1 (a) $\Phi = \Phi_0 e^{-\alpha x}$

$$\frac{\partial P}{\partial t} + \nabla \cdot \Phi = R_{rad} - A \qquad (7.8)$$

Steady-state, no R_{rad}, $A = G_{op}$, 1-D

$$\frac{d\Phi}{dx} = -G_{op}$$

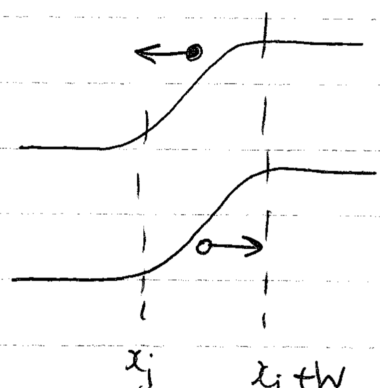
$$\underline{\underline{-\alpha \Phi_0 e^{-\alpha x} = -G_{op}}}$$

This is (7.11)

(b) From equation tool set:

$$-\frac{1}{q}\frac{dJ_e}{dx} = G_{op} - U \qquad (5.24)$$

$U \approx 0$ in depletion region



$$\int_{J_e @ x_j}^{J_e @ (x_j+W)} dJ_e = -q \int_{x_j}^{x_j+W} G_{op}\, dx$$

$$J_e \Big|_{J_e(x_j)}^{0} = -q \int \alpha \Phi_0 e^{-\alpha x} dx$$

$$-J_e(x_j) = q \Phi e^{-\alpha x} \Big|_{x_j}^{x_j+W} = q \Phi_0 \left(e^{-(x_j+W)\alpha} - e^{-\alpha x_j} \right)$$

$$= q \Phi_0 e^{-\alpha x_j} \left[e^{-\alpha W} - 1 \right]$$

$$\underline{\underline{J_e(x_j) = q \Phi_0 e^{-\alpha x_j} \left[1 - e^{-\alpha W} \right]}} \qquad (7.18)$$

(c) $x_j = 0$; $J_e = 1.6 \times 10^{-19} \times 6.25 \times 10^{20} \left[1 - e^{-10^5 \cdot 10^{-4}} \right] = \underline{\underline{100\ A/m^2}}$

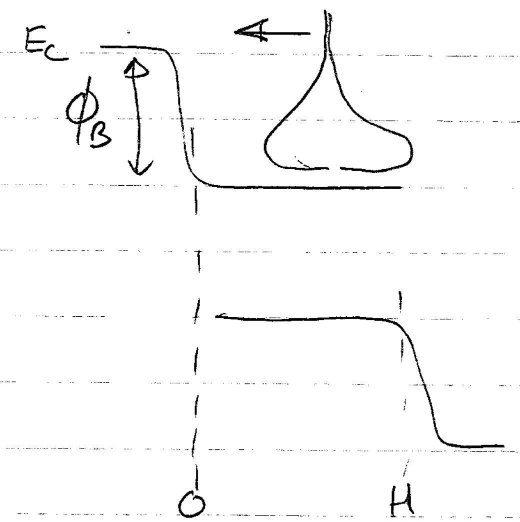
2 (a) From Fig 8.7 (Fig. 1 of exam), luminous efficacy of 68.3 lm/W i.e. $\gamma = 0.1$, occurs in the blue @ $\approx$ 470 nm.

$$E = hc/\lambda = \left[6.63\times10^{-34} \times 3\times10^{8} / 470\times10^{-9}\right] / 1.6\times10^{-19} \quad \text{in eV}$$

$$E \equiv E_g = \underline{\underline{2.645\ eV}}$$

(b) $$\eta_v = \frac{E_g}{qV_a} = \frac{2.645}{2.64} = \underline{\underline{1.002}} \qquad (8.1)$$

(c)



No hole escape @ H

$\therefore\ J_e(H) = J_{Diode}$

$I_e(H) = 10\times10^{-3}$ A

Electron escape current at O due to thermionic emission

$$J_e(x=0) = -q\,\frac{n}{2}\,2v_R\,(-\hat{x})\,e^{-\phi_B/k_BT}$$

$$I_e(0) = 1.6\times10^{-19}\times10^{19}\times10^{5}\ e^{-0.3/0.0259} \times (5\times10^{-3})^2$$

$$= 37.3\ \mu A$$

$$\therefore\ \eta_c = \frac{10\times10^{-3} - 37.3\times10^{-6}}{10\times10^{-3}} = \frac{10 - 0.0373}{10} = \underline{\underline{0.996}}$$

$$\equiv \frac{J_e(H) - J_e(0)}{J_D} \qquad (8.5)$$

2(d) $\Delta n > p_0 \quad \therefore$ high-level injection

$$\tau_{e,rad} = \frac{1}{B(p_0 + \Delta n)} \qquad (3.19)$$

$$= \frac{1}{2\times10^{-10} \times (10^{16} + 10^{19})} = \underline{5\times10^{-10}\ \text{s}}$$

