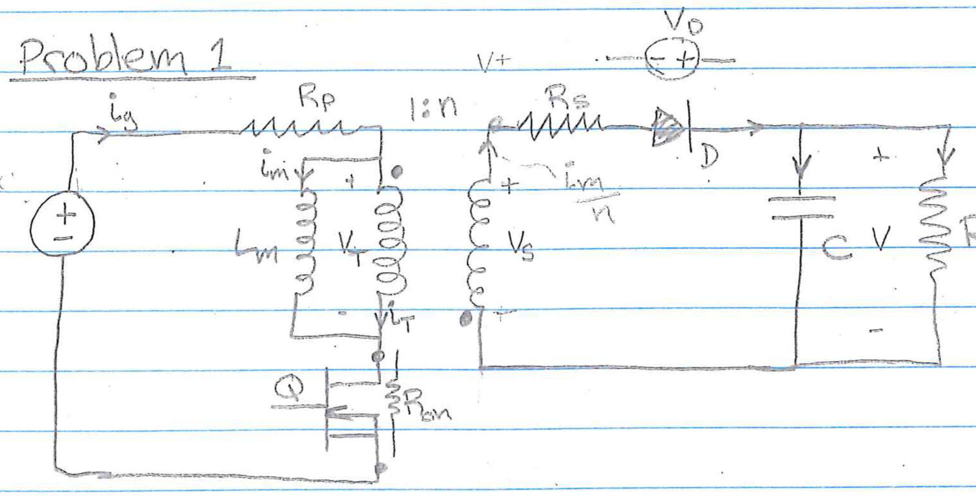
$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \rightarrow V_1 = \frac{N_1}{N_2} V_2 \qquad \frac{i_1}{i_2} = \frac{N_2}{N_1} \Rightarrow i_2 = \frac{N_1}{N_2} i_1 = \frac{i_m}{n}$$

Problem 1



a) Derive the steady-state equivalent circuit model.

Top 1 (Q on)

$V_T = V_g - i_g R_P - i_g R_{on}$

$i_c = -V/R$

$i_g = i_m$

Top 2 (Q off) $\qquad V_S = \left[V + V_D + \frac{i_m}{n} R_s\right]$

$V_T = (-V_S)/n = -\frac{1}{n}\left[V + V_D + \frac{i_m}{n} R_s\right]$

$i_c = i_m/n - V/R$

$i_g = 0$

• Volt-second and charge balance

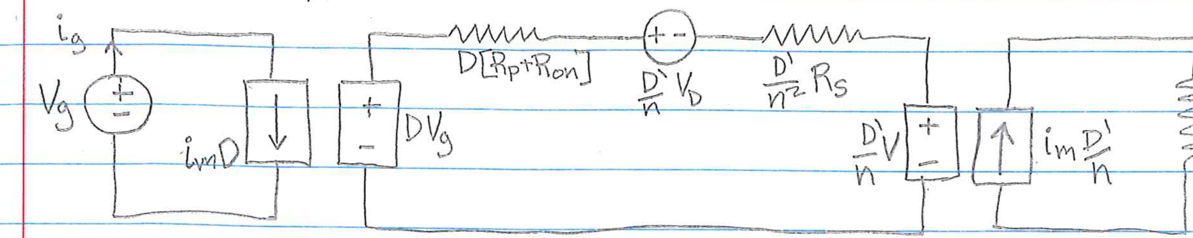
$$\langle V_T \rangle = 0 = D\left[V_g - I_g R_P - I_g R_{on}\right] + \frac{D'}{n}\left[V + V_D + \frac{i_m}{n} R_s\right]$$

$$0 = DV_g - D i_m\left[R_P + R_{on}\right] - \frac{D'}{n} V - \frac{D'}{n} V_D - D' \frac{i_m}{n^2} R_s$$

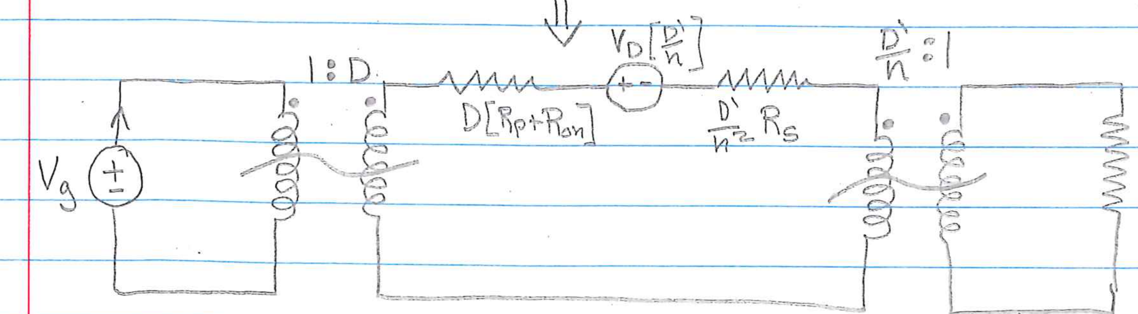
$$\langle i_c \rangle = 0 = -V/R + \frac{i_m}{n} D'$$

