#### EECE 574 - Adaptive Control Overview

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#### Lectures: Thursday 09h30-12h00 Location: MCLD 207

#### • Course Objective:

# To give an overview of the theory and practice of the mainstream adaptive control techniques

- Four assignments: 15% each
- Project: 40%
- Textbook:

K.J. Åström and B. Wittenmark, Adaptive Control, Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1995. (This book is out of print, but is downloadable from the internet)

#### **Related Books**

- N. Hovakimyan, C. Cao, L1 Adaptive Control theory, SIAM Press, Philadelphia, 2010.
- ٠ P. Ioannou and B. Fidan, Adaptive Control Tutorial, SIAM Press, Philadelphia, 2006.
- V. Bobal, J. Bohm, J. Fessl and J. Macacek, Digital Self-Tuning Controllers, Springer-Verlag, Berlin, 2005.
- ۰ Landau, Lozano and M'Saad, Adaptive Control, Springer-Verlag, Berlin, 1998.
- ٠ Isermann, Lachmann and Matko, Adaptive Control Systems, Prentice-Hall, Englewood Cliffs, NJ, 1992.
- Wellstead and Zarrop, Self-Tuning Systems Control and Signal Processing, J. Wiley and Sons, NY, 1991.
- ٠ Bitmead, Gevers and Wertz, Adaptive Optimal Control, Prentice-Hall, Englewood Cliffs, NJ, 1990.
- ۰ Goodwin and Sin, Adaptive Filtering, Prediction, and Control, Prentice-Hall, Englewood Cliffs, NJ, 1984.
- ۲ Liung, and Söderström, Theory and Practice of Recursive Identification, MIT Press, Cambridge, MA, 1983.

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#### **Course Outline**

- Introduction
- Identification
- Control Design
- Self-Tuning Control
- Model-Reference Adaptive Control
- Properties of Adaptive Controllers
- Auto-Tuning and Gain Scheduling
- Implementation and Practical Considerations
- Extensions

## What Is Adaptive Control?

#### • According to the Webster's dictionary, to adapt means:

- to adjust oneself to particular conditions
- to bring oneself in harmony with a particular environment
- to bring one's acts, behaviour in harmony with a particular environment
- According to the Webster's dictionary, adaptation means:
  - adjustment to environmental conditions
  - alteration or change in form or structure to better fit the environment

#### When is a Controller Adaptive?

- Linear feedback can cope with parameter changes (within some limits)
- According to G. Zames<sup>1</sup>:
  - A non-adaptive controller is based solely on a-priori information
  - An adaptive controller is based on a posteriori information as well

# A Narrow Definition of Adaptive Control

- An adaptive controller is a **fixed-structure** controller with adjustable parameters and a mechanism for automatically adjusting those parameters
- In this sense, an adaptive controller is one way of dealing with parametric uncertainty
- Adaptive control theory essentially deals with finding parameter ajustment algorithms that guarantee global stability and convergence

## Why Use Adaptive Control?

- Control of systems with time-varying dynamics
- If dynamics change with operating conditions in a known, predictable fashion, use **gain scheduling**
- If the use of a fixed controller cannot achieve a satisfactory compromise between **robustness** and **performance**, then and **only then**, should adaptive control be used

Use the simplest technique that meets the specifications <sup>2</sup>

<sup>&</sup>lt;sup>2</sup>... or as A. Einstein apparently once said: *"make things as simple as possible, but no simpler"* 

## Feedback and Process Variations

#### Consider the feedback loop:



The closed-loop transfer function is

$$T = \frac{PC}{1 + PC}$$

Differentiating T with respect to P:

$$\frac{dT}{T} = \frac{1}{1 + PC} \frac{dP}{P} = S \frac{dP}{P}$$

T and S are respectively known as the complementary sensitivity and the sensitivity functions. Note that

$$S + T = 1$$

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## Feedback and Process Variations

- The closed-loop transfer function is NOT sensitive to process variations at those frequencies where the loop transfer function L = PC is large
- Generally *L* >> 1 at low frequencies, and *L* << 1 at high frequencies
- However, L >> 1 can only be achieved in a limited bandwidth, particularly when unstable zeros are present

## Judging the Severity of Process Variations

- Difficult to judge impact of process variations on closed-loop behaviour from open-loop time responses
  - Significant changes in open-loop responses may have little effect on closed-loop response
  - Small changes in open-loop responses may have significant effect on closed-loop response
- Effect depends on the desired closed-loop bandwidth
- Better to use frequency responses

Example 1

## Effect of Process Variations

Consider the system given by

$$G(s) = \frac{1}{(s+1)(s+a)}$$

Open loop step responses for a = -0.01, 0, 0.01:



