Carrier concentrations and current

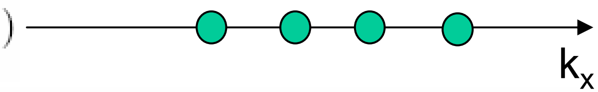
LECTURE 6

- Density of states
- Carrier concentrations at equilibrium
- Maxwellians and hemi-Maxwellians
- Current and energy gradients
- Diffusion current
- Drift current

Sec. 3.3

Density of states

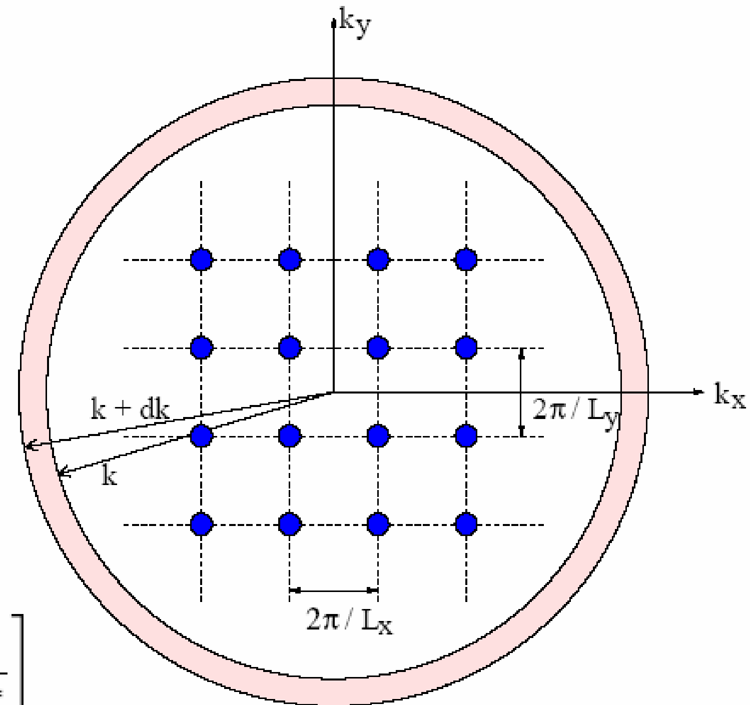
1. Recall spacing of k-states in 1-D: $k = \frac{2\pi n}{Na}\,, \quad (n = 0, \pm 1, \pm 2, \pm 3, \cdots)$



How much k-space does a state occupy in 1-D?

How much k-space does a state occupy in 3-D?

How many states in the k-sphere?



2. Total volumetric density of states is the same whether you integrate over E or |k|

$$\int_0^\infty g(E - E_C)\, d(E - E_C) = \int_0^\infty g(k)\, dk$$

3. Transform E to k.

$$E - E_C = \frac{\hbar^2}{2}\left[\frac{k_x^2}{m_x^*} + \frac{k_y^2}{m_y^*} + \frac{k_z^2}{m_z^*}\right]$$

4. Allow for different m*'s

$$\equiv \frac{\hbar^2}{2}\frac{k^2}{m^*_{e,\mathrm{DOS}}}\,,$$

6. Get g(E)

$$g_C(E - E_C) = \frac{8\pi\sqrt{2}}{h^3}(m^*_{e,\mathrm{DOS}})^{3/2}(E - E_C)^{1/2} \quad \text{for } E \geq E_C$$

What are the units?

Sec. 4.3

Equilibrium carrier concentrations

$$n_0 = \int_{E_C}^{\text{top of band}} g_C(E) f_{FD}(E)\, dE$$
$$p_0 = \int_{\text{bottom of band}}^{E_V} g_V(E) \boxed{}\, dE$$

These are formidable integrals as they involve E in square roots and in exponentials.

To simplify for electrons, for example, set the top of the band at [ ]

Why is it reasonable to do this?

