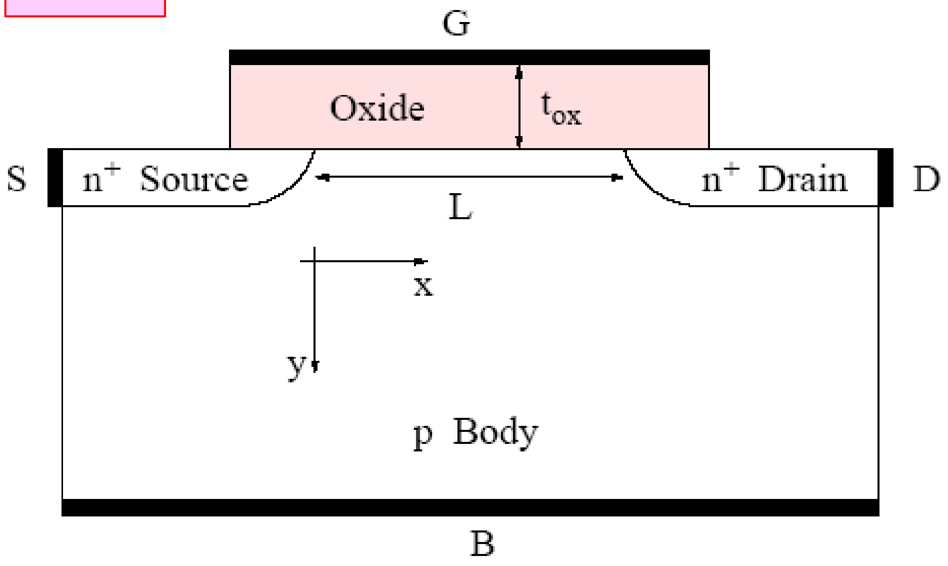
MOSFET models

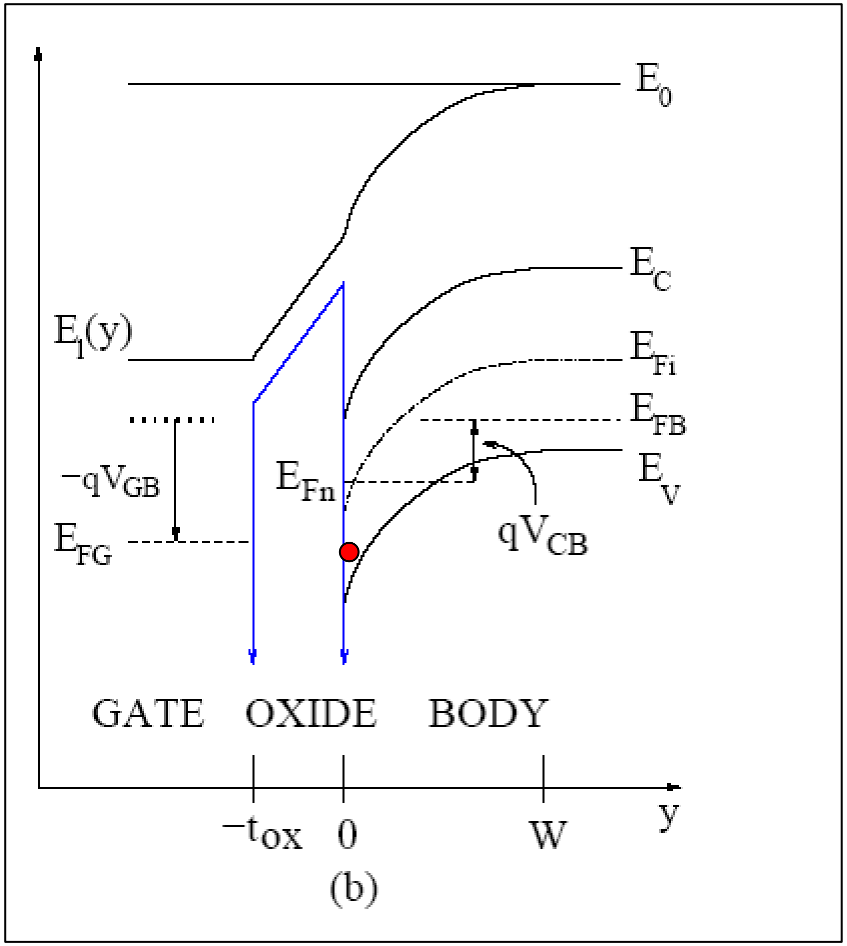
LECTURE 19

- Interim summary of MOSFET
- Surface Potential Model
- Drain I-V characteristic
- SPICE MODEL (LEVEL 1)
- Comparison of PSP and SPICE