#### Figure: Open-loop responses

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#### Effect of Process Variations



Figure: Closed-loop responses for unit feedback

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Example 1

#### Effect of Process Variations



#### Bode Diagram

#### Figure: Open-loop Bode plots

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Example 1

#### Effect of Process Variations



Figure: Closed-loop Bode plots

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#### Effect of Process Variations

Consider now the system

$$G(s) = rac{400(1-sT)}{(s+1)(s+20)(1+sT)}$$

Open-loop responses for T = 0, 0.015, 0.03:



#### Figure: Open-loop responses

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#### Effect of Process Variations



Figure: Unit-feedback closed-loop responses

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#### Effect of Process Variations



#### Figure: Open-loop Bode plots

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#### Effect of Process Variations



Figure: Unit-feedback closed-loop Bode plots

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#### Effect of Process Variations

# Consider now the same system but with a controller C(s) = 0.075/(s+1):



#### Figure: New closed-loop responses

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#### Effect of Process Variations



Figure: New closed-loop Bode plots

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#### Mechanisms for Process Dynamics Changes

#### Nonlinear actuators or sensors

- Nonlinear valves
- pH probes
- Flow and speed variations
  - Concentration control
  - Steel rolling mills
  - Paper machines
  - Rotary kilns

#### Mechanisms for Process Dynamics Changes

- Wide operating range with a nonlinear system
  - Flight control
- Variations in Disturbance Dynamics
  - Wave characteristics in ship steering
  - Raw materials in process industries

#### Gain Scheduling

- In many cases, process dynamics change with operating conditions in a known fashion
  - Flight control systems
  - Compensation for production rate changes
  - Compensation for paper machine speed
- Controller parameters change in a **predetermined** fashion with the operating conditions
- Is gain scheduling adaptive?

#### Gain Scheduling



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## **Development of Adaptive Control**

- Mid 1950s: Flight control systems (eventually solved by gain scheduling)
- 1957: Bellman develops dynamic programming
- 1958: Kalman develops the self-optimizing controller "which adjusts itself automatically to control an arbitrary dynamic process"
- 1960: Feldbaum develops the dual controller in which the control action serves a dual purpose as it is "directing as well as investigating"

## **Development of Adaptive Control**

Mid 60s-early 70s: Model reference adaptive systems



But now came a technical problem that spelled the end. The Honeywell adaptive flight control system began a limit-cycle oscillation just as the plane came out of the spin, preventing the system's gain changer from reducing pitch as dynamic pressure increased. The X-15 began a rapid pitching motion of increasing severity. All the while, the plane shot downward at 160,000 feet per minute, dynamic pressure increasing intolerably. ... As the X-15 neared 65,000 feet, it was speeding downward at Mach 3.93 and experiencing over 15 g vertically, both positive and negative, and 8 g laterally. It broke up into many pieces amid loud sonic rumblings, ... Then an Air Force pilot, ..., spotted the main wreckage ... Mike Adams was dead and the X-15 destroyed.

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## **Development of Adaptive Control**

- Late 60s-early 70s: System identification approach with recursive least-squares
- Early 1980s: Convergence and stability analysis
- Mid 1980s: Robustness analysis
- 1990s: Multimodel adaptive control
- 1990s: Iterative control
- 2000s:  $\mathcal{L}_1$  adaptive control: fast adaptation with guaranteed robustness.

Adaptive Schemes Model Reference Adaptive Control

## Model Reference Adaptive Control

- Performance specifications given in terms of reference model
- Originally introduced for flight control systems (MIT rule)
- Nontrivial adjusment mechanism

Adaptive Schemes Model Reference Adaptive Control

### Model Reference Adaptive Control



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## Self-Tuning Controller

- Model-based tuning consists of two operations:
  - Model building via identification
  - Controller design using the identified model
- Self-tuning control can be thought of as an automation of this procedure when these two operations are performed on-line

## Self-Tuning Controller



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## Self-Tuning vs. Auto-Tuning

#### Self-tuning

- Continuous updating of controller parameters
- Used for truly time-varying plants

#### Auto-tuning

- Once controller parameters near convergence, adaptation is stopped
- Used for time invariant or very slowly varying processes
- Used for periodic, usually on-demand tuning

#### Final Motivation...



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#### Final Motivation...



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#### **Dual Control**

# Dual Control: A Rigorous Approach to Adaptive Control

- Use of nonlinear stochastic control theory to derive an adaptive controller
- No distinction between parameters and state variables Hyperstate
- The controller is a nonlinear mapping from the hyperstate to the control variable



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#### **Dual Control**

## A Rigorous Approach to Adaptive Control

- Can handle very rapid parameter changes
- Resulting controller has very interesting features:
  - Regulation
  - Caution
  - Probing
- Unfortunately solution is untractable for most systems

# Illustration of Dual Control

Consider the simple process

$$y(t+1) = y(t) + bu(t) + e(t+1)$$

where e(t) is zero-mean white noise  $N(0, \sigma)$ , y(t) and u(t) are the output and the input signals.