(e)

$$\tau_e = \left[\frac{1}{\tau_{non\text{-}rad}} + \frac{1}{\tau_{rad}}\right]^{-1} \qquad (3.24)$$

$$= \left[\frac{1}{10^{-9}} + \frac{1}{5\times10^{-10}}\right]^{-1} = 3.33\times10^{-10}\ \text{s}$$

$$\eta_{rad} = \frac{\tau_e}{\tau_{rad}} \quad (8.7) \quad = \frac{3.33\times10^{-10}}{5\times10^{-10}} = \underline{\underline{0.667}}$$

(f) luminous efficiency = wall-plug efficiency × lum. efficacy (8.16)

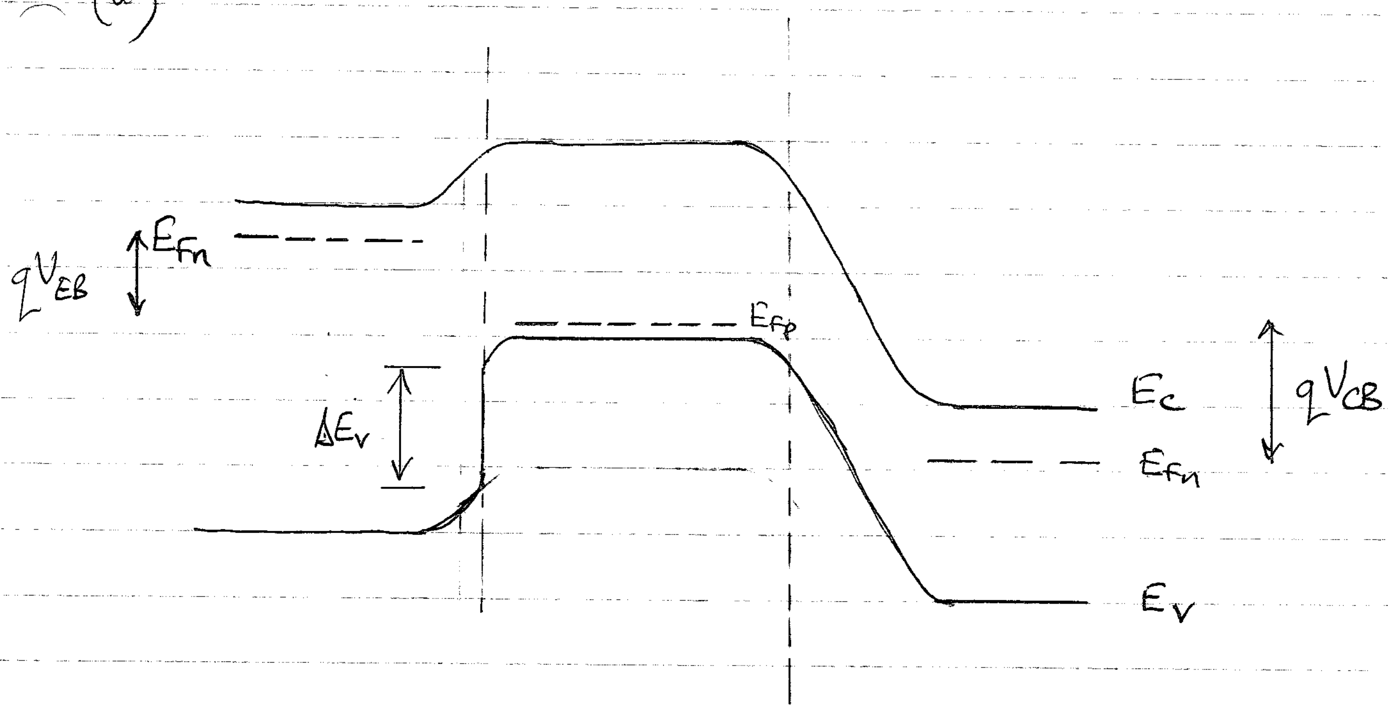
$$40 = WPE \times 68.3 \rightarrow WPE = 0.586$$

$$WPE = \eta_v \eta_c \eta_{rad} \eta_{ext} \qquad (8.12)$$

$$\therefore \eta_{ext} = 0.586 / (1.002 \times 0.996 \times 0.667)$$

$$= \underline{\underline{0.88}}$$

3 (a)



