# Capacitance

LECTURE 14

- charge balance
- steady state, dielectric relaxation time, and transit time
- definition of capacitance
- equivalent circuits for charging current
- MOSFET capacitance
- non-reciprocity

IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 39, NO. 4, APRIL 2004

# RF CMOS Comes of Age

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***Abstract*—All-CMOS radio transceivers and systems-on-a-chip are rapidly making inroads into a wireless market that for years was dominated by bipolar and BiCMOS solutions. It is not a matter of replacing bipolar transistors in known circuit topologies with FETs; the wave of RF CMOS brings with it new architectures and unprecedented levels of integration. What are its origins? What is the commercial impact? How will RF CMOS evolve in the future? This paper offers a retrospective and a perspective.**

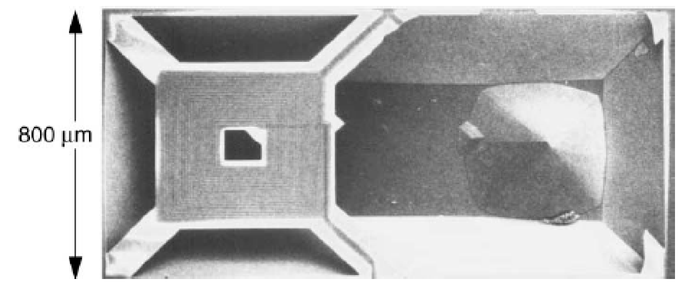



Fig. 1. A suspended spiral inductor of about 100 nH on a heavily doped CMOS substrate. This element enabled the first RF CMOS circuits at 900 MHz.

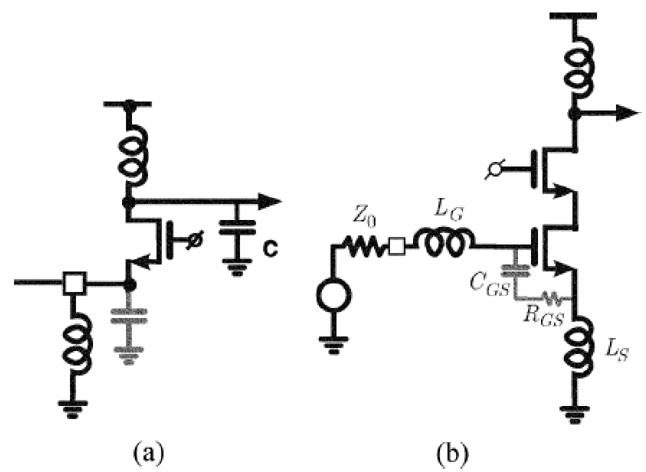



Fig. 2. Low-noise amplifier styles commonly used in CMOS. (a) Common-gate circuit, robust against parasitics, moderate noise figure. (b) Common-source circuit, lowest noise figure.

Sec. 5.2

# Time-dependent equations

e.g., Hydrodynamic Equations for electrons

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_e}{\partial x}$$

$$\frac{\partial J_{e,x}}{\partial t} = \frac{2q}{m^*}\frac{\partial W_{e,x}}{\partial x} + \frac{q^2 n}{m^*}\mathcal{E}_x - \frac{J_{e,x}}{\langle\langle \tau_M \rangle\rangle}$$

$$\frac{\partial W_{e,x}}{\partial t} = \kappa_e \frac{\partial^2 T_e}{\partial x^2} + J_{e,x}\mathcal{E}_x - \frac{k_B}{2}\frac{(T_e - T_L)}{\langle\langle \tau_E \rangle\rangle} .$$