Secs. 10.1, 10.2

Si MOSFET summary





- E_y due to V_{GB} causes depletion and inversion
- E_x due to V_{DS} causes reduction in inversion (effect is greatest near the drain)
- V_{DS} gives rise to a channel voltage $V_{CB}(x)$

Sec.
10.3.1

Estimating the drain current

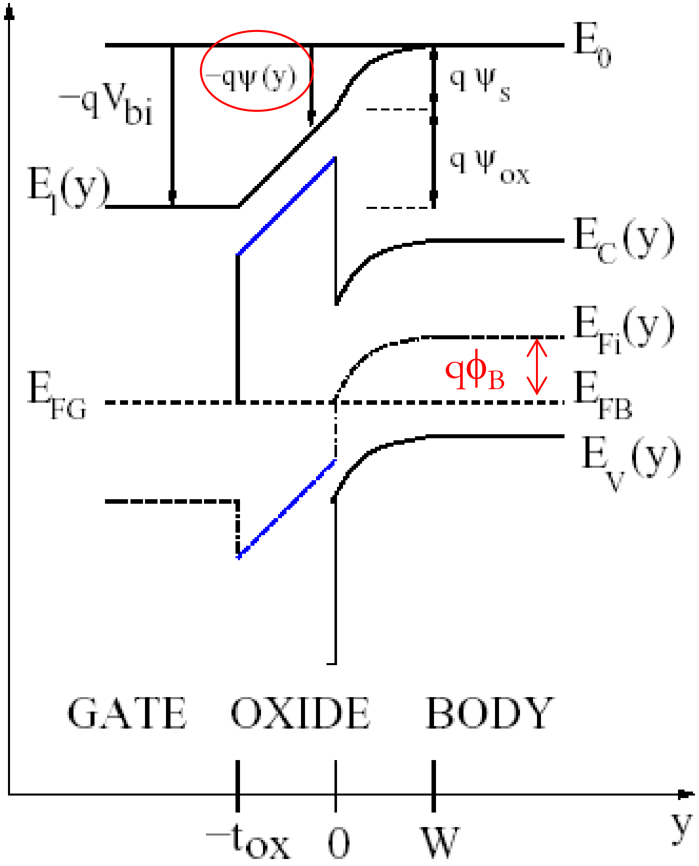
Start with the TOOLBOX

$$
\begin{aligned}
-\nabla^2\psi &= \frac{q}{\epsilon}[\cancel{p} - n + \cancel{N_D} - N_A] \\
J_e &= -qn\mu_e\nabla\psi + qD_e\nabla n \\
&\cancel{J_h = -qp\mu_h\nabla\psi - qD_h\nabla p} \\
\cancel{\frac{\partial n}{\partial t}} &= \frac{1}{q}\nabla\cdot J_e - \cancel{\frac{n-n_0}{\tau_e}} + \cancel{G_{op}} \\
&\cancel{\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla\cdot J_h - \frac{p-p_0}{\tau_h} + G_{op}}
\end{aligned}
$$

- Why can we ignore holes?
- Why can we ignore recombination?
- Why can we not ignore $\nabla\psi$?

Sec. 10.3.1

Surface potential and the PSP model



$$n(y) = n_i e^{(E_{FB} - E_{Fi}(y))/kT}$$

$$n(B) = n_i e^{-q\phi_B/kT} = \frac{n_i^2}{N_A}$$

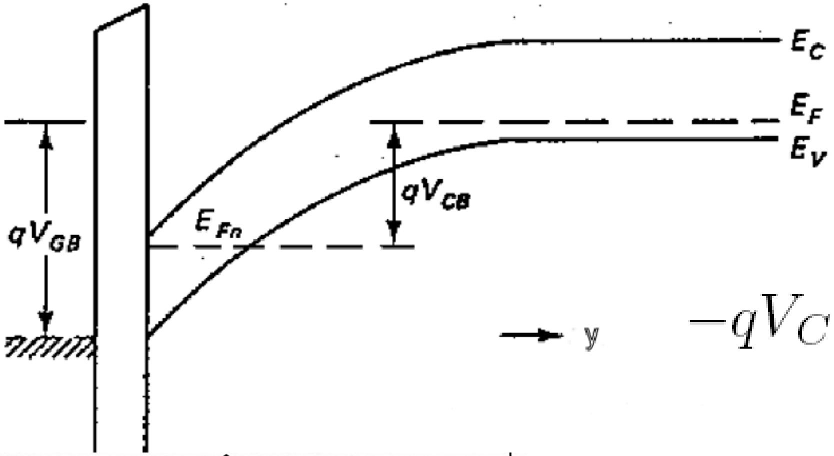
$$\phi_B = \frac{1}{q}(E_{Fi}(B) - E_{FB})$$