$$\langle i_g \rangle = i_m D$$

- Draw the equiv. circuit



$\Downarrow$

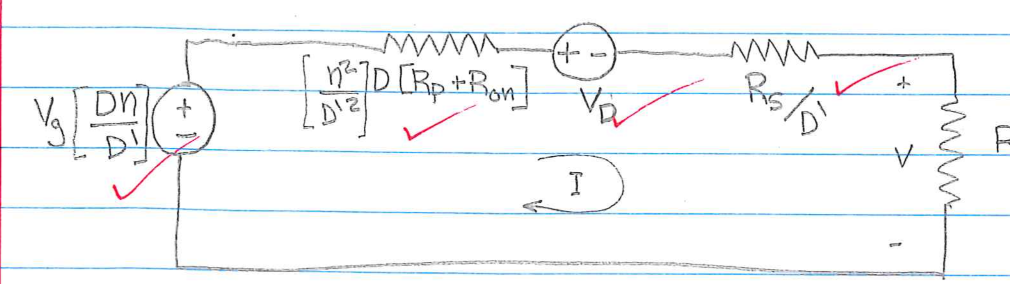


b) Derive the analytical expression for η

- Move all components to output side

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$V_2 = \frac{N_2}{N_1} V_1$



$\rightarrow P_{in}$

$\rightarrow P_{out} = VI$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V \cancel{I}}{V_g \frac{D}{D'} n}$$

10/10

- Express $V(V_g)$;
  - use voltage divider;

$$V = \left(V_g \frac{D}{D'} n - V_D\right)\left[\frac{R}{\frac{n^2 D}{D'^2}[R_p+R_{on}] + \frac{R_s}{D'} + R}\right]$$

$$\boxed{\eta = \frac{\cancel{V_g \frac{D}{D'} n}}{\cancel{V_g \frac{D}{D'} n}} \frac{\left(1 - \frac{V_D}{V_g}\frac{D'}{Dn}\right)}{\frac{\frac{n^2 D}{D'^2}[R_p+R_{on}]}{R} + \frac{R_s}{RD'} + 1}}$$

10/10

c) Derive the max blocking voltage across the mosfet.

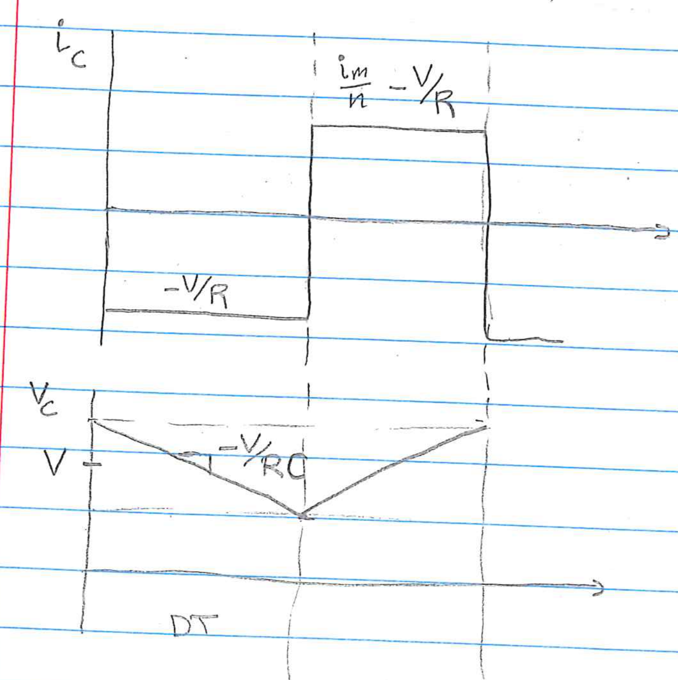
- MOSFET is off in Top 2

$$V_Q = V_g - V_T - 0$$

$$\boxed{V_Q^{max} = V_g + \frac{1}{n}\left[V + V_D + \frac{i_m}{n} R_s\right]}$$ ✓ 5/5

