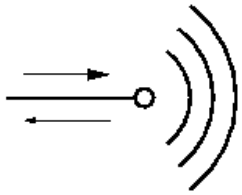


EECE 483 - Antennas and Propagation

Module 1 - Electromagnetic Wave Propagation

Basic antenna concepts; antennas for low, medium and high frequencies; terrestrial and satellite propagation links; environmental effects on electromagnetic radiation. [3-0-0]

Introduction

- The electromagnetic force is one of four fundamental forces (gravity and the strong and weak nuclear forces are the other three); only gravity and electromagnetism manifest themselves at macroscopic scales
- In past courses, you studied:
  - static electric and magnetic fields;
  - time-varying electromagnetic fields;
  - guided waves (transmission lines and waveguides);
  - propagating plane waves.
- If one is to excel in EECE 483, one requires an excellent grasp of the fundamentals of electromagnetics, hence this review.

Assumptions

- In EECE 483, our focus is on the theory and analysis of electromagnetic phenomena that vary sinusoidally with time, *i.e.,* time-harmonic fields, especially those with frequencies between 9 kHz and 275 GHz.
- In this course, we will view such electromagnetic phenomenon from the macroscopic viewpoint, *i.e.,* cases in which:
  - linear dimensions are large compared to atomic dimensions,
  - charge magnitudes are large compared to atomic charges.
- In general, we will not consider mechanical forces associated with electromagnetic fields.
- We will assume that relative velocities between sources and observers are sufficiently low that relativistic effects can be ignored.

Objectives

Upon completion of this module, the EECE 483 student will be able to:

- recount key milestones in the history of electromagnetics,
- describe the key electromagnetic quantities and their physical significance and express Maxwell's equations in both point and integral form,
- solve problems involving plane waves propagating in free space and various material media,
- solve problems involving spherical waves,
- solve problems involving plane waves incident upon material boundaries and,
- solve problems involving diffraction by a knife edge.

Outline

1. A Very Brief History of Electromagnetics
2. Electromagnetic Quantities
3. Maxwell's Equations
4. Wave Propagation in Space
5. Boundary Conditions
6. Wave Propagation at Boundaries

1. A Very Brief History of Electromagnetics

Electromagnetics has a long and rich history:

- Static electric and magnetic fields have been known since antiquity.
- In 1747, the modern concept of electric charge was introduced by Benjamin Franklin.
- In 1764, Joseph Louis Lagrange discovered the divergence theorem in connection with the study of gravitation. In 1813, it was rediscovered in connection with electrostatic fields and became known as Gauss's law.
- In 1820, the relationship between electric current and magnetism was first noted by Oersted. This discovery ushered in a period of intense activity in this area.

- In 1825, Ampere, referred to by some as "the Newton of electricity," published an amazingly complete and modern description of electromagnetism.
- In 1831, the relationship between changing magnetic flux and voltage was first noted by Michael Faraday.
- In 1837, Faraday introduced the notion of the dielectric constant. In 1846, he speculated that light might be transverse electromagnetic waves.
- In 1849, Fizeau repeated Galileo's famous hilltop experiment with a 9 km separation distance and a rapidly rotating toothed wheel. He estimated that c $= 3.15 \times 10^8$ m/s.
- In 1850, William Thomson (Lord Kelvin) introduced the notion of magnetic permeability and susceptibility, along with the concepts of $\mathbf{B}$ and $\mathbf{H}$.

- In 1864, after a decade of work, James Clerk Maxwell presented a complete mathematical description of electromagnetism to the Royal Society.
- In 1888, Heinrich Hertz became the first person to artificially generate and detect radiofrequency waves.
- By the early 1900's, wireless technology (or radio) began to achieve considerable economic and practical significance, especially for maritime communications.
- By the 1920's, broadcast radio had become a potent consumer force.
- Initially limited to lower frequencies, radio technology quickly moved upward to shortwave, VHF/UHF, and microwave frequencies.
- Today, we couldn't imagine a world without technology based upon electromagnetics!