$$n(y) = N_A e^{q[\psi(y) - 2\phi_B]/kT}$$

$$Q_n = \int_0^W -qn(y)\, dy$$

Sec. 10.3.1

Introducing the channel potential $V_C(x)$

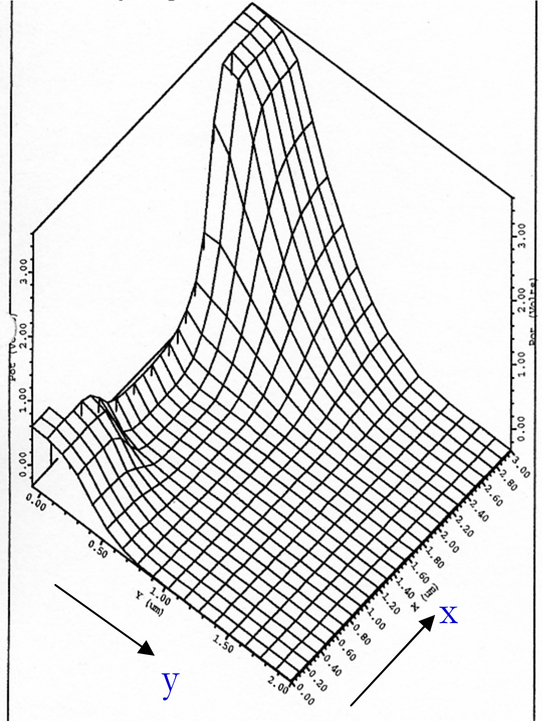


$$-qV_{CB}(x) = E_{Fn}(x) - E_{FB}$$

$$V_{CB}(0) = V_{SB} \quad \text{and} \quad V_{CB}(L) = V_{DB}$$

$$n(x,y) = N_A e^{q[\psi(x,y) - 2\phi_B - V_{CB(x)}]/kT}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{qN_A}{\epsilon_s}\left[1 + e^{q(\psi - 2\phi_B - V_{CB})/kT}\right]$$



$$\frac{\partial^2 \psi}{\partial x^2} \ll \frac{\partial^2 \psi}{\partial y^2}$$

Sec. 10.3.1

One expression for Q_s

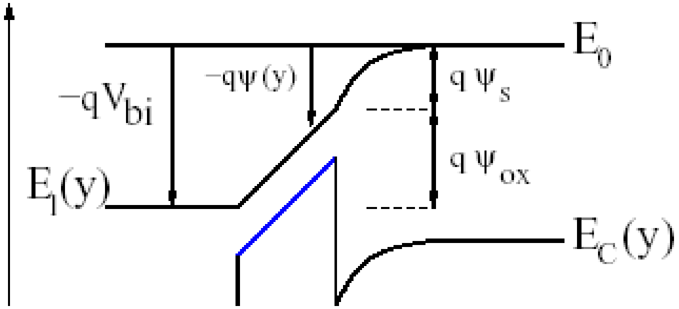
- Intermediate step in solving Poisson for ψ, gives the electric field E
- Gauss' Law relates E to Q_s, the charge per unit area in the semiconductor
- Q_s is f(x) because of $V_{CB}(x)$

$$Q_s = -\sqrt{2q\epsilon_s N_A}\sqrt{\psi_s + \frac{kT}{q}\left[e^{q\psi_s/kT} - 1\right] e^{-q(2\phi_B+V_{CB})/kT}}$$

semiconductor

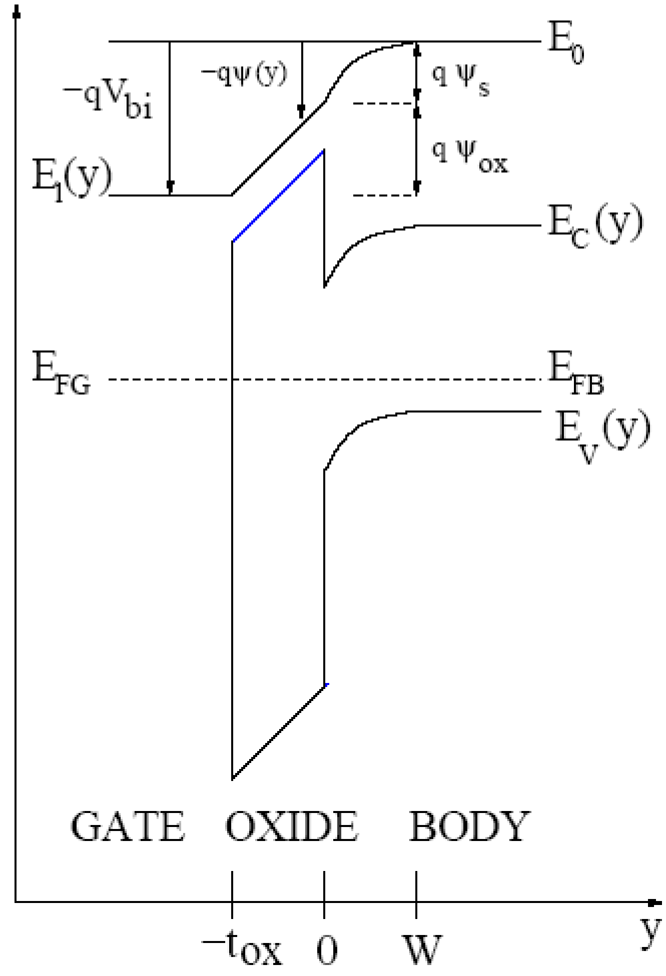
semiconductor, but also, surface
(see fragment below)

