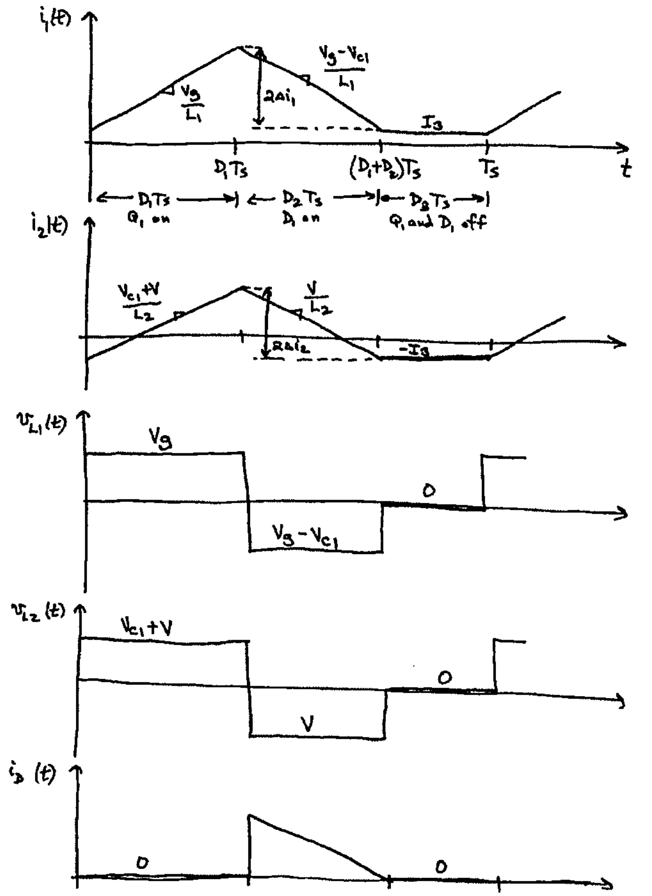
$$i_D(t) = \begin{cases} 0 & \text{during} \quad 0 < t < DT_s \\ i_1(t) + i_2(t) & \text{during} \quad DT_s < t < T_s \end{cases}$$



Note $i_D(T_s) = 0$ (for this operating point at the CCM/DCM boundary)

Note that $i_D(T_s) = 0$ does not necessarily imply that $i_1(T_s) = 0$ and $i_2(T_s) = 0$! Rather, it implies that $i_1(T_s) = -i_2(T_s)$.

If the load resistance is increased, then the dc currents I_1 and I_2 are decreased. The converter enters the discontinuous conduction mode, with the following waveforms:



(solution to part c)

b) Solution of DC conversion ratio $M(D_1, K)$ in DCM

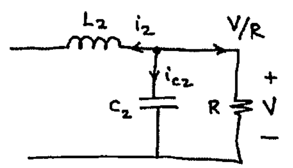
Note: this is a general approach to solving the DCM waveforms, there are easier ways to obtain the same result — see Ch 10

Volt-second balance on L_1 and L_2:

$$\langle v_{L1} \rangle = D_1 V_g + D_2 (V_g - V_{c1}) = 0$$

$$\langle v_{L2} \rangle = D_1 (V_{c1} + V) + D_2 (V) = 0$$

Charge balance on C_2:

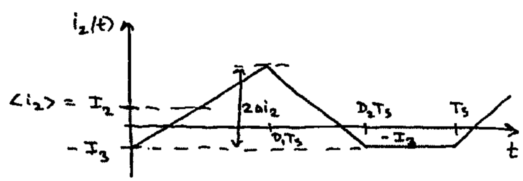


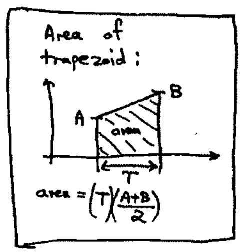
Dc component $\langle i_{c2} \rangle = 0$