- note: this does not take into account voltage ripple

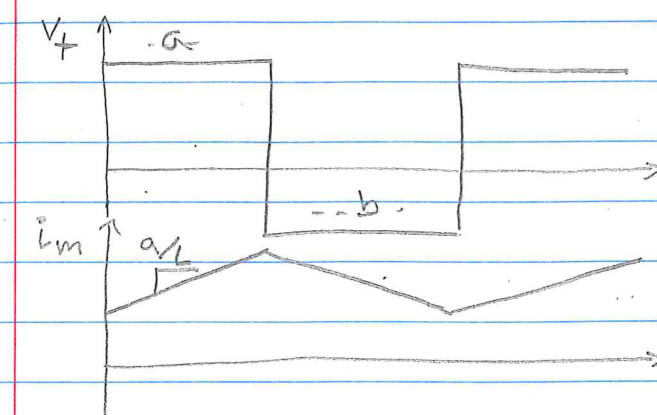
d) Output voltage ripple



$$2\Delta V = \frac{-V}{RC} DT \rightarrow \Delta V = -\frac{DT}{2RC} V$$

- need $V(I_m)$;
  - Use charge balance: $\frac{V}{R} = \frac{I_m}{n} D' \rightarrow V = \frac{i_m R}{n} D'$

- need $i_m(\Delta i_m)$;



$$2\Delta i_m = \frac{[V_g - I_m R_p - I_m R_{on}] DT}{L}$$

$$\frac{2\Delta i_m L}{V_g DT} = -I_m DT[R_p + R_{on}]$$

$$\therefore \Delta V = \left[\frac{-\cancel{DT}}{\cancel{2}\cancel{R}C}\right]\left[\frac{\cancel{R}}{n} D'\right]\left[\frac{-\cancel{2}\Delta i_m L}{V_g \cancel{DT^2}[R_p + R_{on}]}\right]$$

$$\boxed{\Delta V = \frac{D' L \Delta i_m}{C n V_g DT[R_p + R_{on}]}}$$

Bonus.

Problem 2

a) Determine DC operating characteristics

i) Find D;

Top 1

$V_L = V_g$

$i_c = -V/R$

$i_g = i_L$

Top 2

$V_L = V_g - V$

$i_c = i_L - V/R$

$i_g = i_L$

• Volt-second balance;

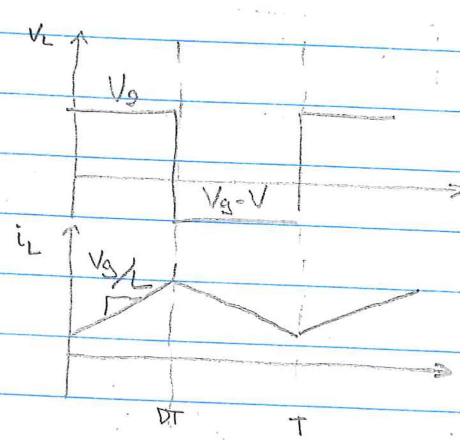
$$\langle V_L \rangle = 0 = V_g - VD' \rightarrow \frac{V}{V_g} = \frac{1}{D'}$$

$$\frac{1}{(1-D)} = \frac{100}{20} \Rightarrow 20 = 100 - 100D$$

$$\therefore \boxed{D = 0.8} ✓$$

ii) Prove CCM

• For CCM $I_L > \Delta i_L$



$$\langle i_c \rangle = 0 = -V/R + D'I_L$$

$$I_L = \frac{V}{RD'}$$

$$2\Delta i_L = \frac{V_g}{L} DT \rightarrow \Delta i_L = \frac{V_g DT}{2L}$$

• For CCM

$$\frac{V}{RD'} > \frac{V_g DT}{2L}$$

$$100 > 40 ✓ \quad \therefore \text{CCM}$$

5/5

b) Find $Z_{out}(s)$, $G_{vg}(s)$, $G_{vd}(s)$

• Use AC small signal equiv circuit.