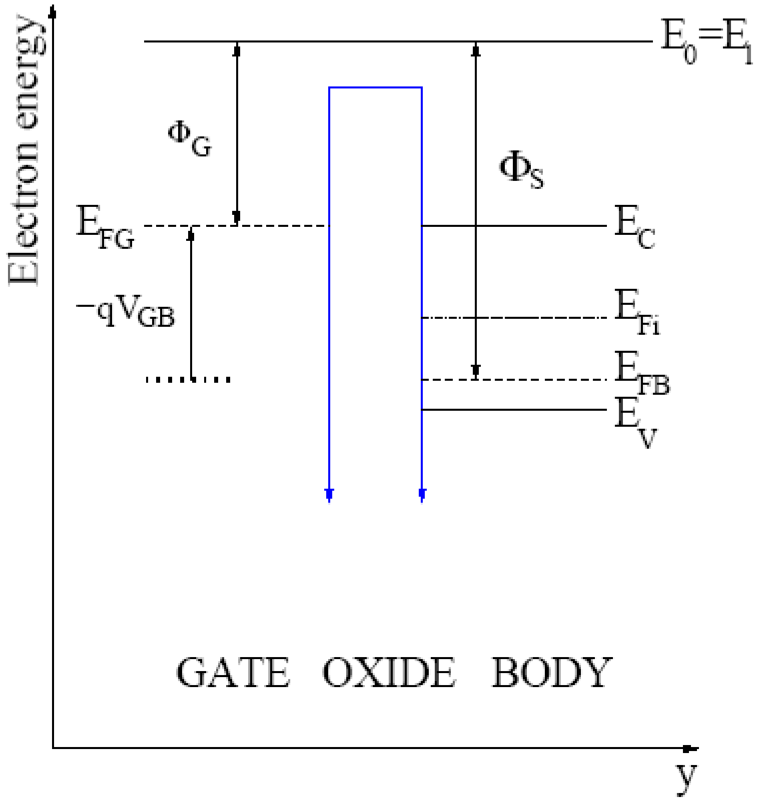
What are the units of Q_s ?



Sec. 10.2.1

$V_{GS} = V_{fb}$, the flatband voltage



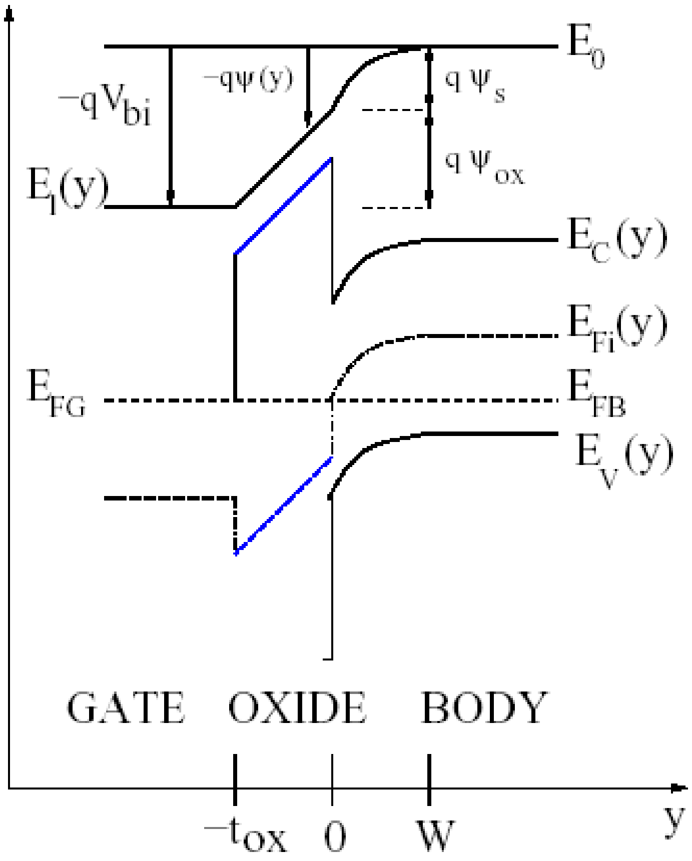


What is the relationship between V_{fb} and V_{bi} ?