Sec. 5.2

# Continuity of charge

This is a prelude to our study of capacitance and RF transistors

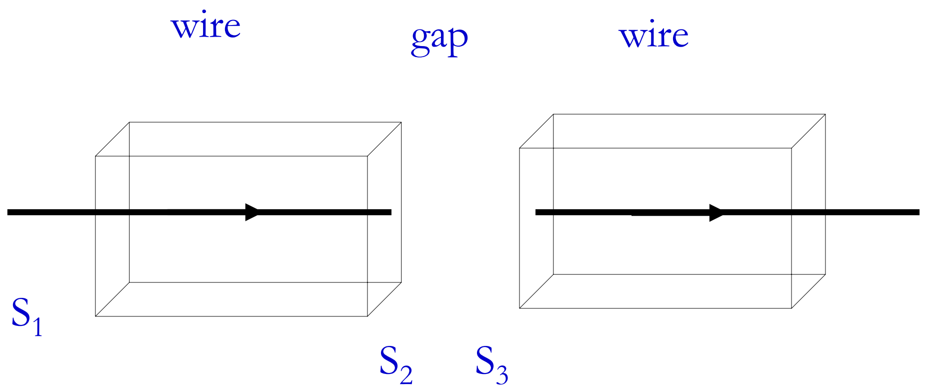
$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_e}{\partial x}$$

This equation expresses the universal truth regarding the conservation of charge.

The fundamentality of this relation can be further appreciated by deriving it from Maxwell's Equations.

Now, we use it as a basis for defining capacitance.

# Example of charging C




Apply charge continuity to surfaces $S_1$ and $S_2$.

What does this say about displacement current?

Apply charge continuity to surfaces $S_1$ and $S_3$.

What does this say about currents in steady-state?

What happens in the transient case?

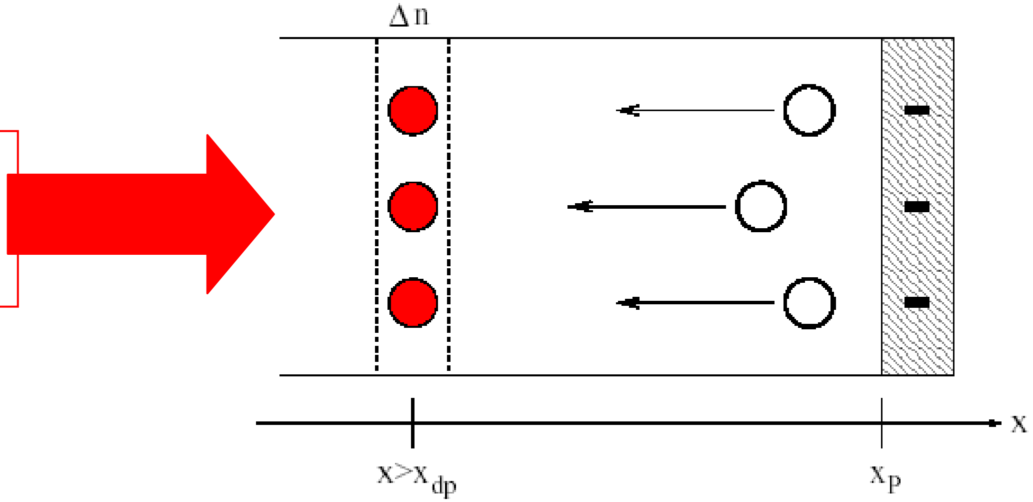
Sec. 6.3.2

# Dielectric Relaxation Time

Apparent charge motion due to charge neutralization.

View the situation as RC-charging and define the time constant.