$$\langle V_L \rangle = L\frac{d\langle i_L \rangle}{dt} = d\langle V_g \rangle + d'[\langle V_g \rangle - \langle V \rangle] = \langle V_g \rangle - d'\langle V \rangle$$

$$L\frac{d\hat{i}}{dt} = (V_g + \hat{v}_g) - (D' - \hat{d})(V + \hat{v})$$

$$\therefore L\frac{d\hat{i}}{dt} = \hat{v}_g - D'\hat{v} + \hat{d}V$$

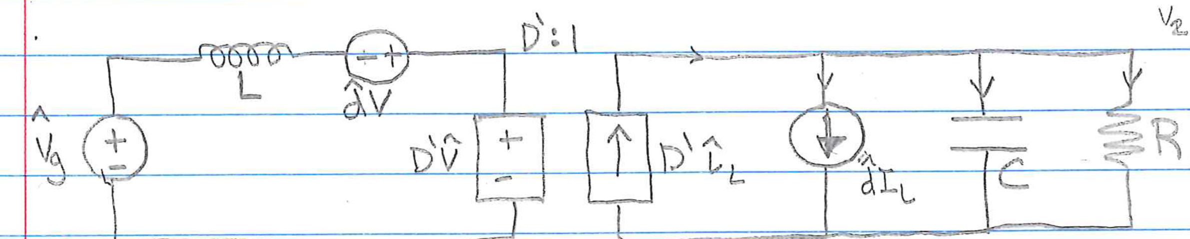
$$\langle i_c \rangle = C\frac{d\langle v \rangle}{dt} = -\frac{\langle v \rangle}{R} + d'\langle i_L \rangle$$

$$C\frac{d\hat{v}}{dt} = \frac{-1}{R}(V + \hat{v}) + (D' - \hat{d})(I_L + \hat{i}_L)$$

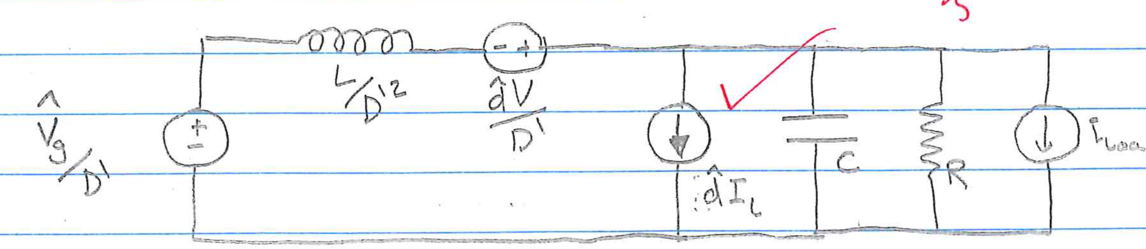
$$\therefore C\frac{d\hat{v}}{dt} = \frac{-\hat{v}}{R} + D'\hat{i}_L - \hat{d}I_L$$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$V_2 = \frac{N_2}{N_1} V_1$

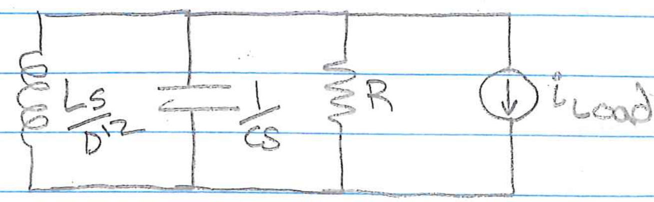


$\Downarrow$



5/5

i) $Z_{out}(s) = \left.\frac{\hat{v}}{i_{load}}\right|_{\hat{v}_g = 0,\ \hat{d} = 0}$



$$R /\!/ \tfrac{1}{Cs} = \frac{R/Cs}{R + 1/Cs} = \frac{R}{RCs+1}$$

$$Z_{out}(s) = sL /\!/ \tfrac{1}{Cs} /\!/ R \qquad L' = L/D'^2$$

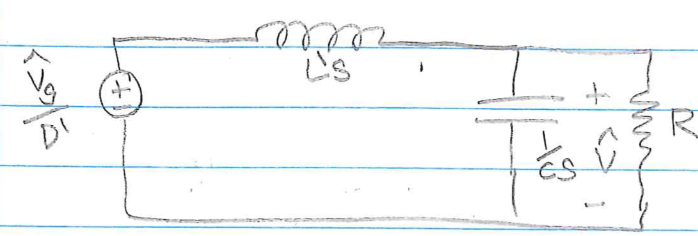
$$= \frac{L's\left[\frac{R}{RCs+1}\right]}{L's + \frac{R}{RCs+1}}$$

$$= \frac{L'sR}{L'RCs^2 + L's + R}$$

$$\therefore \boxed{Z_{out}(s) = \frac{Ls/D'^2}{\frac{LCs^2}{D'^2} + \frac{L}{RD'^2}s + 1}}$$

2/2

ii) $G_{vg}(s) = \frac{v}{v_g}\Big|_{\hat{d}=0,\ i_{load}=0}$



· Use voltage divider;

