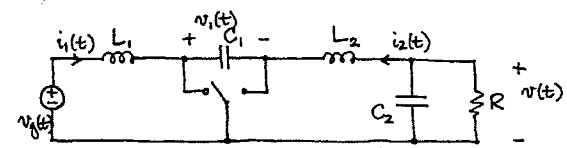
Problem 7.6

Ideal Ćuk converter in CCM:



(a) Averaged nonlinear equations:

$$L_1 \frac{d\langle i_1\rangle_{T_s}}{dt} = \langle v_g\rangle_{T_s} - d'\langle v_1\rangle_{T_s}$$

$$C_1 \frac{d\langle v_1\rangle_{T_s}}{dt} = d'\langle i_1\rangle_{T_s} - d\,\langle i_2\rangle_{T_s}$$

$$L_2 \frac{d\langle i_2\rangle_{T_s}}{dt} = \langle v\rangle_{T_s} + d\,\langle v_1\rangle_{T_s}$$

$$C_2 \frac{d\langle v\rangle_{T_s}}{dt} = -\langle i_2\rangle_{T_s} - \frac{\langle v\rangle_{T_s}}{R}$$

Small signal AC equations:

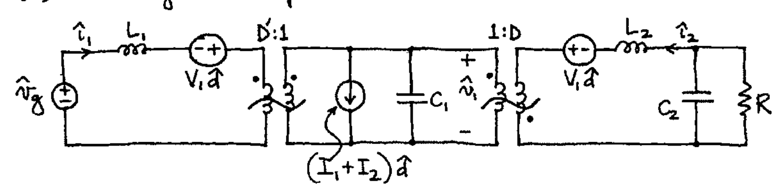
$$L_1 \frac{d\hat{i}_1}{dt} = \hat{v}_g - D'\hat{v}_1 + V_1\hat{d}$$

$$C_1 \frac{d\hat{v}_1}{dt} = D'\hat{i}_1 - D\hat{i}_2 - (I_1+I_2)\hat{d}$$

$$L_2 \frac{d\hat{i}_2}{dt} = \hat{v} + D\hat{v}_1 + V_1\hat{d}$$

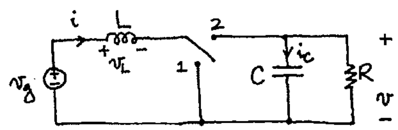
$$C_2 \frac{d\hat{v}}{dt} = -\hat{i}_2 - \frac{\hat{v}}{R}$$

(b) Small signal AC equivalent circuit

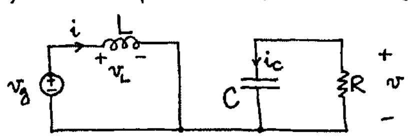


Problem 7.1

Ideal boost converter in CCM:



(a) Switch in position 1, $0 < t \le dT_s$



$$L\frac{di}{dt} = v_L = v_g$$

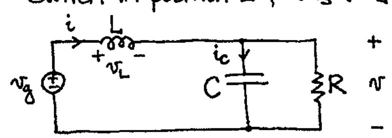
$$C\frac{dv}{dt} = i_C = -\frac{v}{R}$$

Small-ripple approximation:

$$L\frac{d\langle i(t)\rangle_{T_s}}{dt} = \langle v_g(t)\rangle_{T_s}$$

$$C\frac{d\langle v(t)\rangle_{T_s}}{dt} = -\frac{\langle v(t)\rangle_{T_s}}{R}$$

Switch in position 2, $dT_s < t \le T_s$



$$L\frac{di}{dt} = v_L = v_g - v$$

$$C\frac{dv}{dt} = i_C = i - \frac{v}{R}$$

Small ripple approximation:

$$L\frac{d\langle i(t)\rangle_{T_s}}{dt} = \langle v_g(t)\rangle_{T_s} - \langle v(t)\rangle_{T_s}$$

$$C\frac{d\langle v(t)\rangle_{T_s}}{dt} = \langle i(t)\rangle_{T_s} - \frac{\langle v(t)\rangle_{T_s}}{R}$$