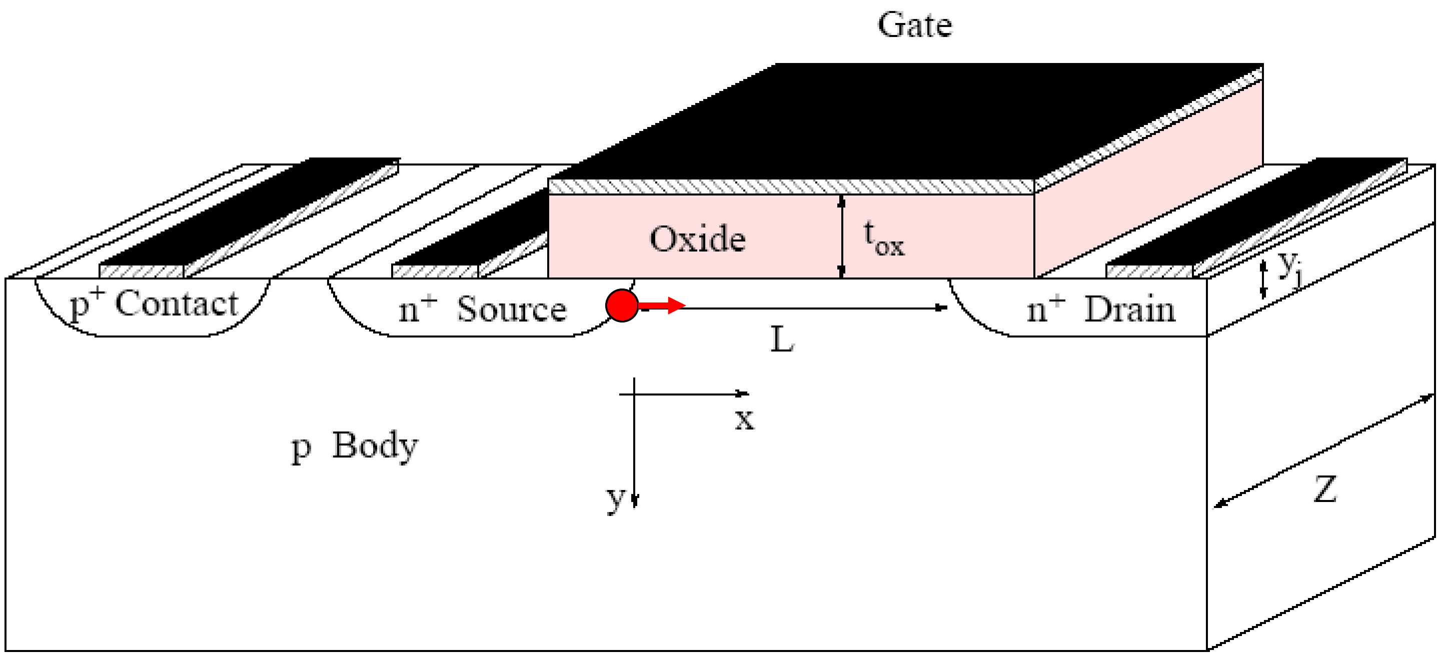
Electrons injected from n-region




What is the comparable situation in the channel of a FET?

Sec. 12.1

# MOSFET Transit Time




How long before the Drain "experiences" the electron?

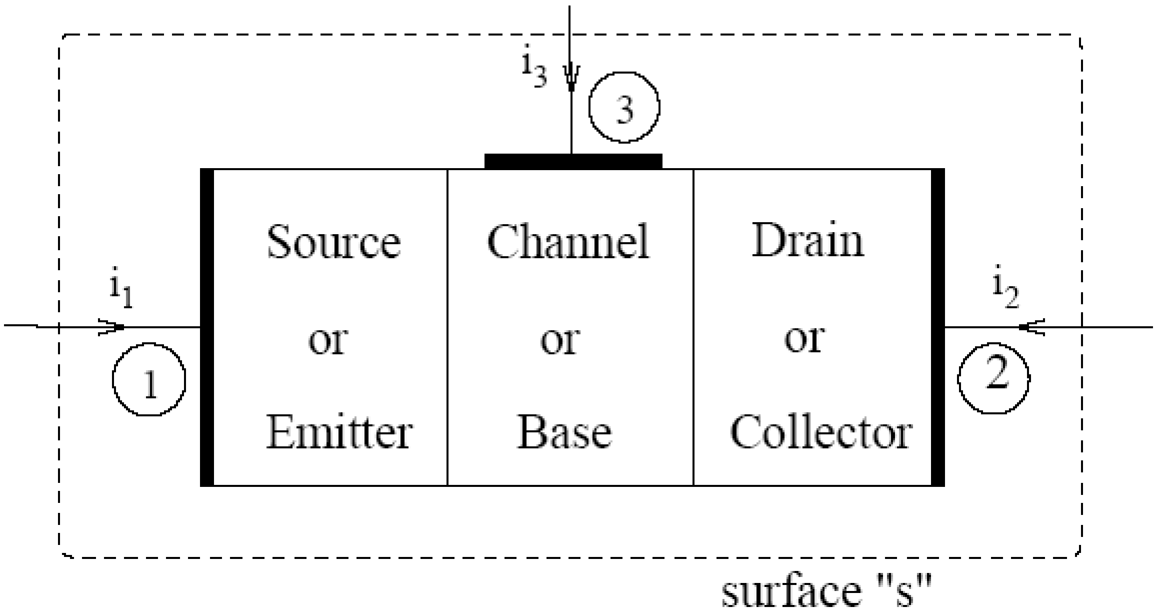
Sec. 12.1

# Steady-state

Steady state

$$\frac{\partial Q_1}{\partial t} + \frac{\partial Q_2}{\partial t} + \frac{\partial Q_3}{\partial t} = 0$$