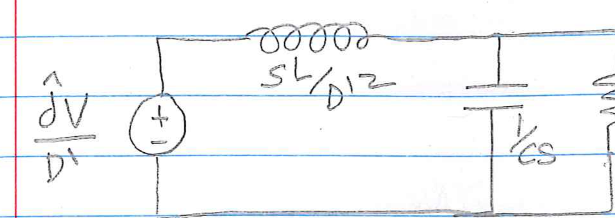
$$\hat{v} = \frac{\hat{v}_g}{D'}\left[\frac{R/(RCs+1)}{L's + R/(RCs+1)}\right]$$

$$= \frac{\hat{v}_g}{D'}\left[\frac{R}{L'RCs^2 + L's + R}\right]$$

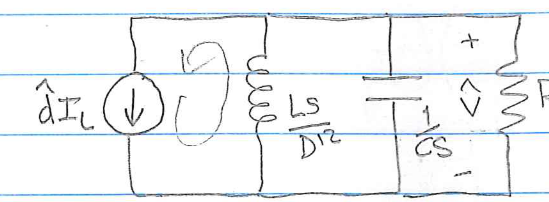
$$\therefore \boxed{G_{vg}(s) = \frac{1}{D'}\ \frac{1}{\frac{LCs^2}{D'^2} + \frac{Ls}{RD'^2} + 1}} ✓$$

iii) $G_{vd}(s) = \frac{\hat{v}}{\hat{d}}\Big|_{\hat{v}_g=0,\ i_{load}=0}$

· must use superposition



$$\frac{\hat{v}}{\hat{d}} = \frac{V}{D'}\left[\frac{1}{\frac{LCs^2}{D'^2} + \frac{Ls}{D'^2R} + 1}\right]$$



$$\frac{\hat{v}}{\hat{d}} = I_L \frac{-Ls/D'^2}{\frac{LCs^2}{D'^2} + \frac{L}{RD'^2}s + 1}$$

$$\therefore \boxed{G_{vd} = \frac{V/D' + I_L\, Ls/D'^2}{\frac{LCs^2}{D'^2} + \frac{Ls}{RD'^2} + 1}}$$

1/2

14/15

c) i) $V H(s) = V_{ref} \rightarrow H(s) = \frac{5}{100} = \frac{1}{20}$ ✓

· Select $R_1 = R_2$ = large

· Voltage divider on input

$V_o = 100 \frac{R_2}{R_1+R_2} \Rightarrow \frac{R_2}{R_1+R_2} = \frac{1}{20}$ $\therefore R_2 = 10k$, $R_1 + R_2 = 200k$ ✓

$\therefore \boxed{R_1 = 190k}$

R_3 is to balance opamp?

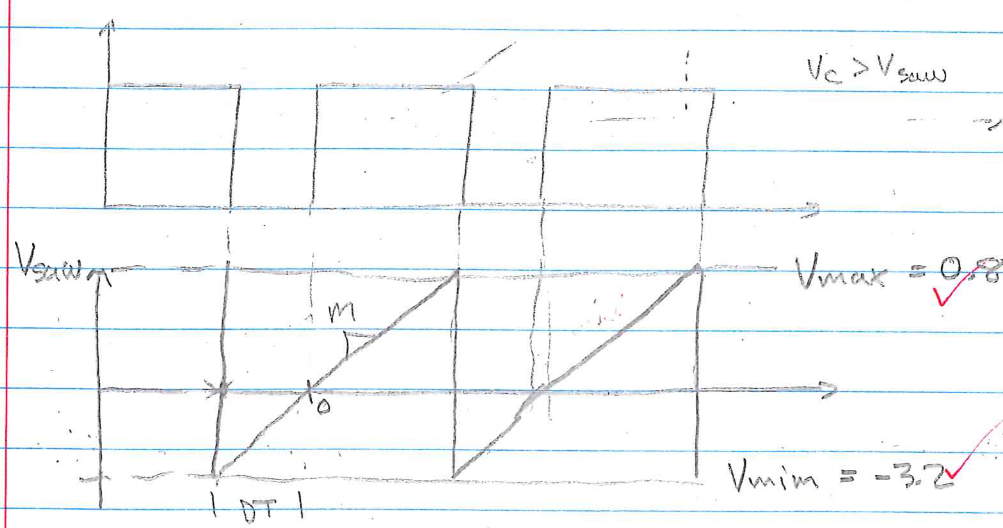
$R_3 = R_1 /\!/ R_2$

2/3

ii) • Draw the PWM waveform

• At DC steady-state desire $V_{comp} = 0$

$D = 0.8$



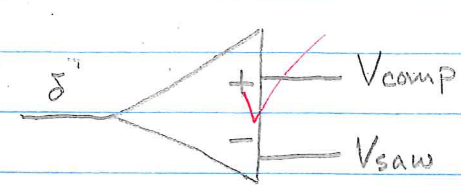
$V_{saw}(0) = V_{min}$ $\qquad$ $V_{saw}(DT) = 0 = V_{min} + MDT$