**One-stage control** 

Find u(t) that minimizes

$$I_1 = E[y^2(t+1)|y(t), u(t)]$$

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## Certainty Equivalence Controller

In case *b* is known, the solution is trivial:

$$\min I_1 = \min [y(t-1) + bu(t-1)]^2 + \sigma^2 = \sigma^2$$

since e(t) is independent of y(t-1), u(t-1) and b.

$$u(t) = -\frac{y(t)}{b} \qquad \qquad I_{1 \text{opt}} = \sigma^2$$

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## Certainty Equivalence Controller

Now, assume that *b* is unknown. We now have an estimate  $\hat{b}$  with covariance  $p_b$ If least-squares is used:

$$\hat{b} = \left\{ \sum_{s=1}^{t} [y(s) - y(s-1)]u(s-1) \right\} / \sum_{s=1}^{t} u^2(s-1)$$

$$p_b = \sigma^2 / \sum_{s=1}^t u^2(s-1)$$

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## Certainty Equivalence Controller

The most direct way to control the system is simply to replace *b* by  $\hat{b}$  in the controller above, thus ignoring the uncertainty:

$$u_{ce}(t) = -rac{y(t)}{\hat{b}}$$

then

$$I_{\rm 1ce} = \sigma^2 + \frac{p_b}{\hat{b}^2} y^2 (t-1)$$

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## **Cautious Controller**

Performing the minimization of  $I_1$  actually gives:

$$u(t) = -\frac{\hat{b}}{\hat{b}^2 + \rho_b} y(t)$$

and the minimum performance index

$$I_{1 \text{caut}} = \sigma^2 + \frac{p_b}{\hat{b}^2 + p_b} y^2 (t-1)$$

## Cautious Controller

 Because p<sub>b</sub> is positive, the cautious controller has a smaller gain than the certainty equivalence one, which by ignoring uncertainty may be at times too **bold**

#### • Turn-off phenomenon:

- When the uncertainty *p*<sup>*b*</sup> is large, controller gain is small and so does the control action
- So, unless an external perturbation is added to the input, no learning can take place and the uncertainty p<sub>b</sub> cannot be reduced
- This highlights the importance of **probing** signals

#### N-stage control

Find u(t) that minimizes

$$I_N = E[\sum_{1}^{N} y^2(t+i)|y(t), u(t)]$$

- By using the N-stage control problem with N > 1, it can be shown that the effect of present inputs on the future values of  $\hat{b}$  and  $p_b$  enters the minimization of  $I_N$
- Indeed it is sometimes beneficial to sacrifice short term performance by sending a probing signal to reduce the uncertainty, and thus improve performance in the long term

- Using dynamic programming, a functional equation (Bellman equation) can be derived
- However, this equation can only be solved numerically and for very simple cases
- For large *N*, the control tends towards a steady-state control law

#### **Dual Controller**

Define

$$\eta = rac{\mathbf{y}}{\sigma} \quad \beta = rac{\hat{\mathbf{b}}}{\mathbf{p}_{\mathbf{b}}} \quad \mu = -rac{\hat{\mathbf{b}}\mathbf{u}}{\mathbf{y}}$$

μ = 1 corresponds to the certainty-equivalence controller
μ = β<sup>2</sup>/(1 + β<sup>2</sup>) corresponds to the cautious controller
Dual controller for large *N* is:

$$\mu = \frac{\beta^2 - 0.56\beta}{\beta^2 + 0.08\beta + 2.2} + \left(\frac{1.9\beta}{\beta^4 + 1.7}\right)\frac{1}{\eta}$$

#### **Dual Controller**



#### Figure: Dual control map

Guy Dumont (UBC)

EECE 574 Overview

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## Properties of the Dual Controller

- Dual control finds the best compromise between
  - boldness
  - caution
  - probing
- $\bullet \ Low \ uncertainty \rightarrow boldness \ prevails$
- Large uncertainty + large control error  $\rightarrow$  caution prevails
- Large uncertainty + small control error  $\rightarrow$  probing prevails

# Implications for Adaptive Control

- The dual controller is in general impossible to compute
- Most current adaptive control methods enforce certainty equivalence
- Thus, learning is **passive** rather than active
- **Passive** learning is a shortcoming of current adaptive control methods
- Practical methods of active learning attractive for
  - Commissioning of adaptive controllers
  - Adaptive control of processes with rapidly time-varying dynamics