Generic transistor




Using Terminal 1 as reference.

$$\begin{aligned}
\frac{\partial Q_1}{\partial t} &= \frac{\partial V_{21}}{\partial t}\frac{\partial Q_1}{\partial V_{21}} + \frac{\partial V_{31}}{\partial t}\frac{\partial Q_1}{\partial V_{31}} \\
\frac{\partial Q_2}{\partial t} &= \frac{\partial V_{21}}{\partial t}\frac{\partial Q_2}{\partial V_{21}} + \frac{\partial V_{31}}{\partial t}\frac{\partial Q_2}{\partial V_{31}} \\
\frac{\partial Q_3}{\partial t} &= \frac{\partial V_{21}}{\partial t}\frac{\partial Q_3}{\partial V_{21}} + \frac{\partial V_{31}}{\partial t}\frac{\partial Q_3}{\partial V_{31}} .
\end{aligned}$$

What is another name for the LH sides?

Sec. 12.1

# Defining capacitance

Hold $V_{21}$ constant:

$$\frac{\partial V_{31}}{\partial t}\left[\frac{\partial Q_1}{\partial V_{31}}+\frac{\partial Q_2}{\partial V_{31}}+\frac{\partial Q_3}{\partial V_{31}}\right]=0\,.$$

For this to be true for any $\partial V_{31}/\partial t$, we must have

$$\frac{\partial Q_3}{\partial V_{31}}=-\frac{\partial Q_1}{\partial V_{31}}-\frac{\partial Q_2}{\partial V_{31}}\,,$$

from which capacitances are defined:

$$C_{33}=C_{13}+C_{23}\,.$$

What is the interpretation of the double-subscripted notation?

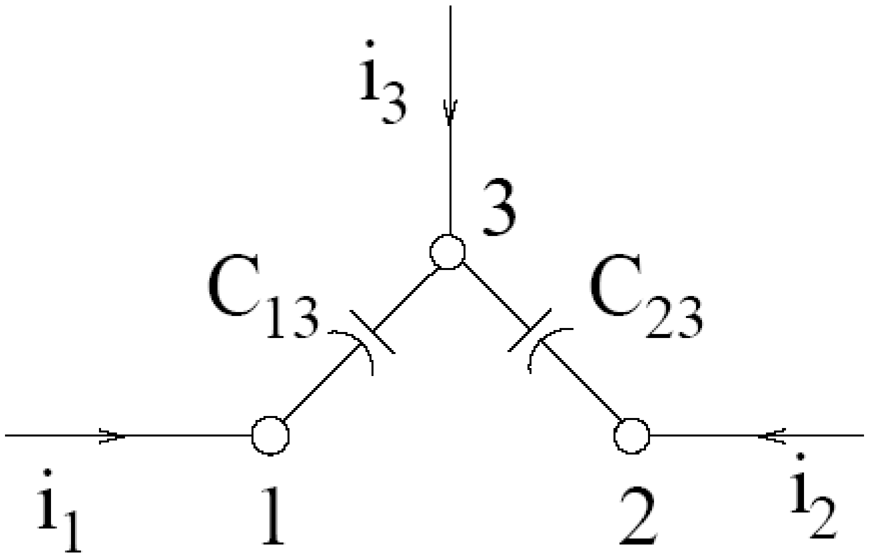
The notation gives:

$$\begin{aligned}
C_{jk} &= -\frac{\partial Q_j}{\partial V_k} && \text{if } k\neq j\\
C_{jk} &= +\frac{\partial Q_j}{\partial V_k} && \text{if } k=j\\
C_{jj} &= \sum_{k\neq j} C_{jk}=\sum_{k\neq j} C_{kj}\,.
\end{aligned}$$