Nonlinear averaged equations:

$$L\frac{d\langle i\rangle_{T_s}}{dt} = d\langle v_g\rangle_{T_s} + d'\left(\langle v_g\rangle_{T_s} - \langle v\rangle_{T_s}\right)$$

$$= \langle v_g\rangle_{T_s} - d'\langle v\rangle_{T_s}$$

$$C\frac{d\langle v\rangle_{T_s}}{dt} = d\left(-\frac{\langle v\rangle_{T_s}}{R}\right) + d'\left(\langle i\rangle_{T_s} - \frac{\langle v\rangle_{T_s}}{R}\right)$$

$$= d'\langle i\rangle_{T_s} - \frac{\langle v\rangle_{T_s}}{R}$$

(b) Perturb and linearize. Let

$$\langle v_g\rangle_{T_s} = V_g + \hat{v}_g(t)$$

$$d(t) = D + \hat{d}(t)$$

$$\langle i\rangle_{T_s} = I + \hat{i}(t)$$

$$\langle v\rangle_{T_s} = V + \hat{v}(t)$$

then

$$L\frac{d}{dt}\left(I + \hat{i}(t)\right) = V_g + \hat{v}_g(t) - \left(D' - \hat{d}(t)\right)\left(V + \hat{v}(t)\right)$$

Large signal DC components:

$$0 = V_g - D'V$$

1st order AC terms:

$$L\frac{d\hat{i}}{dt} = \hat{v}_g(t) - D'\hat{v}(t) + V\hat{d}(t)$$

$$C\frac{d}{dt}\left(V + \hat{v}(t)\right) = \left(D' - \hat{d}(t)\right)\left(I + \hat{i}(t)\right) - \frac{V + \hat{v}(t)}{R}$$

Large signal DC components

$$0 = D'I - V/R$$

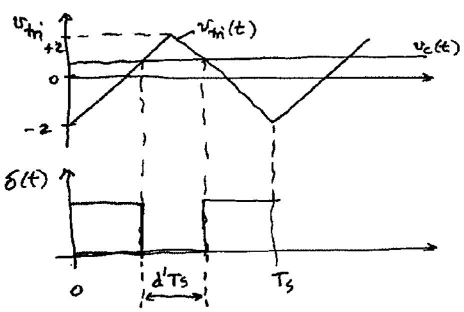
1st order AC terms:

$$C\frac{d\hat{v}}{dt} = D'\hat{i}(t) - I\hat{d}(t) - \frac{\hat{v}(t)}{R}$$

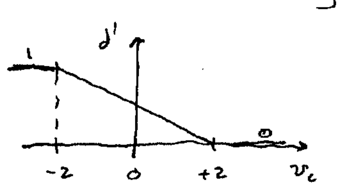
Problem 7.14

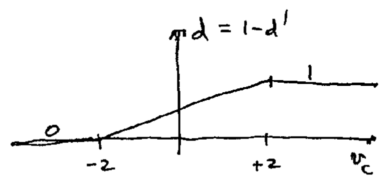
a) $T_s = 50\mu s$ so $f_s = \frac{1}{50\mu s} = 20\text{ kHz}$

b)



$d' = 0$ when $v_c = +2$
$d' = 1$ when $v_c = -2$
d' varies linearly with v_c



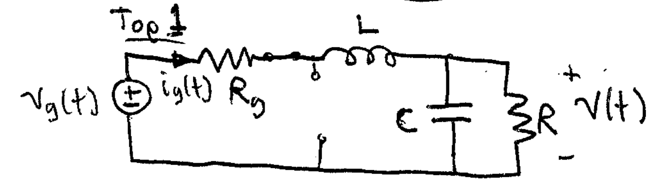


$$d = \begin{cases} 1 & \text{for } v_c \geq 2V \\ \frac{v_c}{4} + \frac{1}{2} & \text{for } -2V \leq v_c \leq +2V \\ 0 & \text{for } v_c \leq -2V \end{cases}$$

The gain $\frac{\hat{d}}{\hat{v}_c}$ is $\frac{1}{4}$, valid for $-2 \leq v_c \leq +2$ (c)

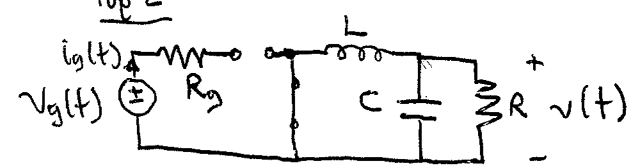
Problem 7.8 a, b

Top 1



$, 0 < t \leq DT_s$

Top 2



$, DT_s < t < T_s$

Top 1

$* \; v_L(t) = L \cdot \frac{di(t)}{dt} = V_g - i_L R_g - v(t) \qquad * \; i_c(t) = C \frac{dv(t)}{dt} = i_L - v/R$

$* \; i_g = i_L$

Top 2

$* \; v_L(t) = L \cdot \frac{di(t)}{dt} = -v(t) \qquad * \; i_c(t) = C \cdot \frac{dv(t)}{dt} = i_L - v/R \qquad * \; i_g = 0$