$V_{saw}(T) = V_{max}$ $\qquad$ $\Rightarrow V_{min} = -MDT$

$M = \frac{(4)}{T} = 8\times10^5$

N/V

$$\therefore \boxed{\begin{aligned} V_{min} &= -3.2\,V \\ V_{max} &= 0.8\,V \end{aligned}}$$



• Express the PWM TF;

$V_{saw}(dT) = V_{min} + mdT = V_c \rightarrow \frac{\hat{d}}{\hat{v}_c} = mT(D+\hat{d})$

$\frac{\hat{d}}{\hat{v}_c} = \frac{1}{mT}$

$$\therefore \boxed{G_{pwm}(s) = \frac{1}{4}}$$

iii) $T_v(s) = H(s)\,G_{pwm}(s)\,G_{vd}(s)$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{20}\right)\frac{\frac{V}{D'}\left(1 - s\frac{L}{D'V}\right)}{\left(\frac{LCs^2}{D'^2} + \frac{L}{RD'}s + 1\right)}$$

$$T_v(s) = G_{c0}\frac{(1 - s/\omega_z)}{\left(\frac{s^2}{\omega_p^2} + \frac{1}{Q\omega_p} + 1\right)}$$

$T_v(0) = G_{c0} = \left(\frac{1}{4}\right)\left(\frac{1}{20}\right)\left(\frac{100}{0.2}\right) \rightarrow \boxed{T_v(0) = 6.25 = 15.9\ dB}$

$\omega_z = \frac{D'^2 V}{L} = \frac{0.2(100)}{(1\times10^{-6})} \Rightarrow \boxed{\omega_z = 2\times10^7\ rad/s}$

4×10^6 rad/s

$\omega_p^2 = \omega_0^2 = \left(\frac{D'^2}{LC}\right) = \frac{0.2^2}{(1\times10^{-6})(400\times10^{-6})} \rightarrow \boxed{\omega_0 = 10{,}000\ rad/s}$

$\omega_p = \frac{D'}{\sqrt{LC}}$

20 = 26db

$Q\omega_p = \frac{RD'^2}{L} \rightarrow Q = \frac{RD'^2}{L}\left(\frac{\sqrt{LC}}{D'}\right) = D'R\sqrt{\frac{C}{L}} \rightarrow \boxed{Q = \cancel{100} = \cancel{40}\ dB}$

3.5/5 $\qquad$ −.5

iv) Select the plot that corresponds to $T_u(s)$;

$\boxed{T1}$ 2.5/ 10/10

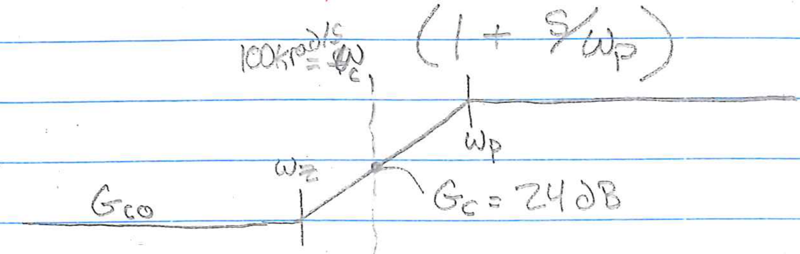
d)

ii) ~~To reject disturbance we need high (∞) steady-state gain and high gain at low freq. To achieve the a PI controller can be used.~~

i) Desire a cross-over frequency of 16kHz (100krad/s). To do this the $T_u(s)$ must be shifted up by ~24dB. The stability at this point is marginal ∴ Phase margin needs to be increased. 4/4

ii) A PD controller must be selected b/c phase margin must be added to ensure stability at the desired crossover frequency of 16kHz. With this controller there will still be some disturbances at low freq. To ~~remove~~ further limit these a PID should be used 4/4

iii) $G_c(s) = G_{co} \dfrac{(1 + s/\omega_z)}{(1 + s/\omega_p)}$

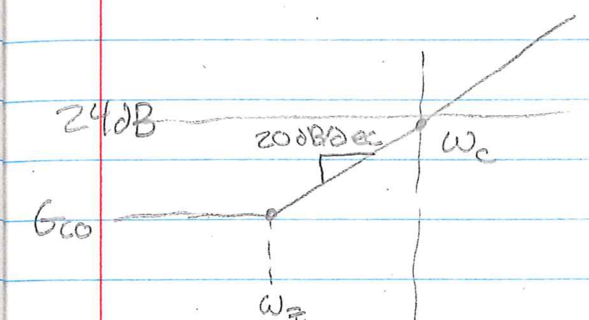


• Select the following values for $\omega_c = 100$k rad/s;

$\boxed{\omega_z = 10 \text{ krad/s}}$

$\boxed{\omega_p = 1 \text{ M rad/s}}$

dB = 20 log (Mag)



... must remember how to work w/ log plots...