Sec.
10.3.1

Another expression for Q_s

$$Q_s(x) = -Q_G(x) = -C_{ox}(V_{GB} - V_{fb} - \psi_s(x))$$



This comes from:

$V_{GB} - V_{fb} = \psi_{ox}(x) + \psi_s(x)$

and

$Q_G = C_{ox}\,\psi_{ox}$

Sec.
10.3.1

Implicit expression for ψ_s

Combine the two equations for Q_s :

$$Q_s = -\sqrt{2q\epsilon_s N_A}\sqrt{\psi_s + \frac{kT}{q}\left[e^{q\psi_s/kT} - 1\right] e^{-q(2\phi_B + V_{CB})/kT}}$$

$$Q_s(x) = -Q_G(x) = -C_{ox}(V_{GB} - V_{fb} - \psi_s(x))$$

The result is:

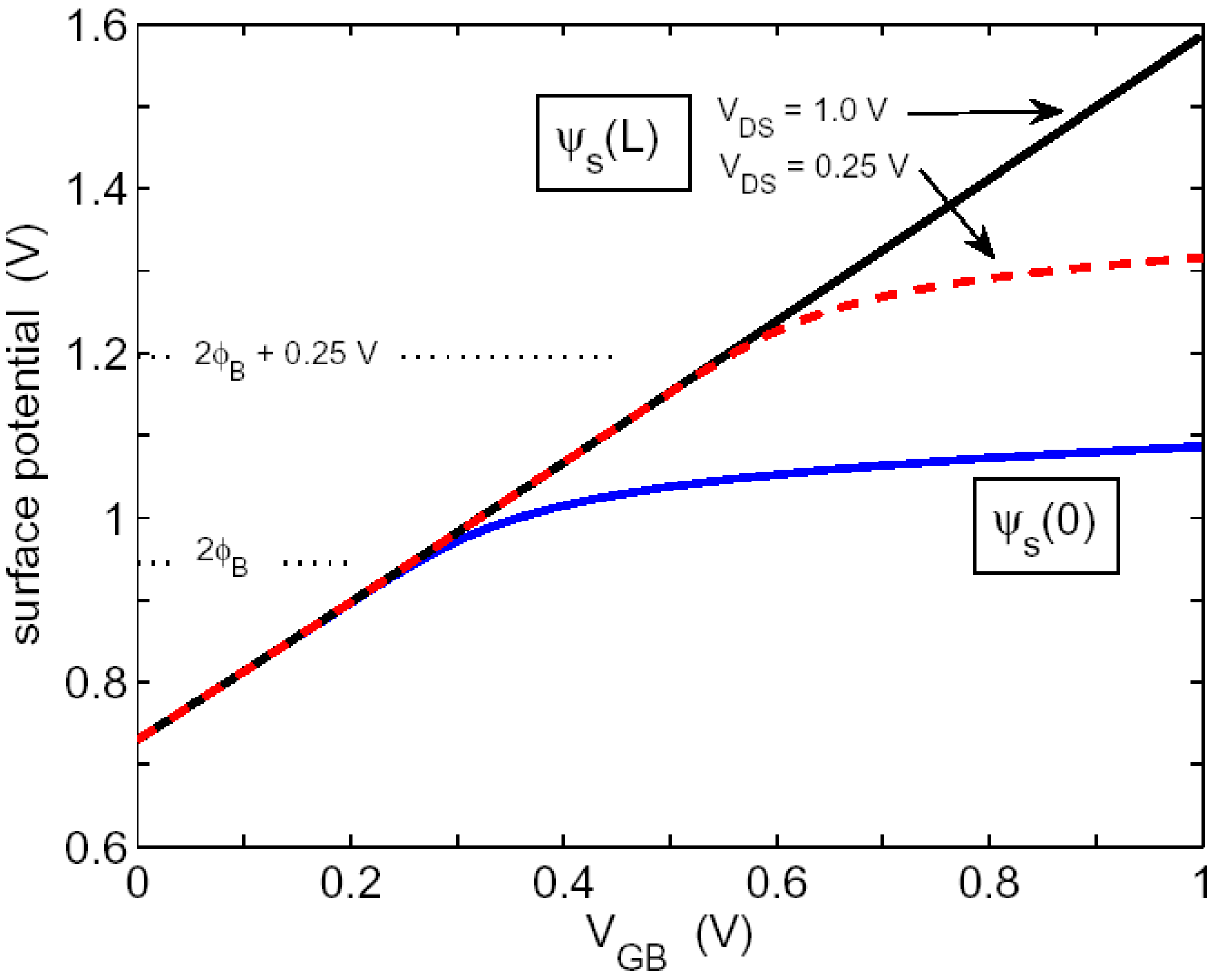
$$\psi_s = V_{GB} - V_{fb} - \gamma\sqrt{\psi_s + \frac{kT}{q}\left[e^{q\psi_s/kT} - 1\right] e^{-q(2\phi_B + V_{CB})/kT}}$$