The result is $n_0 = N_C F_{1/2}(a_F)$

where $N_C = 2\left(\frac{2\pi m^*_{e,\text{DOS}} k_B T}{h^2}\right)^{3/2}$ is the [ ] in the CB

and $\mathcal{F}_{1/2}(a_F)$ is a [ ] integral

and $a_F = (E_F - E_C)/k_B T.$ depends on [ ] and type.

Sec. 4.3

Maxwell-Boltzmann statistics

A very convenient approximation to $\mathcal{F}_{1/2}(a_F)$ arises if $a_F <$

$$\mathcal{F}_{1/2}(a_F) \to \exp(a_F)\,,$$

which then enables (4.10) to be written concisely as

$$n_0 = N_C \exp\left(\frac{E_F - E_C}{k_B T}\right)\,.$$

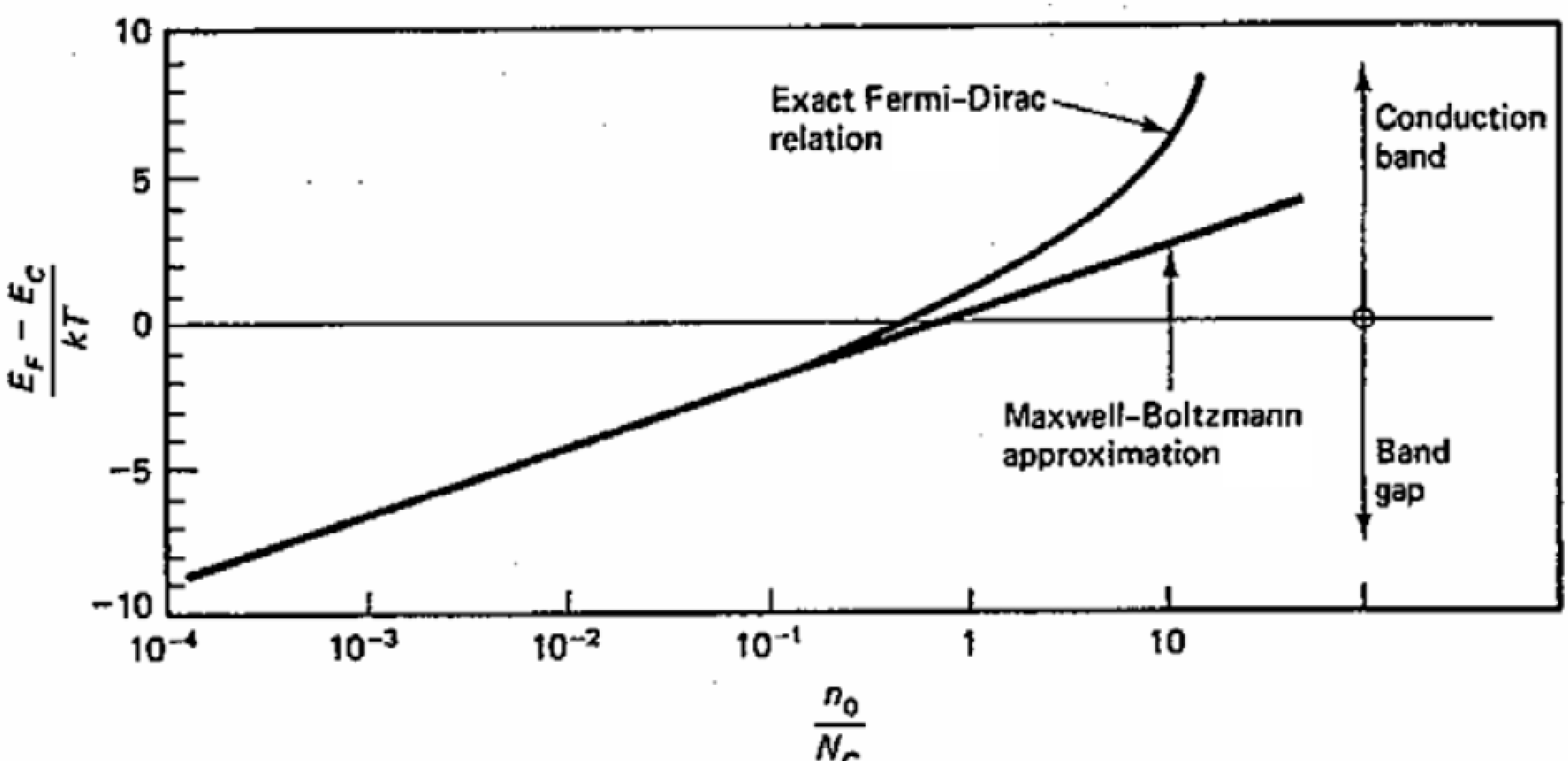
This equation could have been derived directly using the Maxwell-Boltzmann distribution function.