Top 1

$$\underbrace{\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}}_{K} \begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \underbrace{\begin{bmatrix} -R_g & -1 \\ 1 & -1/R \end{bmatrix}}_{A_1} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_1} [v_g]$$

$$[i_g] = \underbrace{[1 \;\; 0]}_{C_1} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \underbrace{[0]}_{E_1} [v_g]$$

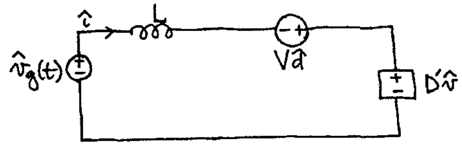
Top 2

$$\underbrace{\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}}_{K} \begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & -1/R \end{bmatrix}}_{A_2} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{B_2} [v_g]$$

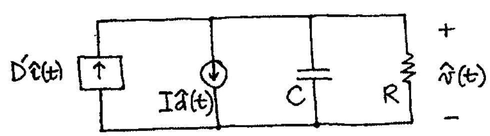
$$[i_g] = \underbrace{[0 \;\; 0]}_{C_2} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \underbrace{[0]}_{E_2} [v_g]$$

Problem 7.2

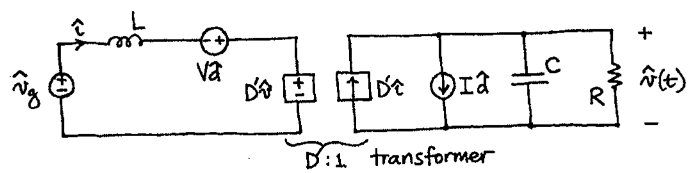
$$L\frac{d\hat{i}(t)}{dt} = -D'\hat{v}(t) + V\hat{d}(t) + \hat{v}_g(t)$$

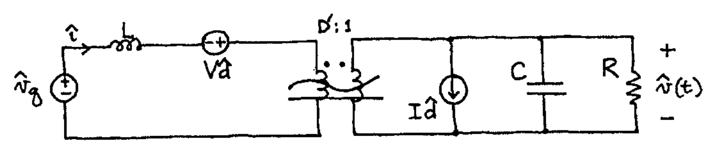


$$C\frac{d\hat{v}(t)}{dt} = D'\hat{i}(t) - I\hat{d}(t) - \frac{\hat{v}(t)}{R}$$



Combine circuits:





Averaged small-signal AC equivalent circuit model of the ideal CCM boost converter.

7.8 (cont)

Remember:
$(D+D'=1)$

* Averaged matrices

- $A = DA_1 + D'A_2 = \begin{bmatrix} -DR_g & -1 \\ 1 & -1/R \end{bmatrix}$; - $C = DC_1 + D'C_2 = [D \quad 0]$
- $B = DB_1 + D'B_2 = \begin{bmatrix} D \\ 0 \end{bmatrix}$; - $E = DE_1 + D'E_2 = [0]$
- $(A_1 - A_2) = \begin{bmatrix} -R_g & 0 \\ 0 & 0 \end{bmatrix}$
- $(B_1 - B_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $[C_1 - C_2] = [1 \quad 0]$

* AC small signal equations

a) $$K \cdot \frac{d}{dt}\mathbb{X}(t) = A\hat{\mathbb{X}} + B\hat{\mathbb{u}} + [(A_1-A_2)\mathbb{X} + (B_1-B_2)\mathbb{U}]\,\hat{d}(t)$$

$$\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}\frac{d}{dt}\begin{bmatrix} \hat{i}_L \\ \hat{v}_c \end{bmatrix} = \begin{bmatrix} -DR_g & -1 \\ 1 & -1/R \end{bmatrix}\begin{bmatrix} \hat{i}_L \\ \hat{v}_c \end{bmatrix} + \begin{bmatrix} D \\ 0 \end{bmatrix}[\hat{v}_g] + \begin{bmatrix} -R_gI_L + V_g \\ 0 \end{bmatrix}\hat{d}(t)$$

$$[\hat{i}_g] = [D \quad 0]\begin{bmatrix} \hat{i}_L \\ \hat{v} \end{bmatrix} + [I]\,\hat{d}(t) + [0]\,\hat{v}_g(t)$$

b) Equivalent circuit model

* $L\frac{d\hat{i}_L}{dt} = \hat{v}_L(t) = -DR_g\hat{i}_L - \hat{v}_c + D\hat{v}_g - R_gI_L\hat{d} + V_g\hat{d}$
- $C\frac{d\hat{v}_c}{dt} = \hat{i}_c = \hat{i}_L - \frac{\hat{v}}{R}$
- $\hat{i}_g = D\hat{i}_L + I\hat{d}$