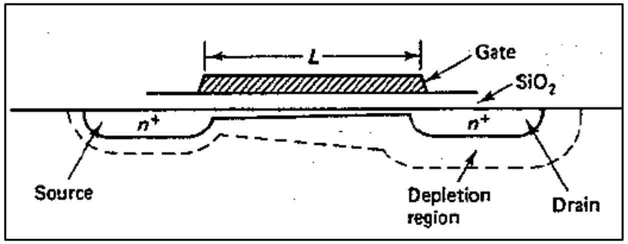
$\gamma = \sqrt{2q\epsilon_s N_A}/C_{ox}$ is called the Why?

See next slide for a plot of surface potential *vs.* gate bias

Sec. 10.3.1

Surface potential *vs.* gate bias





- Mark y=0 and y=L on the figure.
- At the source, initially the relation is linear. Why?
- At higher bias ψ_s increases less. Why?
- Same behaviour at the drain, but there is a dependence on V_{DS}. Why?
- Mark on the graph the regions of moderate- and strong- inversion. Note they are not the same for S and D, and they depend on V_{DS}.

Sec. 10.3.2

The Drain Current

$$Q_s(x) = Q_n(x) + Q_b(x)$$

$$Q_s(x) = -Q_G(x) = -C_{ox}(V_{GB} - V_{fb} - \psi_s(x))$$

$$Q_b(x) = -qN_AW(x) = -\sqrt{2\epsilon_s q N_A \psi_s(x)}$$

Charge Sheet Approximation & Depletion Approximation

$$J_e \quad = \quad -qn\mu_e \nabla\psi + qD_e\nabla n$$

DDE

What are the units of J?