$$f_{MB}(E) =$$

Physically, what is the difference between FD and MB statistics?

Sec. 4.3

Fermi-Dirac *vs.* Maxwell-Boltzmann



- M-B works well for electrons if $(E_C-E_F) >$

- i.e., if $n_0 < 0.4\ N_C$

- We will use M-B for most of this course.

- N_C for Si at 300K is $/cm^3$

Ludwig Boltzmann

Sec. 4.5

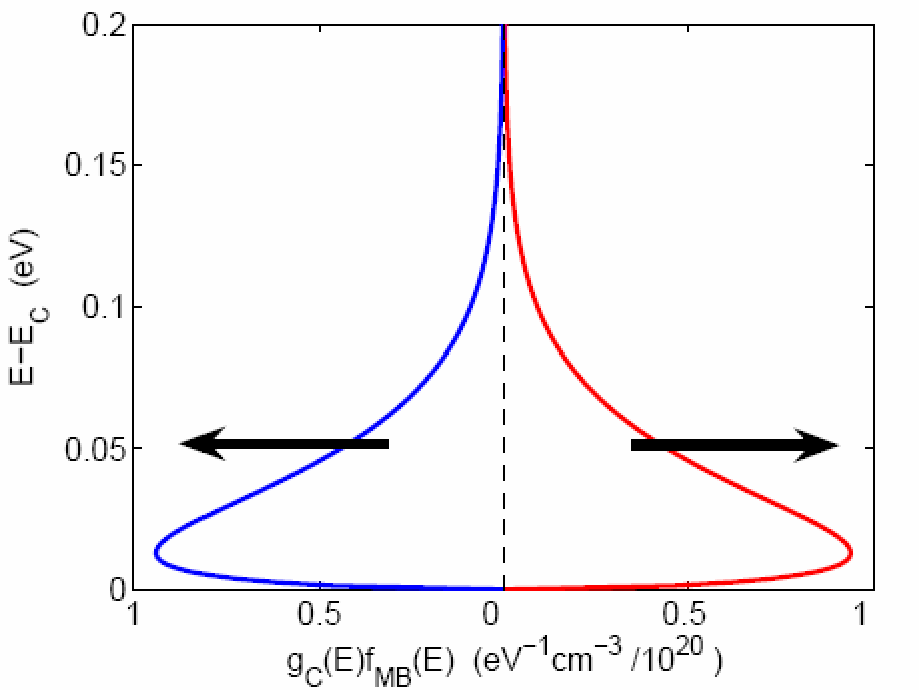
The hemi-Maxwellian

Full Maxwellian distribution
$$f_{MB}(E) = e^{-\frac{E-E_F}{kT}} \equiv e^{-\frac{E-E_C+E_C-E_F}{kT}} = \square\, e^{-\frac{E-E_C}{kT}}.$$