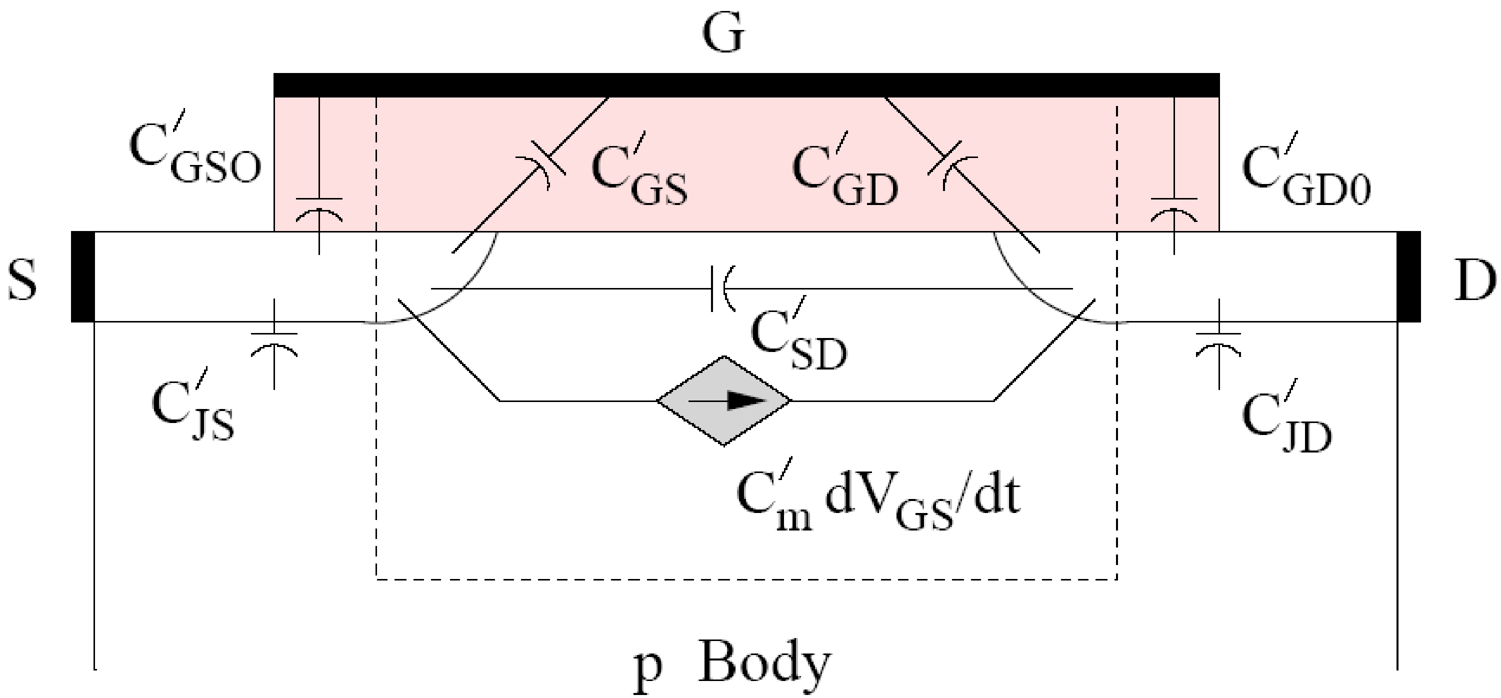
# Equivalent circuit representation

$$0 = -C_{13}\frac{\partial V_{31}}{\partial t} - C_{23}\frac{\partial V_{31}}{\partial t} + C_{33}\frac{\partial V_{31}}{\partial t}$$