$$I_e = ZQ_n\mu_e\frac{d\psi_s}{dx} - Z\frac{kT}{q}\mu_e\frac{dQ_n}{dx}$$

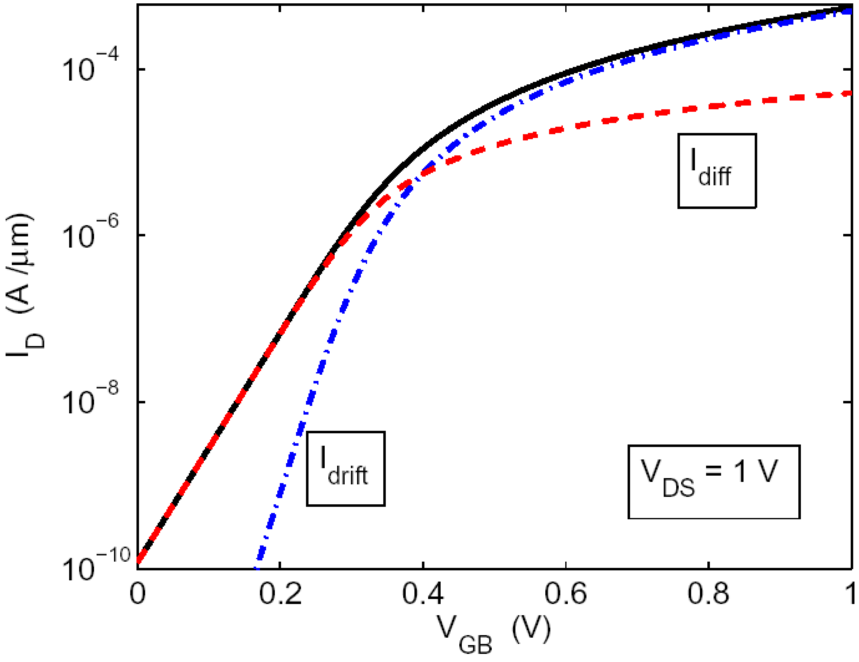
Non-degenerate Einstein relation

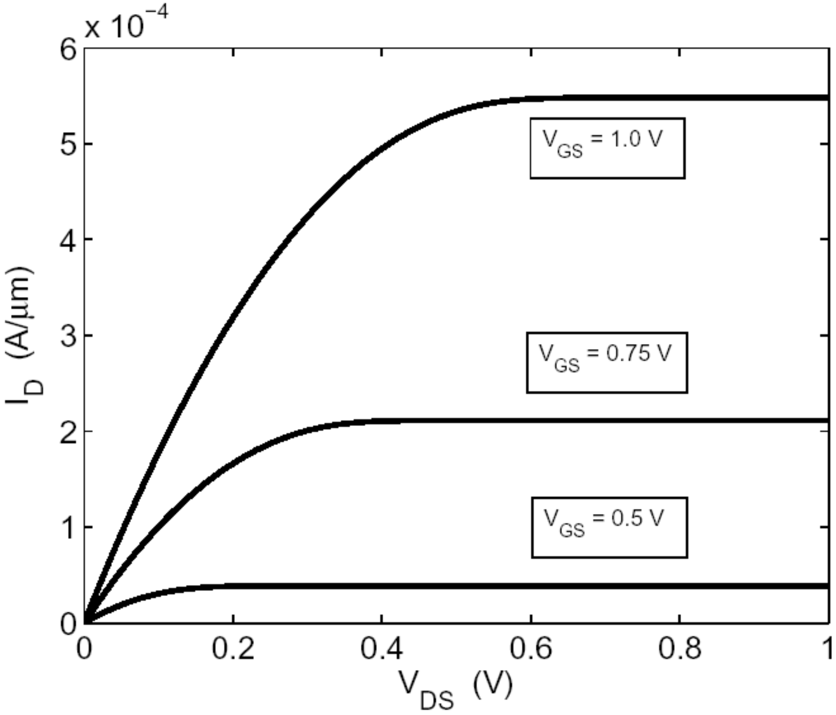
$$\int_0^L I_D\,dx \equiv \boxed{-}\int_0^L I_e\,dx = -Z\int_{\psi_s(0)}^{\psi_s(L)} \mu_e Q_n\,d\psi_s + Z\frac{kT}{q}\int_{Q_n(0)}^{Q_n(L)} \mu_e\,dQ_n$$

IEEE convention

Sec. 10.3.2

Gate- and Drain- I-V characteristics



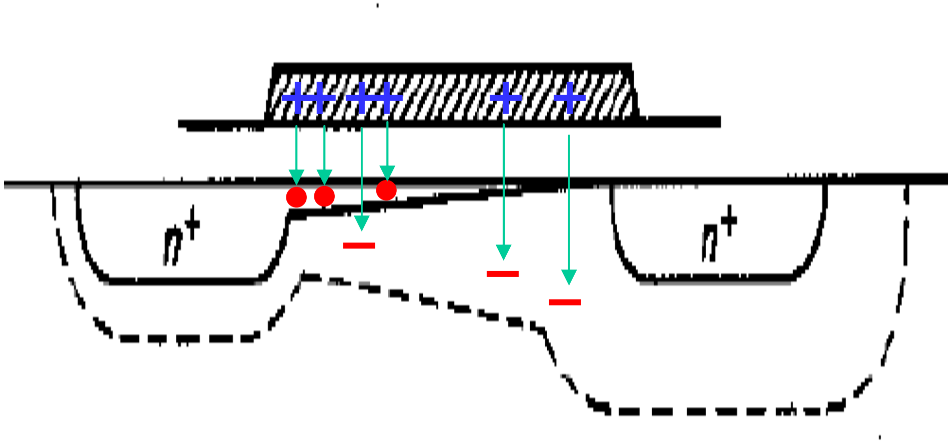


- Diffusion dominates in weak inversion
- Drift dominates in strong inversion

- Drain characteristic in strong inversion

Sec. 10.3.3

Saturation and loss of strong inversion at the drain



In Saturation:

- Q_n(L) becomes very small. (I=Qv, and v(L) is large).
- Field lines from gate terminate on acceptors in body.
- There is "no" charge at x=L that can be influenced by the drain.
- I_D saturates.
- Drain end of channel is NOT in strong inversion,
- but SPICE models assume that it is !