Electron concentration
$$n_0(E) = g_C(E) f_0(E)$$

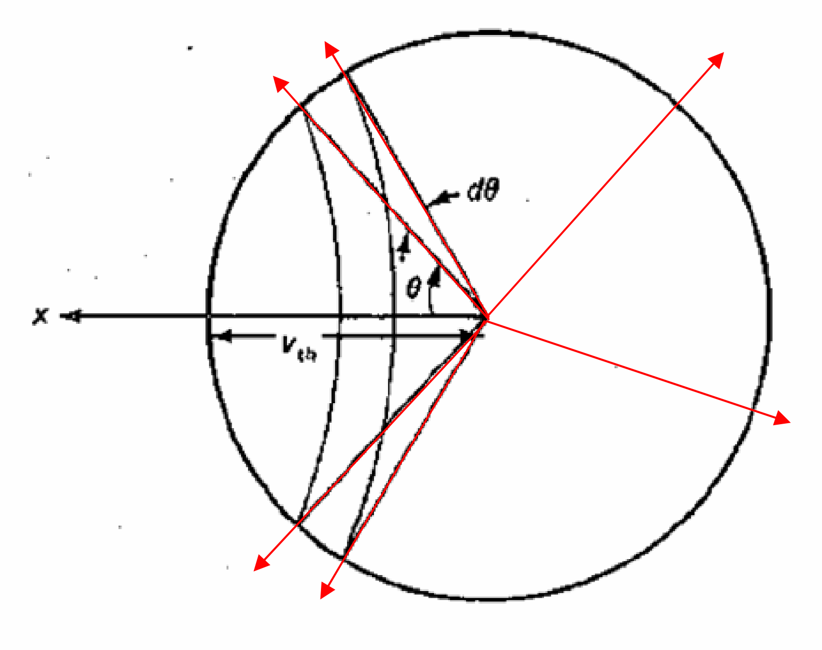
Counter-propagating hemi-M's for n_0=1E19/cm^3

What is the current?



Sec. 4.5

Mean thermal speed and mean unidirectional thermal velocity



Mean thermal speed

$$v_{th} = \frac{\int_0^\infty v n_0(v)\, dv}{\int_0^\infty n_0(v)\, dv}$$

Why is $v_{th} \neq 0$?

The result for the mean unidirectional velocity (for MB stats) is $\frac{1}{4}$ [ ]

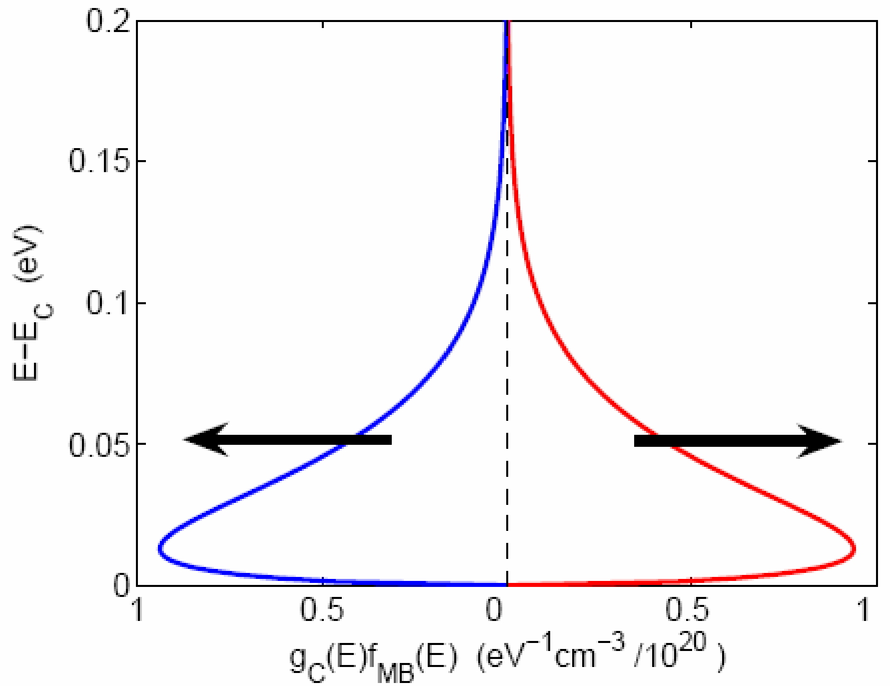
Why is $\langle v_x \rangle \neq 0$?