$$\equiv$$



# MOSFET capacitance



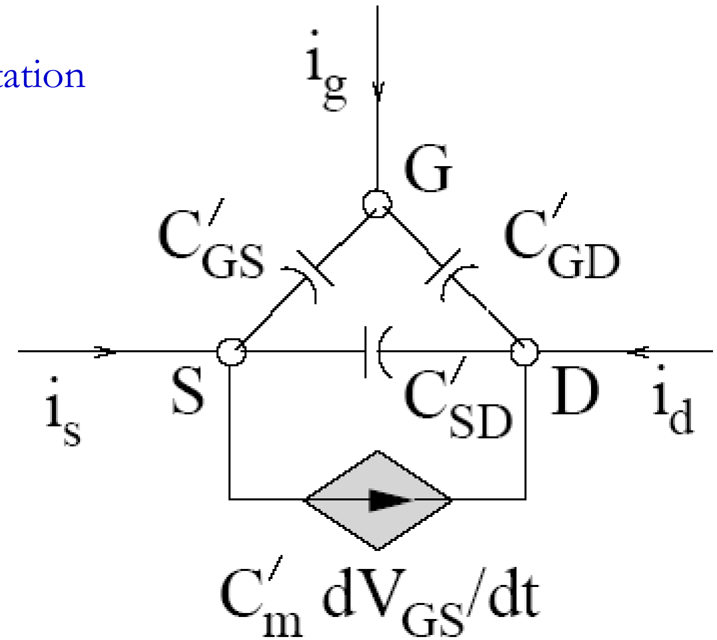
Sec. 12.2.1

# MOSFET equivalent circuit

$$\begin{pmatrix} i_S \\ i_D \\ i_G \end{pmatrix} = \begin{pmatrix} C'_{SS} & -C'_{SD} & -C'_{SG} \\ -C'_{DS} & C'_{DD} & -C_{DG} \\ -C'_{GS} & -C'_{GD} & C'_{GG} \end{pmatrix} \begin{pmatrix} \partial V_S/\partial t \\ \partial V_D/\partial t \\ \partial V_G/\partial t \end{pmatrix}$$

How many independent C's?

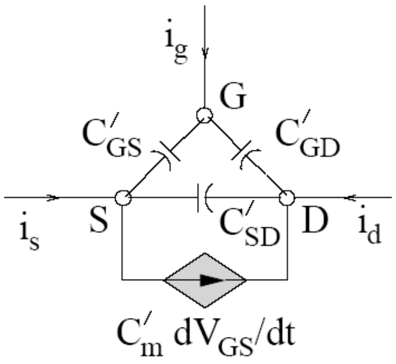
Equivalent circuit representation



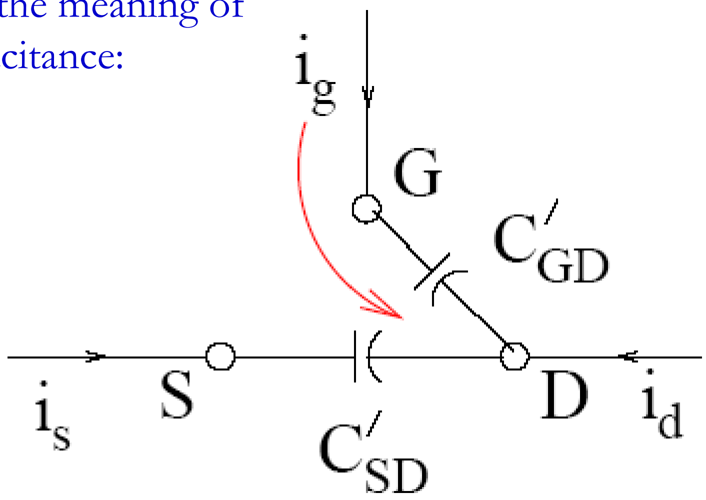

What is $C_m$ ?