$24 = G_{co} + \times 10^{(\frac{24}{20})}$

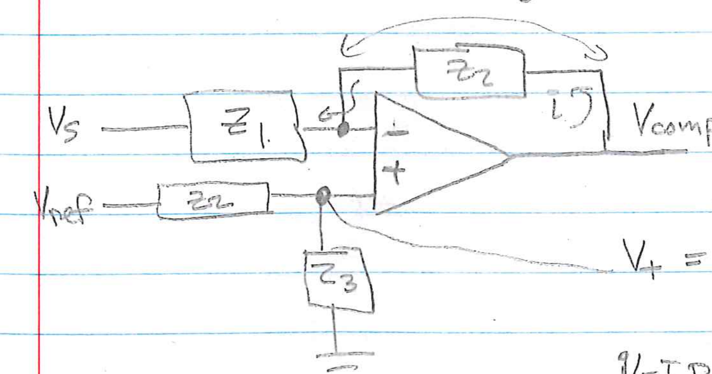
∴ $\boxed{G_{co} = 8.15 \text{ dB}}$ 3/4

$G_{co} = 4$dB

$G_{co} = 1.58$ good work though.

iV) • Calculate Resistor and Cap. Values

$G_c = G_{co} \dfrac{(1 + s/\omega_z)}{(1 + s/\omega_o)}$



$V_+ = \dfrac{Z_3}{Z_2 + Z_3} = V_-$ 1.5/3

$V = IR \Rightarrow I = \dfrac{V_s - V_-}{Z_1}$

$V_{comp} = \dfrac{V_- - V_{comp}}{Z_2}$ ... times up

12.5/15

$i_L(DT) = i_L(0) + m_1 dT = i_c$

$\hat{i}_c = 0 = \hat{i}_L(0) + m_1 T\hat{d} \rightarrow \hat{d} = -\frac{\hat{i}_L(0)}{m_1 T}$

$\hat{i}_L(T) = \hat{i}_L(0) + \cancel{T}\left(\frac{-\hat{i}_{L0}}{m_1 \cancel{T}}\right)(m_1 - m_2)$

$\hat{i}_L(0)\ \cancel{\hat{i}_L(0)} + -\cancel{\hat{i}_{L0}} + \frac{m_2}{m_1}\hat{i}_{L0}$

$\downarrow$

$\hat{i}_L(T) = \hat{i}_L(0)\left[\frac{m_2}{m_1}\right]$

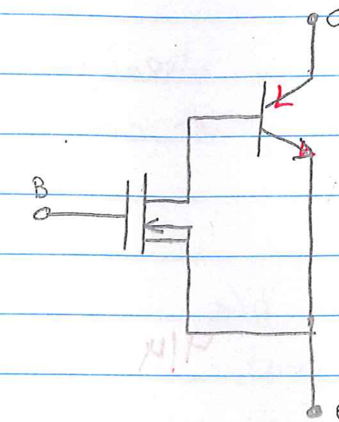
$\hat{i}_L(T) = \hat{i}_L(0)\left[\frac{D}{D'}\right]$

$i_L(nT) = \hat{i}_L(0)\left[\frac{D}{D'}\right]^n$ ∴ for stability $\frac{D}{D'} < 1$

$\frac{D}{D'} < 1 \Rightarrow D < 1-D \Rightarrow 2D < 1 \Rightarrow \boxed{D < 0.5}$

Problem 3

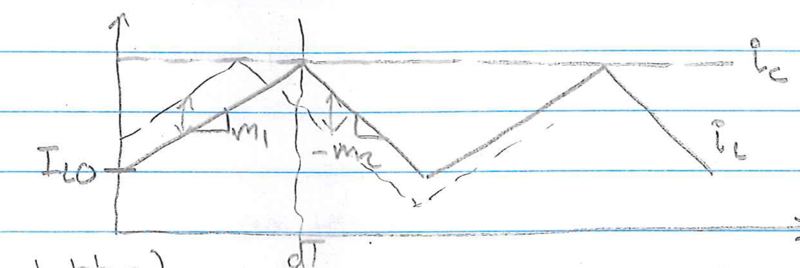
a)