What is it for a hemi-Maxwellian?

What is m^*_{th} ?

Sec. 4.5.2

Current due to a hemi-Maxwellian distribution



$< v_x > \equiv 2v_R =$ ☐ cm/s for Si

$$\vec{J}_{e,\rightarrow} = -q\frac{n_0}{2} \square = \square$$ A/cm² for n_0=1E19 /cm³

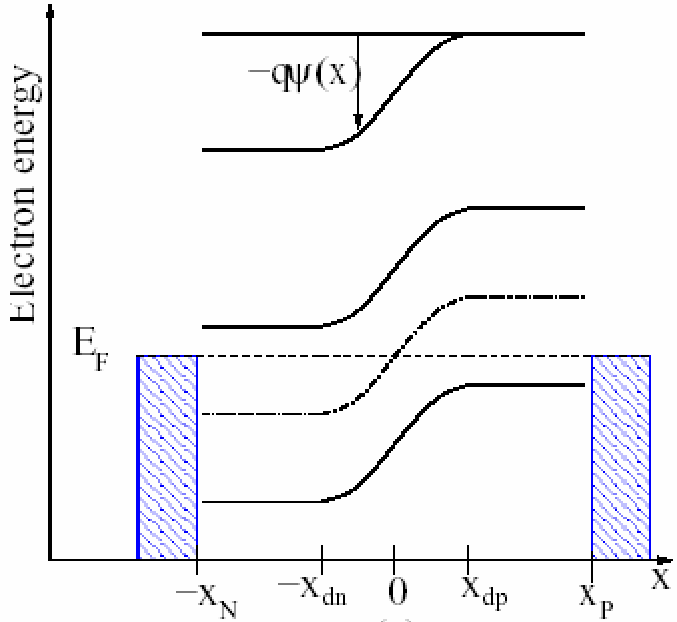
$$\vec{J}_{e,\text{total}} = ??$$

Current cancellation at equilibrium in np-junction

Sec. 6.3

Note the electron reservoirs (contacts).

Label the lines (ignore the stippled one).

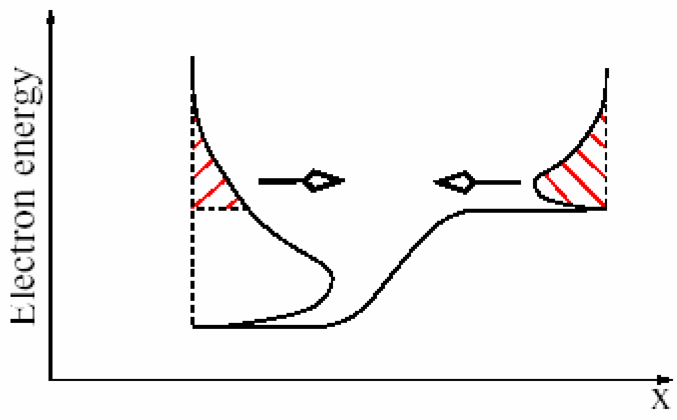


(a)

Why are the hemi-Maxwellians different sizes?

Why is the larger one only partly shaded?

Identify the "drift" and "diffusion" currents



(c)

Sec. 5.1

Current and energy

Current, the net flow of charge, is due to gradients in energy density.

Gradient in PE density $-qn\nabla\psi$

Gradient in KE density $\nabla(nu) \equiv n\nabla u + u\nabla n$

What are these currents called?