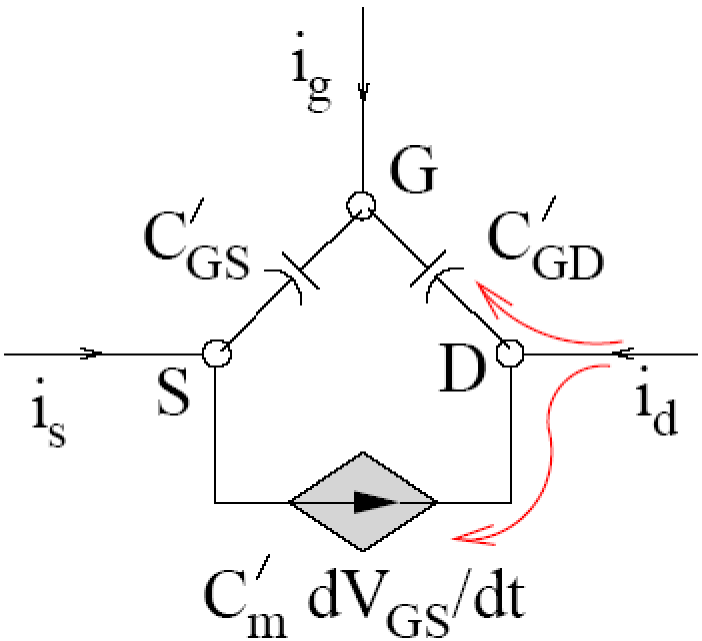
Sec. 12.2.1

# Transcapacitance




Consider 2 cases to illustrate the meaning of transcapacitance:




What does each circuit represent?

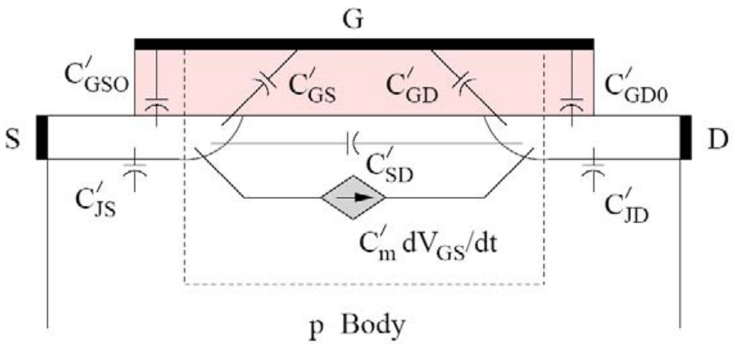
# Non-reciprocity

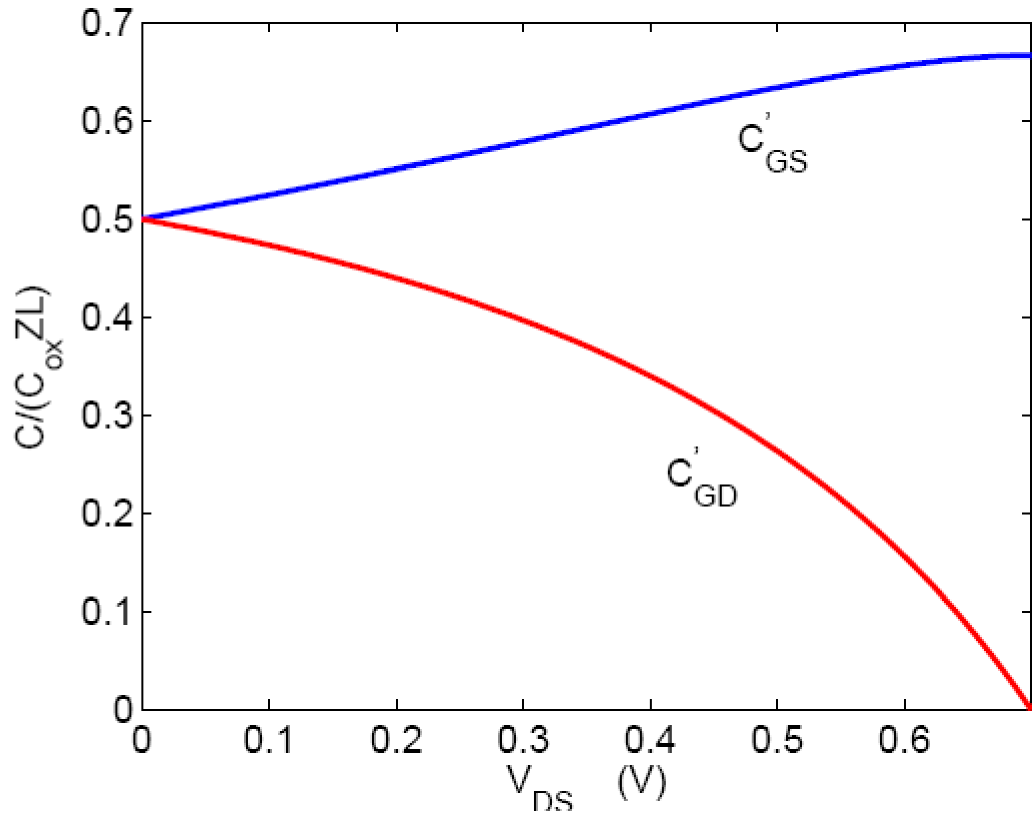
Capacitances are not necessarily reciprocal.

