HBTs: high-frequency attributes

LECTURE 17

- Recapitulation of equivalent circuits
- Two components of Emitter-Base capacitance
- Figures of merit
- \( f_T \): definition and derivation
- Design for high \( f_T \)
- \( f_{\text{max}} \): definition and derivation
- Design for high \( f_{\text{max}} \)
Sec. 9.4

DC Equivalent circuit

\[ J_c = -q n_0 B \left( e^{\frac{q V_{BE}}{kT}} - e^{\frac{q V_{BC}}{kT}} \right) \left( \frac{W_B}{D_c} + \frac{1}{v_R} \right)^{-1} \]

\[ I_{B,rec} = A q n_0 B \left[ (e^{\frac{V_{BE}}{V_{th}}} - 1) + \left( e^{\frac{V_{BC}}{V_{th}}} - 1 \right) \right] \left\{ \frac{1}{\frac{2\tau_e}{W_B} + \frac{1}{2v_R}} \right\} \]
Sec. 14.2

AC small-signal equivalent circuit

Small-signal base current

\[ i_3 = \frac{\partial I_3}{\partial v_{31}} v_{31} + \frac{\partial I_3}{\partial v_{21}} v_{21} \]

\[ = g_{33} v_{31} + g_{32} v_{21} \]

2-generator circuit

1-generator circuit

\[ (g_{23} - g_{32}) v_{31} \]

\[ g_{22} + g_{32} \]
Sec. 14.3

**Hybrid-π equivalent circuit**

HBT with parasitic R’s and C’s identified

Omit small-value components

Hybrid-π equivalent circuit under AC short-circuit at the output.

\[ g_\pi \equiv g_{33} \]
\[ g_m \equiv g_{23} \]
\[ C_\pi \equiv C_{EB} \]
\[ C_\mu \equiv C_{CB} \]
\[ \Delta V_{BE} = \Delta Q_{E,j} \]

- \( \Delta Q_{E,j} \) is the change in charge entering the device through the emitter and creating the new width of the depletion layer (narrowing it in this example),
- in response to a change in \( V_{BE} \) (with E & C at AC ground).
- It can be regarded as a parallel-plate cap.

What is the voltage dependence of this cap?
• $\Delta Q_{E,b}$ is the change in charge entering the device through the emitter and resting in the base (the black electrons),

• in response to a change in $V_{BE}$ (with E & C at AC ground).

• It’s not a parallel-plate cap, and we only count one carrier.
Emitter-base base-storage capacitance: evaluation

\[ Q_{E,b}(V_{BE}) = -q \frac{1}{2} W_B A \left[ n_{0p} \exp \left( \frac{V_{BE}}{V_{th}} \right) - n(W_B) \right] - q W_B A n(W_B) \]

Take \( \frac{\Delta Q_{E,b}}{\Delta V_{BE}} \rightarrow \frac{dQ_{E,b}}{dV_{BE}} \)

Hence \( C_{EB,b} \)

\[ n(0,V_{BE1}) = n_{0p} \exp \left( \frac{V_{BE1}}{V_{th}} \right) \]

\[ n(0,V_{BE,2}) = n_{0p} \exp \left( \frac{V_{BE2}}{V_{th}} \right) \]

What is the voltage dependence of \( C_{EB,b} \)?
Sec. 14.2

HF figures of merit

Represent transistor by its small-signal equivalent circuit

Consider frequency dependence of some current gain

Consider frequency dependence of some power gain

What are the associated figures of merit?
Sec. 14.4

\textbf{f}_T \textit{ from hybrid-pi equivalent circuit}

- \( f_T \) is measured under AC short-circuit conditions.

- We seek a solution for \(|ic/ib|^2\) that has a single-pole roll-off with frequency.
- Why?
  - Because we wish to extrapolate at -20 dB/decade to unity gain.
Extrapolated $f_T$

- **Assumption:**
  \[ i_b R_e \ll i_c (R_e + R_c) \]

- **Conditions:**
  \[ \omega^2 \ll \frac{g_m^2}{C^2 \mu} \]
  \[ \omega^2 \ll \frac{1}{C^2 \mu R_{ec}^2} \]
  \[ \omega^2 \gg \frac{g^2}{(C_\pi + C_\mu (1 + R_{ec} g_m))^2} \]

- **Current gain:**
  \[ \left| \frac{i_c}{i_b} \right|^2 = \frac{g_m^2}{\omega^2 (C_\pi + C_\mu (1 + g_m R_{ec}))^2} \]

- **Extrapolated $f_T$:**
  \[ 2\pi f_T = \]
Improving $f_T$

$$2\pi f_T = \frac{g_m}{C_\pi + C_\mu (1 + g_m R_{ec})}$$

- III-V for high $g_m$
- Highly doped sub-collector and supra-emitter to reduce $R_{ec}$
- Dual contacts to reduce $R_c$ and $R_B$
- Lateral shrinking to reduce C's

What is required to get $f_T > 200$ GHz?
Sec. 14.6

Circuit for derivation of \( f_{\text{max}} \)

Justify these omissions

Add source and load

Model with Thévenin equivalent circuit

\[
\begin{align*}
V_{\text{Th}} &= \frac{v_{\text{in}} j \omega C_\mu - g_m}{j \omega C_\mu (1 + g_m R_b) - \omega^2 C_\mu C_\pi R_b} \\
Z_{\text{Th}} &= \frac{R_b^{-1} + j \omega C_T}{j \omega C_\mu (g_m + R_b^{-1}) - \omega^2 C_\mu C_\pi}
\end{align*}
\]
Developing an expression for $f_{\max}$

Further conditions:

$$\omega^2 \ll \frac{g_m^2}{C^2 \mu} \quad \omega^2 \gg \frac{g_{\pi}^2}{C^2 \pi}$$

$$\omega^2 \ll \frac{g_m^2}{C^2 \pi} \quad \omega^2 \ll \frac{(g_m + R_b^{-1})^2}{C^2 \pi}$$

Conjugately match at the output

$$\Re(Z_{Th}) \approx \frac{C_T}{C_{\mu} g_m}$$

$$P_{out,\max} = \frac{|v_{Th}|^2}{4 \Re(Z_L)} \approx \frac{|v_{in}|^2}{4 R_b^2} \frac{g_m}{\omega^2 C_{\mu} C_T}$$

Conjugately match at the input

$$P_{in,\max} = \frac{|v_S|^2}{4 \Re(Z_{in})} = \frac{|v_{in}|^2}{R_b}$$
Sec. 14.6

**Improving $f_{\text{max}}$**

$$\text{MAG} = \frac{P_{\text{out, max}}}{P_{\text{in, max}}} = \frac{g_m}{4\omega^2 C_{\mu} C_T R_b}$$

$$f_{\text{max}} = \sqrt{\frac{f_{T,i}}{8\pi C_{\mu} R_b}}$$

- Pay even more attention to $R_b$ and $C_{\mu}$
Sec. 14.6.1

**Base-spreading resistance**

What is $\rho$?

$$ P_{\text{left}}(y) = I_B(y)^2 R $$

$$ P_{\text{left}} = \int_0^h \frac{I_B^2}{4} \left(1 - \frac{y}{h}\right)^2 \frac{\rho \, dy}{A} $$

What is $\rho$?

$$ P = \frac{I_B^2}{12} R_{B,QNB} \equiv I_B^2 R_{B,sp} $$

What is $R_{B,QNB}$?

Do you now see why HBTs have helped enable portable wireless products?

What is required to get $f_{\text{max}} > 300$ GHz?