Probability Review and Distributions

Lecture 4

What will we learn ?

- Review of basic probability
- Conditional probability and Baye's theorem
- Series-Parallel and Non-series-parallel systems
- Standby redundancy

Why learn this stuff (again) ?

- Probability & statistics is at the heart of what we do in reliability
  - No such thing as a perfectly reliable system
  - Engineering design requires quantification of trade-offs and design choices
  - Failures are rare events (hopefully), so you need the tools of probability to reason about them
  - Modeling errors using statistical distributions is the de-jure model for evaluating systems

Basic definitions (Review)

- Experiment
  - Any process with an uncertain outcome
- Sample space
  - The set of outcomes of the experiment
- Trial
  - A single performance of the experiment
- Event
  - A subset of the sample space (i.e., a set of trials)

Probability axioms

- We assign to each sample s in the sample space, a probability value P such that:
  - Let P(A) be the probability measure associated with event A. Then, for any event A, P(A) >= 0
  - P(S) = 1
  - P (A U B) = P(A) + P(B), where A and B are *mutually exclusive events (*A Π B = Φ).

- *Note that the above formulation does not rely on the countability/finiteness of the sample space S*

Discrete Sample Spaces

- A discrete sample space is one in which the set of samples is either finite or countably infinite
  - Example: Number obtained by rolling a die
- Probability can be defined on discrete sample spaces as the fraction of favorable outcomes to the total number of outcomes. Requires:
  - Identifying the sample space (harder than it seems)
  - Identifying the set of favorable outcomes
  - Identify the events of interest

Conditional Probability

- Probability that event A occurs given that event B has occurred is given as P(A | B).

$P(A \mid B) = P(A \sqcap B) / P(B)$, if $P(B) =/= 0$

Intuitively, we are scaling our expectations of event A if we know that event B has occurred, based on the joint occurrence of the two events and the likelihood of B occurring alone.

Independent Events

- Two events are independent if and only if

$P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$

If you plug in the numbers in the formula for conditional probability, this boils down to:

$$P(A \Pi B) = P(A)P(B)$$

Baye's Rule

- If A and B are two events, then

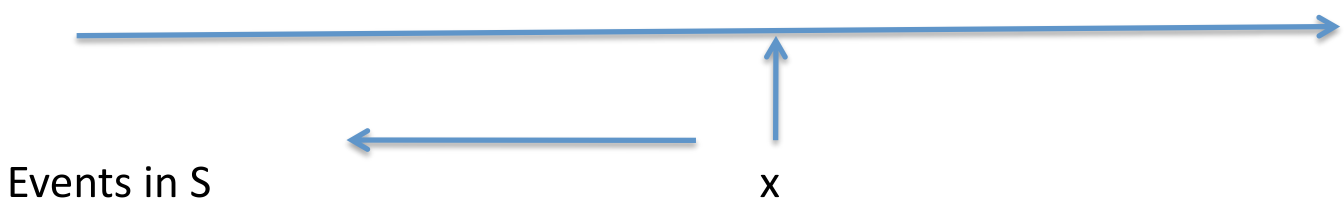
$$\mathbf{P(B \mid A) = P(\ B \prod A) \ / \ P(A) = P(A/B)\ P(B) \ / \ P(A)}$$

Where, $\mathbf{P(A) = P(A|B)P(B) + P(A|B')P(B')}$

Note: The above formula generalizes naturally to 'n' events – B_1, B_2, … B_n, where the B_is are mutually exclusive and collectively exhaustive

Random Variable

- A random variable on a probability space (S,P) is a function X: S → R that assigns a real value X(s) to each sample point s in S, such that for each real number x, the set {s | (X(s) <= x } is an event, that is a subset of S.



Distribution Function

- The Cumulative Distribution Function F of a random variable X is defined to be

  $F(x) = P(X <= x), \qquad -inf < x < inf$

It satisfies the following properties:

1. $0 <= F(x) <= 1, \; -inf < x < inf$
2. F(x) is a monotone non-decreasing function
3. $Lim_{x \to -inf} F(x) = 0$ and $Lim_{x \to inf} F(x) = 1$

Density Function

For a continuous random variable, X, the function f = dF(x) / dx, denots the probability density function (pdf). The CDF becomes:

F(x) = P (X <= x) = $\int_{-\infty}^{x} f(t)dt$

Properties of pdf:

1. f(x) >= 0, for all x,

$$\int_{-\infty}^{\infty} f(t)dt = 1$$

Summary

- Review of basic probability
  - Definitions
  - Conditional probability
  - Random variable
  - Baye's rule
  - Pdf
  - Cdf