- Advantage over MOSFET:
  - Higher blocking voltage
  - Higher efficiency b/c minority carrier.
- Advantage over BJT:
  - Higher efficiency B/C no Base current

3.5/4

c)



steady-state

$I(DT) = I_{L0} + m_1 DT$

$i_c = I_{L0} + m_1 DT$

$I(T) = I(DT) - m_2 D'T$

$I_{L0} = \cancel{I_{L0}} + m_1 DT - m_2$

$D\cancel{T} m_1 = m_2 D'\cancel{T}$

$\frac{m_1}{m_2} = \frac{D'}{D}$

- perturbed

$i_L(dT) = i_L(0) + m_1 dT = i_c$

$i_c = i_L(dT) = I_L(0) + \hat{i}_L(0) + m_1 T(D + \hat{d})$

$= I_{L0} + \hat{i}_L(0) + m_1 TD + m_1 T\hat{d}$

$i_L(T) = i_L(dT) - m_2 d'T$

$i_L(T) = I_{L0} + \hat{i}_L(0) + m_1 T(D + \hat{d}) - m_2 T(D' - \hat{d})$

$\cancel{I_{L0}} + \hat{i}_L(T) = \cancel{I_{L0}} + \hat{i}_L(0) + \cancel{m_1 TD} + m_1 T\hat{d} - \underbrace{\cancel{m_2 TD'}}_{m_1 D/D'} - m_2 T\hat{d}$

$i_L(T) = i_{L0}\left(\frac{m_1}{m_2}\right)$

$\hat{i}_L(T) = \hat{i}_{L0} + m_1 T\hat{d} - m_2 T\hat{d}$

$= \hat{i}_{L0} + T\hat{d}(m_1 - m_2)$

- Eliminate $\hat{d}$; $\hat{i}_{L0} =$

4/4

d) • Simpler dynamics to control. $T_u(s)$ is lower order ✓

• Can take advantage ✓ of circuitry already likely present which monitors Transistor current.

4/4

g) High Efficiency is desired b/c

• Don't want to generate heat 4/4

• Don't want to waste power. Want the load to use / dissipate the power not the converter

f) Top 1

$V_T = V_g$

$i_c = i_s - V/R$

• ~~Assume i_m is neglible~~

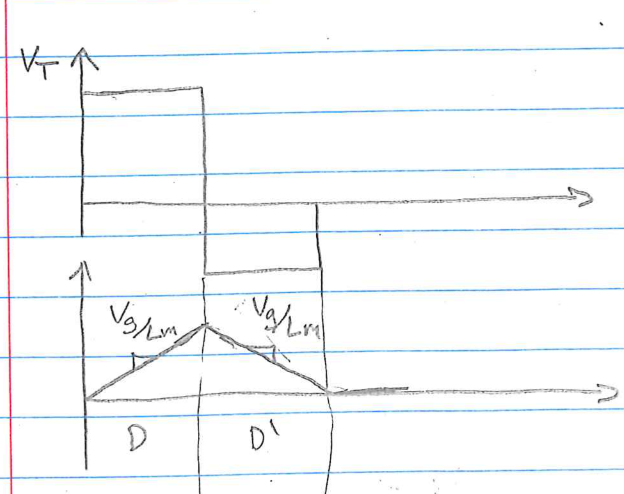
• Condition for stability is that the i_m must reset

Top 2

$V_T = -V_g$

Top 3 ($i_m < 0$)

$V_T = 0$



• For this setup $m_1 = -m_2$

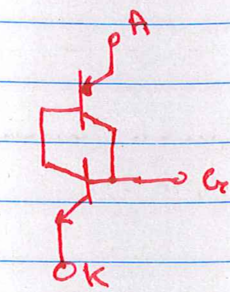
∴ for i_m to properly reset

$m_1 < m_2$

∴ $\boxed{D < 0.5}$

4/4

(B)



• Cannot be turned off b/c of a positive feedback loop between the two transistors.

(e) • Required for Safety. The transformer acts as an interface that restricts Powerflow.

• Put inside the device b/c it operates at a higher frequency, which allows us to use a much smaller transformer core → Less flux required (according to faraday's Law).

(h) The ripple in the inductor is greater than it's average current. A switch in the device prevents reverse current flow, so the circuit enters a topology where there is no current through the inductor (and thus, no voltage across it).

The tendancy of a circuit to enter this state is related to the size of the inductor. If we are okay with the gain that our converter produces in DCM we can use a smaller (i.e. Less inductive) inductor.

DCM operation