Sec. 5.5.2

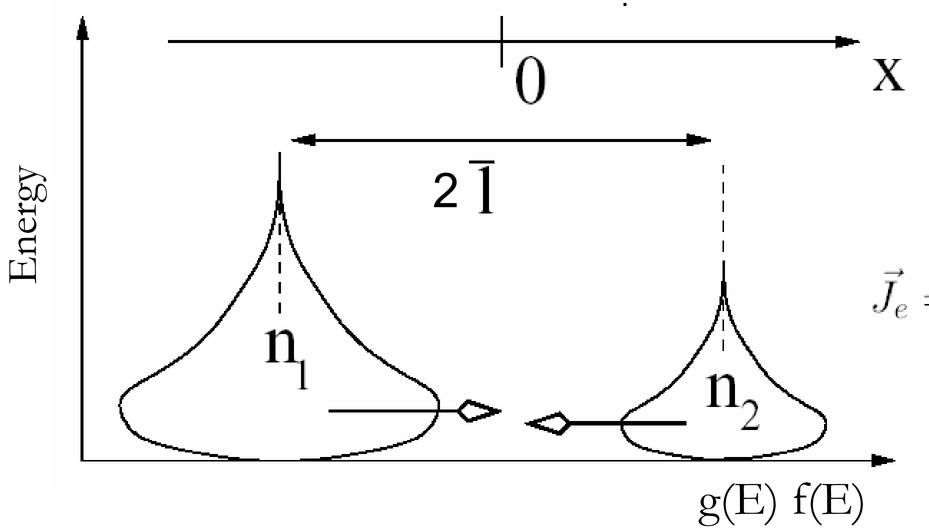
Diffusion current

What does this ad have to do with current in a semiconductor?

The driving force in each case is a [ ] gradient.

There's a net flow of electrons from L $\rightarrow$ R

Even at equilibrium ??



The diffusion current density is

$$\vec{J}_e = -q\frac{n_1}{2}2\vec{v}_R + (-q)\frac{n_2}{2}(-2\vec{v}_R) = -q(\frac{n_1}{2} - \frac{n_2}{2})2\vec{v}_R$$

Do Taylor $\longrightarrow$ $q\bar{l}2v_R\frac{d\vec{n}}{dx} \equiv q\,\boxed{}\,\frac{d\vec{n}}{dx}$

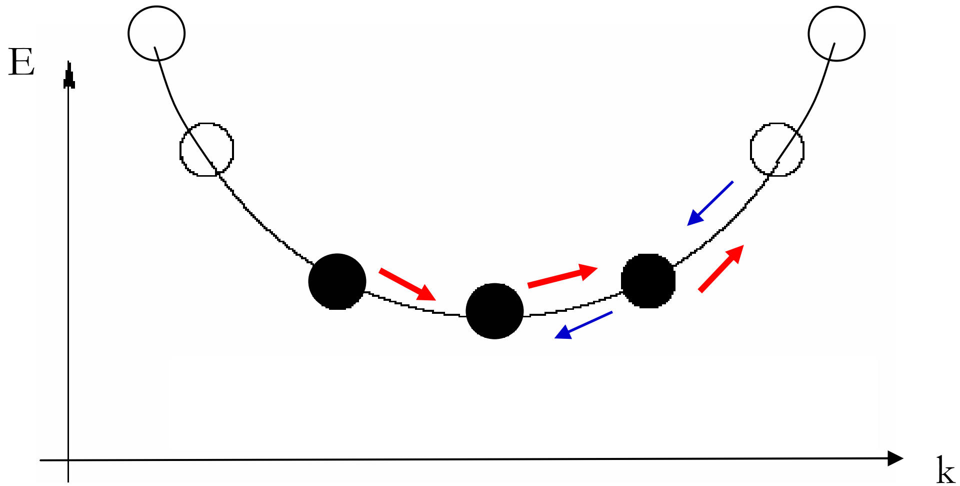
Sec. 5.5.1

Drift Current

The driving force in this case is the [ ] gradient.

$$\vec{F} = m^* \vec{a} = \pm q \vec{\mathcal{E}} = [\ \ ]$$

- Electrons accelerate
- Collisions occur



These events disturb the distribution from that of a true Maxwellian, but if the disturbance is not too great, we can still view the distribution as being very close to its equilibrium form, but with the FULL MAXWELLIAN having a net velocity in the "direction" of the field, the [ ] velocity.