2. Electromagnetic Quantities

$\mathbf{E}$ - electric field strength (V/m)

$\mathbf{H}$ - magnetic field strength (A/m)

$\mathbf{D}$ - electric flux density (C/m^2)

$\mathbf{B}$ - magnetic flux density (Wb/m^2 = T)

$\mathbf{J}$ - electric current density (A/m^2)

ρ_v - electric charge density (C/m^3)

- Review questions:
  - What is the physical significance of each quantity?
  - Which of these quantities can be directly observed or measured?

Constitutive Relations for Linear Isotropic Media

The constitutive relations give the relationship between field strength and flux density, or between field strength and current density.

$$\begin{aligned} \mathbf{D} &= \epsilon_r \epsilon_0 \mathbf{E} \\ \mathbf{B} &= \mu_r \mu_0 \mathbf{H} \\ \mathbf{J} &= \sigma \mathbf{E} \end{aligned}$$

ϵ_0 permittivity of free space (F/m)
ϵ_r relative permittivity of a material (unitless)
μ_0 permeability of free space (H/m)
μ_r relative permeability of a material (unitless)
σ conductivity (siemens/m)

Physical Constants and Derived Values

$$\epsilon_0 \approx \frac{1}{36\pi} \times 10^{-9} \ \ \mathrm{F/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \ \ \mathrm{H/m}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \ \ \mathrm{m/s}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\,\Omega$$

Scalar Electric Potential

- Electric field strength $\mathbf{E}$ can be obtained from knowledge of the configuration of electric charge.
- In many cases, it is more convenient to employ a scalar electric potential or voltage defined by
$$V = \int_v \frac{\rho\, dv}{4\pi\epsilon_0 R}\,.$$
- $\mathbf{E}$ is given by the negative of the gradient of V, *i.e.*,
$$\mathbf{E} = -\nabla V\,.$$
- The scalar electric potential is based upon a zero reference at infinity.

Vector Magnetic Potential

- In a similar way, magnetic field strength $\mathbf{H}$ can be obtained from knowledge of the configuration of electric curents.
- A vector magnetic potential $\mathbf{A}$ can be defined such that

$$\nabla \times \mathbf{A} = \mathbf{B} = \mu \mathbf{H} \, .$$

- As such, $\mathbf{A}$ serves as an intermediate quantity from which $\mathbf{B}$ and hence $\mathbf{H}$ can be calculated.
- This definition is consistent with the requirement that $\nabla \cdot \mathbf{B} = 0$.
- Why? Recall that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$.

- For the case of a current filament, the magnetic vector potential at a particular point is given by

$$\mathbf{A} = \oint \frac{\mu I d\ell}{4\pi R} \, .$$

- Here, R is the distance from the current element $Id\ell$ to the point at which the vector magnetic potential is being calculated.
- The vector magnetic potential is also based upon a zero reference at infinity.

3. Maxwell's Equations

- James Clerk Maxwell's unification of electric and magnetic field theory, first presented to the Royal Society in 1864, is one of the great triumphs of mathematical physics. It allowed him to state that:

  "We have strong reason to conclude that light itself - including radiant heat and other radiation, if any - is an electromagnetic disturbance in the form of waves propagated through the electro-magnetic field according to electro-magnetic laws"

- Maxwell expressed his results using Hamilton's quaternions, but our modern vector notation is due to Josiah Willard Gibbs.
- Review question: What were Maxwell's main contributions, and which aspects were anticipated by others?

Maxwell's equations - General Set

- If the electromagnetic field is well-behaved (*i.e.,* its derivatives are continuous), it is convenient to express Maxwell's equations in point form:

$$\begin{aligned}
\nabla \cdot \mathbf{D} &= \rho \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\partial \mathbf{B}/\partial t \\
\nabla \times \mathbf{H} &= \mathbf{J}_c + \partial \mathbf{D}/\partial t
\end{aligned}$$

- These can be related to the integral forms of the equations via the divergence theorem and Stokes' theorem.