Do board-note example of $C_{GD}$ and $C_{DG}$

Sec. 12.2.1

# Intrinsic capacitances and $C_{ox}$

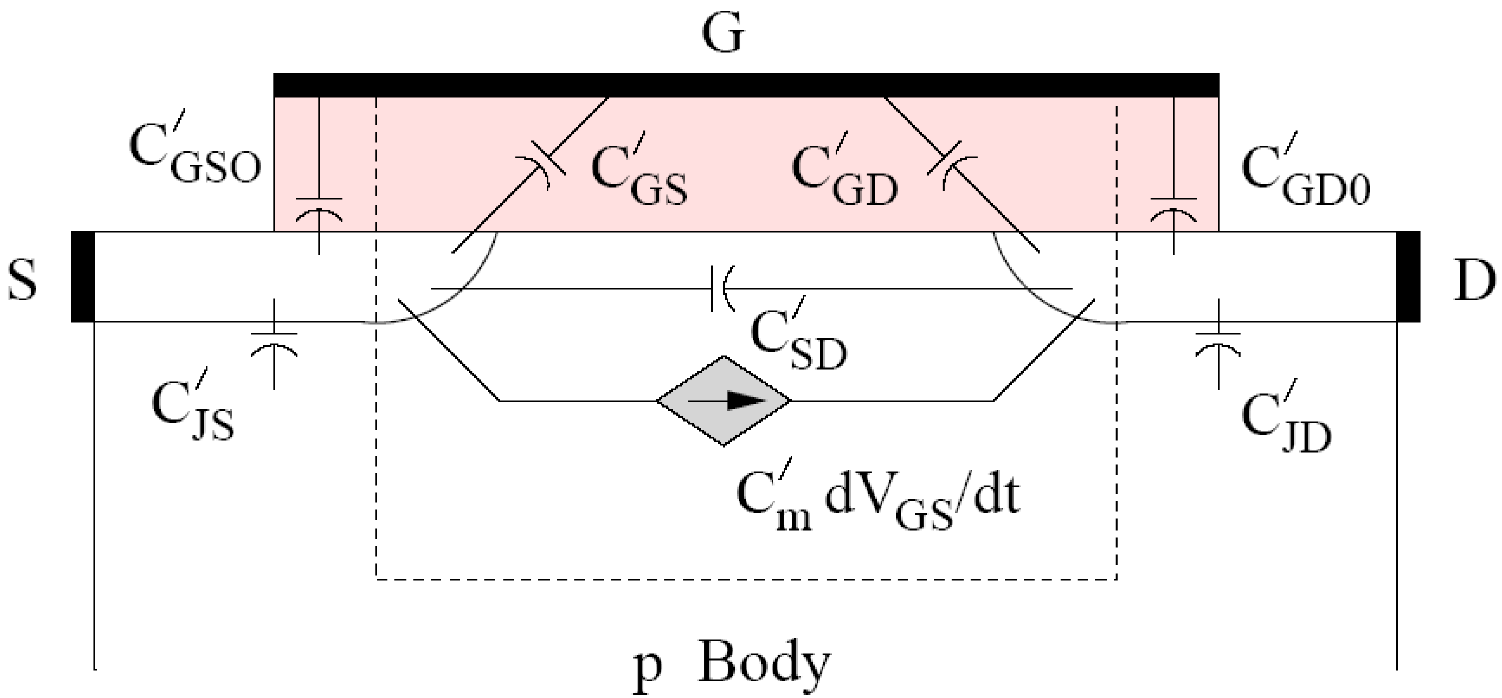



What are the reasons for these trends?

Sec. 12.3.1

# Extrinsic MOSFET capacitance




Are the extrinsic capacitances reciprocal?