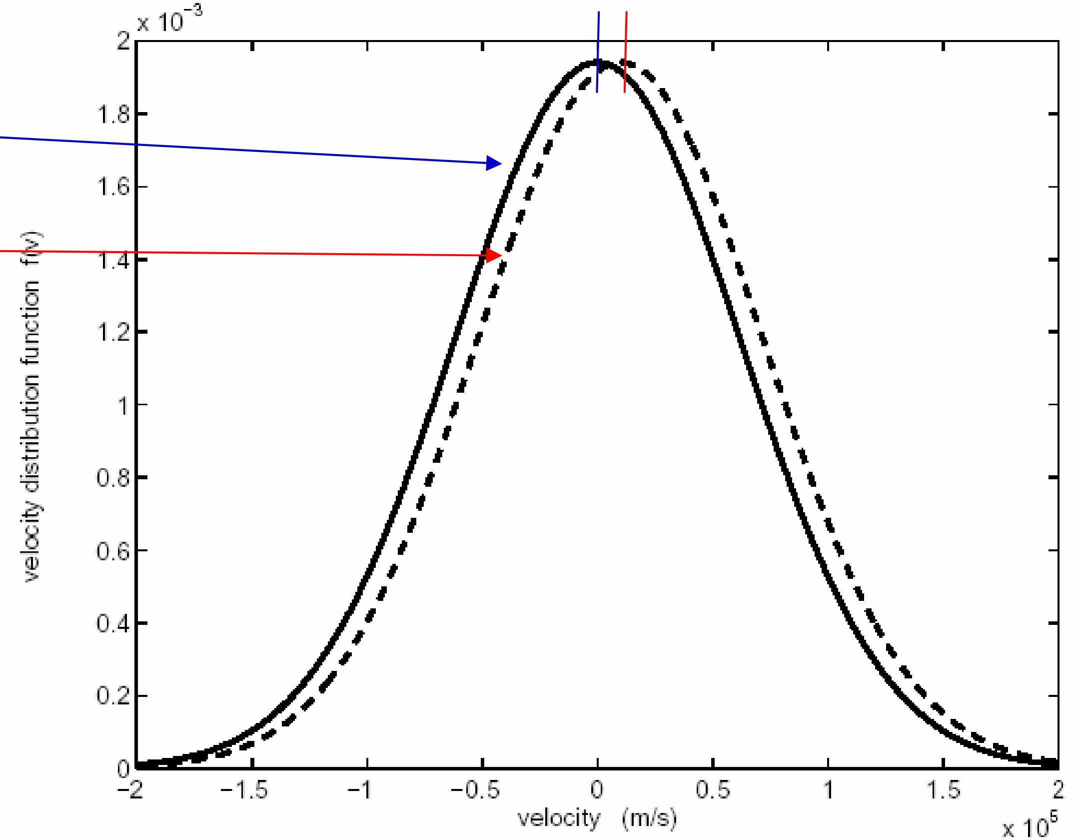
How does this scrum resemble a displaced Maxwellian?

Sec. 5.5.1

Displaced Maxwellian

Equilibrium:

n_0=6.25E16 /cm^3



- Mark v_d on the graph

Drift current density:

$$
\begin{aligned}
\vec{J}_{e,\text{drift}} &= \vec{J}_{e,\text{drift}\rightarrow} + \vec{J}_{e,\text{drift}\leftarrow} \\
&= -q\frac{n}{2}\left(2\vec{v}_R + \vec{v}_{de}\right) + \left(-q\frac{n}{2}\left(-2\vec{v}_R + \vec{v}_{de}\right)\right) \\
&= \boxed{}
\end{aligned}
$$

What is the current density in this example?