(b) $\Delta E_g = \Delta E_c + \Delta E_V$, $\Delta E_c = \Delta \chi = 0$ * ∴ No extra barrier for J_E

∴ $\Delta E_V = \Delta E_g = 1.89 - 1.42 = 0.47\,eV$

* This large barrier suppresses the hole back-injection current.
∴ J_B is reduced

∴ with J_E unaffected & J_B reduced, β can be high

(c) $f_{max} = \sqrt{\dfrac{f_{Ti}}{8\pi R_b C_{BC}}}$ (14.33) , $f_{Ti} = \dfrac{g_m}{2\pi(C_{BE}+C_{BC})}$ (14.16)

$$g_m = \left|\frac{\partial I_c}{\partial V_{BE}}\right| = \frac{q}{V_{th}} n_{0B} e^{V_{BE}/V_{th}} \cdot \frac{1}{W_B/D_e + \frac{1}{v_R}}$$

High doping in the base is the important feature, and is made possible by ΔE_V. However, it lowers n_{0B}, which counteracts

3c, cont) the reduction in W_B, so there's not much advantage re: g_m.

$C_{EB,j}$ may be slightly affected because of $\varepsilon_N \neq \varepsilon_p$ (6.42)
$C_{EB,b}$ reduced because of allowed narrow base.
The big factor is R_b. This is a lateral R (Fig. 14.5), and is reduced directly by N_B (14.37).

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4 (a) Fig. 10.14 shows that LEVEL 1 overestimates I_{Dsat} for $L = 100$ nm, so will definitely overestimate for $L = 65$ nm.
Level 1 assumes $\mu = \mu_0$, i.e. $v = \mu_0 \mathcal{E}_x$

So, as $\mathcal{E}_x \uparrow$ with $L \downarrow$ v becomes unreasonably high.
In LEVEL 49, v is restricted to $v_{max} = v_{sat}$, thereby reducing the saturation current.

(b) In triode mode, $I_D = Z C_{ox}\left[V_{GS} - V_T - m\frac{V_{DS}}{2}\right] \cdot \mu_{eff} \frac{V_{DS}}{L + \mu V_{DS}/v_{sat}}$ (10.47)

$$\equiv k\left[V_{GS} - V_T - m\frac{V_{DS}}{2}\right] \qquad k = \text{constant}$$

when $I_D = 0$, $V_{GS} = V_T + \frac{m V_{DS}}{2} = 0.44\ V$

$m = 1 + \frac{\gamma}{2\sqrt{2\phi_B}}$ (10.36), $\gamma = \frac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_A}$ (10.16) $\phi_B = V_{th} \ln \frac{N_A}{n_i}$ (10.5)

$\rightarrow \gamma = \quad V^{1/2}$, $2\phi_B = \quad V$, $m = 1.23$

$$\therefore V_T = 0.44 - \frac{1.23 \times 0.2}{2} = \underline{\underline{0.317\ V}}$$

4 (c) $\quad$ Long-channel $V_T = V_{fb} + 2\phi_B + \gamma\sqrt{2\phi_B}$

$$V_{fb} = \Phi_G - \Phi_S \equiv -E_g/2 - \phi_B = -1.07\ \text{V}$$
$$2\phi_B = 1.00\ \text{V}$$
$$\gamma\sqrt{2\phi_B} = 0.46\ \text{V}$$

$\therefore$ $\underline{\underline{V_T\ (\text{long channel}) = 0.40\ \text{V}}}$

This is, less ~~than~~ the measured value because it does not account for the V_T reduction ~~that~~ occurs in practice in short FETs due to the short-channel effect.

(d) To keep V_T at a high-enough value so as not to produce excessively high $I_{\text{sub-threshold}}$.
This counters the effect of t_{ox}: its scaling down increases C_{ox} and reduces V_T.

Future FETs will be very small, so the actual number of dopant atoms will be small.
e.g. 5 atoms in $5\times10\times10$ nm^3 FET with $N_A = 10^{19}$ cm^{-3}.