The Divergence Theorem

- Consider a closed surface S which encloses a volume v.
- The Divergence Theorem states that the integral of the normal component of a vector field $\mathbf{F}$ over the closed surface S to a volume integral of a scalar field f which is the divergence of $\mathbf{F}$.
- This allows us to turn a volume integral into a surface integral, and vice versa.

$$\oint_S \mathbf{F} \cdot d\mathbf{S} = \int_v (\nabla \cdot \mathbf{F}) dv$$

$$e.g., \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v (\nabla \cdot \mathbf{D}) dv = Q_{\text{encl}}$$

Stokes' Theorem

- Consider an open surface S whose boundary is a closed curve C.
- Stoke's theorem states that the integral of the tangential component of a vector field $\mathbf{F}$ around C is equal to the integral of the normal component of $\nabla \times \mathbf{F}$ over S.

$$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

$$e.g., \oint_C \mathbf{A} \cdot d\ell = \int_S \mathbf{B} \cdot d\mathbf{S} = \Phi$$

Maxwell's equations - General Set

- The restriction on continuous derivatives can be relaxed if we use the integral form of Maxwell's equations:

$$
\begin{aligned}
\oint_S \mathbf{D} \cdot d\mathbf{S} &= \int_v \rho \, dv \\
\oint_S \mathbf{B} \cdot d\mathbf{S} &= 0 \\
\oint_C \mathbf{H} \cdot d\ell &= \int_S \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S} \\
\oint_C \mathbf{E} \cdot d\ell &= \int_S -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}
\end{aligned}
$$

Maxwell's Equations - Free Space Set

- In free space, there are no charges and currents. Accordingly, the point form of Maxwell's equations reduces to:

$$
\begin{aligned}
\nabla \cdot \mathbf{D} &= 0 \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\partial \mathbf{B} / \partial t \\
\nabla \times \mathbf{H} &= \partial \mathbf{D} / \partial t
\end{aligned}
$$

Maxwell's Equations - Free Space Set

- Similarly, in free space, the integral form of Maxwell's equations (which some regard as more physically intuitive) reduce to:

$$
\begin{aligned}
\oint_S \mathbf{D} \cdot d\mathbf{S} &= \int_v \rho \, dv \\
\oint_S \mathbf{B} \cdot d\mathbf{S} &= 0 \\
\oint_C \mathbf{H} \cdot d\mathbf{l} &= \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S} \\
\oint_C \mathbf{E} \cdot d\mathbf{l} &= \int_S -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}
\end{aligned}
$$

Why Do Maxwell's Equations Take the Form They Do?

- Even if we knew *nothing* about electromagnetics, we could still predict that a complete description of the electromagnetic field is given by equations of the form:

$$
\begin{aligned}
\nabla \cdot \mathbf{D} &= \text{something} \\
\nabla \cdot \mathbf{B} &= \text{something} \\
\nabla \times \mathbf{E} &= \text{something} \\
\nabla \times \mathbf{H} &= \text{something}
\end{aligned}
$$

- How do we know this?

- *Ans.* Helmholtz's theorem tells us that this set of field equations will be complete!
- Once the divergence and curl of a vector field are known, that field is completely specified.
- The "somethings" on the right hand side are simply experimental observations.
- Interestingly, unlike Newton's description of gravity, Maxwell's description of electromagnetics is consistent with special relativity.

Displacement Current

- For static fields,
$$\nabla \times \mathbf{H} = \mathbf{J}_c \, .$$
- For time-varying fields, this would imply that the continuity equation would be
$$\nabla \cdot \mathbf{J}_c = \nabla \cdot (\nabla \times \mathbf{H}) = 0$$
instead of the correct
$$\nabla \cdot \mathbf{J}_c = -\frac{\partial \rho}{\partial t} \, .$$
- James Clerk Maxwell postulated that
$$\nabla \times \mathbf{H} = \mathbf{J}_c + \mathbf{J}_D$$
where $\mathbf{J}_D = \partial \mathbf{D} / \partial t$.