Sec. 10.4.1

Development of SPICE Level 1 model

From PSP:

$$I_{D,\mathrm{drift}} = \frac{Z}{L}\mu_{\mathrm{eff}}C_{ox}\left[(V_{GB}-V_{fb})(\psi_s(L)-\psi_s(0))-\frac{1}{2}(\psi_s(L)^2-\psi_s(0)^2)-\frac{2}{3}\gamma(\psi_s(L)^{3/2}-\psi_s(0)^{3/2})\right]$$

Make strong-inversion assumptions

$$\left\{\begin{aligned}\psi_s(0) &= 2\phi_B + V_{SB}\\ \psi_s(L) &= 2\phi_B + V_{DB}\\ \psi_s(L)-\psi_s(0) &= V_{DS}\\ \psi_s(L) &= 2\phi_B + V_{SB} + V_{DS}\\ & V_{DS} \ll 2\phi_B + V_{SB}\end{aligned}\right.$$

Use Binomial Expansion

$$\boxed{I_D = ZC_{ox}\left[V_{GS}-V_T-m\frac{V_{DS}}{2}\right]\cdot\mu_{\mathrm{eff}}\frac{V_{DS}}{L}}$$

Threshold voltage

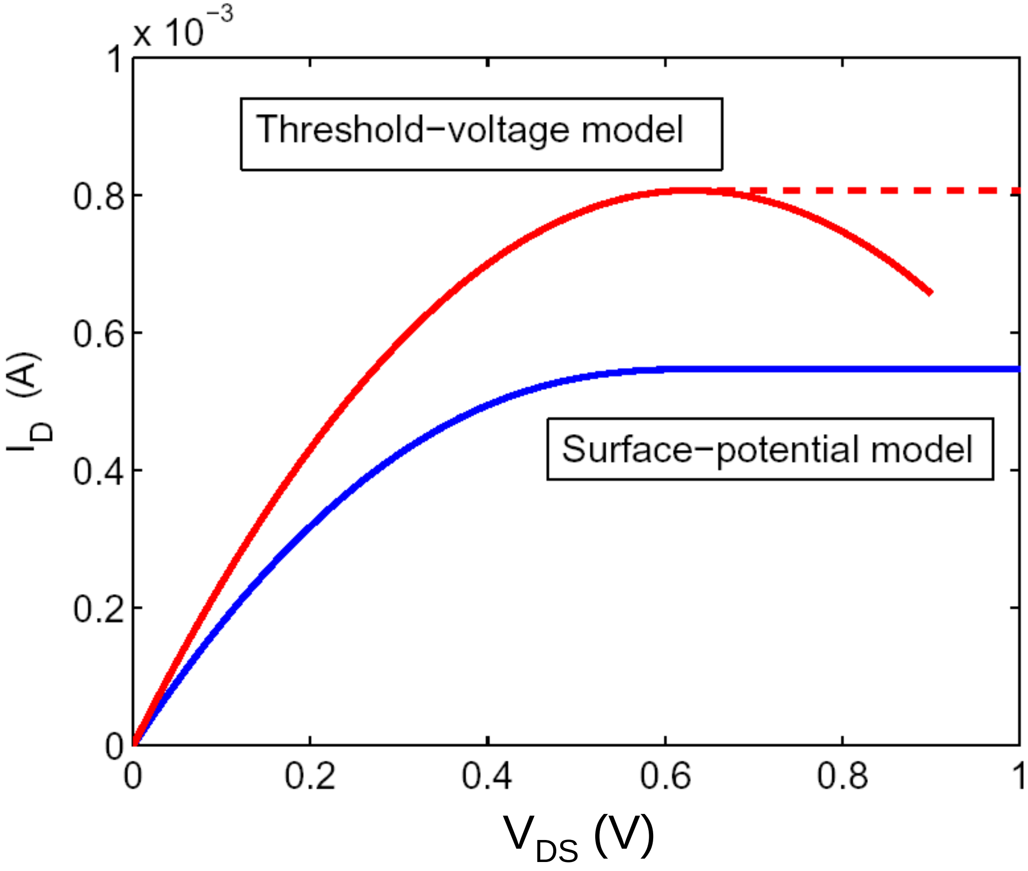
$$V_T = V_{fb} + 2\phi_B + \gamma\sqrt{2\phi_B + V_{SB}}$$

Body-effect coefficient

$$m = 1 + \frac{\gamma}{2\sqrt{2\phi_B + V_{SB}}}$$

Sec. 10.4.3

Comparison of PSP and SPICE



$$I_D = ZC_{ox}\left[V_{GS} - V_T - m\frac{V_{DS}}{2}\right] \cdot \mu_{\text{eff}}\frac{V_{DS}}{L}$$

Differentiate wrt V_{DS} to find V_{DSsat}

$$V_{DS} \equiv V_{DSsat} = \boxed{}$$

Substitute into I_D to get I_{Dsat}

$$I_{Dsat} = \frac{Z}{L}C_{ox}\mu_{\text{eff}}\boxed{}$$

- SPICE LEVEL 1 has the correct form for the drain characteristic, but is not very accurate.
- However, its use of V_T is very helpful in formulating a simple algorithm for MOSFET operation.