Therefore, the dc load current $-\frac{V}{R}$ is equal to the dc component of inductor current $\langle i_2 \rangle = I_2$

$$I_2 = -\frac{V}{R}$$

Must compute $I_2 = \langle i_2 \rangle$. Inductor current waveform from previous page:





$$\langle i_2 \rangle = \frac{1}{T_s} \int_0^{T_s} i_2(t)\,dt = \frac{1}{T_s} \cdot \left[(D_1 T_s) \frac{(-I_3) + (-I_3 + 2\Delta i_2)}{2} + (D_2 T_s) \frac{(-I_3) + (-I_3 + 2\Delta i_2)}{2} + (D_3 T_s)(-I_3) \right]$$

note $D_1 + D_2 + D_3 = 1$

simplify:

$$I_2 = -I_3 + (D_1 + D_2)\Delta i_2$$

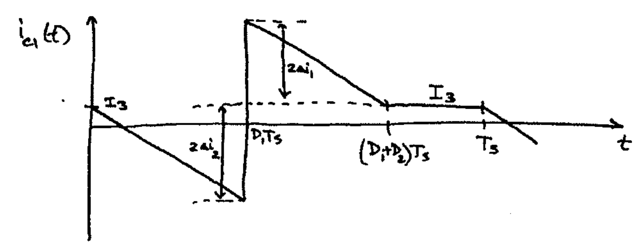
so

$$-I_3 + (D_1 + D_2)\Delta i_2 = -\frac{V}{R}$$

with $\Delta i_2 = \dfrac{V_{c1} + V}{2 L_2} D_1 T_s$

Charge balance on C_1:

$$i_{c1}(t) = \begin{cases} -i_2(t) & \text{during } 0 < t < D_1 T_s \\ i_1(t) & \text{during } D_1 T_s < t < (D_1 + D_2) T_s \\ i_1(t)\ (= -i_2(t) = I_3) & \text{during } (D_1 + D_2) T_s < t < T_s \end{cases}$$



capacitor charge balance:

$$\langle i_{c1} \rangle = 0 = \frac{1}{T_s} \int_0^{T_s} i_{c1}(t)\,dt = \frac{1}{T_s} \cdot \left[D_1 T_s \frac{I_3 + (I_3 - 2\Delta i_2)}{2} + D_2 T_s \frac{(I_3 + 2\Delta i_1) + I_3}{2} + D_3 T_s I_3 \right]$$

Simplify:

$$0 = I_3 - D_1 \Delta i_2 + D_2 \Delta i_1$$

Summary of DCM equations:

(i) $0 = (D_1 + D_2) V_g - D_2 V_{c1}$ (volt-sec balance on L_1)

(ii) $0 = (D_1 + D_2) V + D_1 V_{c1}$ (volt-sec balance on L_2)

(iii) $-I_3 + (D_1 + D_2) \Delta i_2 = -\frac{V}{R}$ (charge balance on C_2)

(iv) $0 = I_3 - D_1 \Delta i_2 + D_2 \Delta i_1$ (charge balance on C_1)

with $\Delta i_1 = \frac{V_g}{2L_1} D_1 T_s$ (v)

$\Delta i_2 = \frac{V_{c1} + V}{2L_2} D_1 T_s$ (vi)

(expressions for inductor current ripples)

Algebra: solve for $M = \frac{V}{V_g}$

from (i): $V_{c1} = \frac{D_1 + D_2}{D_2} V_g$

from (ii): $V = -\frac{D_1}{D_1 + D_2} V_{c1} = -\frac{D_1}{D_2} V_g \Rightarrow M = \frac{V}{V_g} = -\frac{D_1}{D_2}$

Eliminate I_3 from (iii) and (iv):

$$I_3 = (D_1 + D_2) \Delta i_2 + \frac{V}{R} = D_1 \Delta i_2 - D_2 \Delta i_1$$

Solve for D_2:

$$D_2 (\Delta i_1 + \Delta i_2) = -\frac{V}{R} \Rightarrow D_2 = -\frac{V}{R} \frac{1}{\Delta i_1 + \Delta i_2}$$

Substitute (v) and (vi):

$$D_2 = -\frac{V}{R} \frac{1}{(D_1 T_s)\left(\frac{V_g}{2L_1} + \frac{V_{c1} + V}{2L_2}\right)} = -\frac{V}{R D_1 T_s} \frac{2}{\frac{V_g}{L_1} + \frac{V_g}{L_2}}$$

$$= -\frac{V}{V_g} \frac{2 L_1 \| L_2}{D_1 R T_s} \Rightarrow D_2 = -\frac{MK}{D_1}$$

Now substitute this expression for D_2 into the previous expression for M:

$$M = -\frac{D_1}{D_2} = -\frac{D_1}{\left(-\frac{MK}{D_1}\right)}$$

← note that, since D_1 and D_2 are positive, M must be negative

Solve for $M(D_1, K)$:

$$M^2 = \frac{D_1^2}{K} \Rightarrow M(D_1, K) = \pm \frac{D_1}{\sqrt{K}}$$ select minus sign

$$\boxed{M(D_1, K) = -\frac{D_1}{\sqrt{K}}}$$

c) see page 4