5 (a) Read current: $I_{Dsat} = \frac{Z}{L} \mu C_{ox} \frac{(V_{GS}-V_T)^2}{2m}$ (10.38)

$$\equiv \frac{Z}{L} K \frac{(V_{GS}-V_T)^2}{2m}$$

$$10^{-3} = 1 \cdot K \cdot \frac{(1.25-0.25)^2}{2\times 1}$$

$$\therefore \; K = 2\times 10^{-3} \; A/V^2$$

(b) $n = \log_2 \left(\frac{I_{max}}{\Delta I} + 1 \right)$ (15.3)

$$\therefore \; 2^2 = \frac{10^{-3}}{\Delta I} + 1 \quad \Rightarrow \quad \Delta I = \frac{10^{-3}}{3} \; A$$

(c) From (10.38) using K as above

$$\frac{dI_{Dsat}}{dV_T} = \frac{Z}{L} \frac{K}{2m} \cdot 2(V_{GS}-V_T)(-1) = -\frac{Z}{L}\frac{K}{m}(V_{GS}-V_T)$$

$$\therefore \; \Delta V_T = -\Delta I \cdot \frac{Lm}{ZK(V_{GS}-V_T)} = \frac{-\Delta I}{K(V_{GS}-V_T)}$$

(d) $\Delta V_T = -\frac{\Delta Q_F}{C_{ox,top}}$ (15.2) ; $C_{ox,top} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.9\times 8.85\times 10^{-12}}{18\times 10^{-9}}$

$$= 1.918\times 10^{-3} \; F/m^2$$

$$\Delta N_F = \frac{-\Delta Q_F A}{-q} = \frac{\Delta V_T \cdot C_{ox} \cdot A}{-q} = \frac{\Delta I \, C_{ox} \cdot A}{qK(V_{GS}-V_T)}$$

$$= \frac{10^{-3} \times 1.918\times 10^{-3} \times (100\sqrt{10}\times 10^{-9})^2}{3\times 1.6\times 10^{-19} \times 2\times 10^{-3} \times 1} = 200 \text{ electrons}$$

More on 5(c)

$$I_{Dsat} = \frac{Z}{L} K \frac{(V_{GS}-V_T)^2}{2m}, \quad = \text{in this case} \quad \frac{K}{2}(V_{GS}-V_T)^2$$

Calculus gives $\Delta I_{Dsat} = -\Delta V_T \cdot K(V_{GS}-V_T)$

Non-Calculus; $\Delta I = \frac{K}{2}\left[(V_{GS}-V_{T_1})^2 - (V_{GS}-V_{T0})^2\right]$

$$= \frac{K}{2}\left[\cancel{V_{GS}^2} - 2V_{GS}V_{T_1} + V_{T_1}^2 - \cancel{V_{GS}^2} + 2V_{GS}V_{T0} - V_{T0}^2\right]$$

$$= \frac{K}{2}\left[2V_{GS}(V_{T0}-V_{T_1}) + V_{T_1}^2 - V_{T0}^2\right]$$

Let $V_{T_1} - V_{T0} \doteq \Delta V_T$

$$= \frac{K}{2}\left[-2V_{GS}\Delta V_T + \Delta V_T^2 + \cancel{V_{T0}^2} + 2\Delta V_T V_{T0} - \cancel{V_{T0}^2}\right]$$

$$= \frac{K}{2}\left[2\Delta V_T(-V_{GS}+V_{T0}) + \Delta V_T^2\right]$$

$$= -\Delta V_T(V_{GS}-V_T)\cdot K + \frac{K\,\Delta V_T^2}{2}.$$

Calculus refers to $\lim_{\Delta x \to 0}$, i.e. neglect the ΔV_T^2 term.

If it is included: $\frac{\Delta I\, 2}{K} = -\Delta V_T\, 2(V_{GS}-V_T) + \Delta V_T^2$

$$\Delta V_T^2 - \Delta V_T\left(2(V_{GS}-V_T)\right) - \frac{2\Delta I}{K} = 0$$

$$\Delta V_T^2 - 2\Delta V_T - 1/3 = 0$$

$$\Delta V_T = \frac{2 \pm \sqrt{4 + 4/3}}{2} = \frac{2 \pm \sqrt{16/3}}{2} = \frac{2 - \frac{4}{1.732}}{2} = \underline{\underline{-0.1547\,V}}$$

Compare with Calculus: $\underline{\underline{\Delta V_T = -0.1667\,V}}$