- With the inclusion of the displacement current density, we can show that the continuity equation is satisfied.
- Assuming the time dependence $e^{j\omega t}$ for $\mathbf{E}$, the total current density is given by

$$\begin{aligned}\mathbf{J} &= \mathbf{J}_c + \mathbf{J}_D \\ &= \sigma\mathbf{E} + \frac{\partial}{\partial t}(\epsilon\mathbf{E}) \\ &= \sigma\mathbf{E} + j\omega\epsilon\mathbf{E}\,.\end{aligned}$$

- It can thus be shown that

$$\frac{\mathbf{J}_c}{\mathbf{J}_D} = \frac{\sigma}{\omega\epsilon}\,.$$

Interpreting Maxwell's Equations

- The electric and magnetic divergence equations indicate that free electric charges exist, but free magnetic charges do not.
- The magnetic curl equation (Ampere's Law) indicates that a constant current will give rise to a magnetic field that is everywhere perpendicular to it.
- Both the magnetic and electric curl equations indicate that a time-varying magnetic field gives rise to a time-varying electric field and vice-versa.
- The curl equations further indicate the electric and magnetic field vectors will be oriented perpendicular to each other at a given point in space.
- Furthermore, time-varying electric and magnetic fields cannot exist independently of each other, or in isolation.

4. Wave Propagation in Space

1. Time-Harmonic Electromagnetic Fields
2. The Plane Wave Solution
3. Solutions for Perfect Dielectrics
4. Solutions for Partially Conducting Media
5. Solutions for Good Conductors
6. Spherical Waves

4.1 Time-Harmonic Electromagnetic Fields

- If one assumes a time-factor $e^{j\omega t}$ and a source-free region, Maxwell's equations reduce to:

$$\begin{aligned}
\nabla \cdot \mathbf{H} &= 0 \\
\nabla \cdot \mathbf{E} &= 0 \\
\nabla \times \mathbf{H} &= \mathbf{J}_c + j\omega\mathbf{D} = (\sigma + j\omega\epsilon)\mathbf{E} \\
\nabla \times \mathbf{E} &= -j\omega\mathbf{B} = -j\omega\mu\mathbf{H}
\end{aligned}$$

- The coupling between the electric and magnetic fields is obvious!
- Look for a plane wave solution in which the wave propagates in the $+z$ direction, and for which $\mathbf{E}$ and $\mathbf{H}$ do not vary in x or y.

4.2 The Plane Wave Solution

- The Helmholtz equations associated with a plane wave propagating in the $\pm z$ direction:

$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0$$

$$\frac{\partial^2 E_y}{\partial z^2} + k^2 E_y = 0$$

  where $k = \omega\sqrt{\mu_0 \epsilon_0} = \omega/c = 2\pi/\lambda$.

- The two cases refer to horizontally and vertically polarized waves, respectively.
- Use superposition of the results to obtain the general case - an arbitrarily polarized wave.

- The solution to the first equation is

$$\begin{aligned} E_x &= A\, e^{-jkz} + B\, e^{+jkz} \ \ \mathrm{V/m} \\ H_y &= \frac{1}{Z_0}\left(A\, e^{-jkz} + B\, e^{+jkz}\right) \ \ \mathrm{A/m}. \end{aligned}$$

- The e^{-jkz} solution is a plane wave travelling in the $+z$ direction.
- The e^{+jkz} solution is a plane wave travelling in the $-z$ direction.
- For each wave separately, $|E_x/H_y| = Z_0$.

- The power flux density is given by the time-averaged Poynting vector,

$$\mathbf{S} = \frac{1}{2}\,\mathrm{Re}\{\mathbf{E} \times \mathbf{H}^*\}\ (\mathrm{W/m^2}).$$

- For the e^{-jkz} solution,

$$\mathbf{S} = \frac{A^2}{2Z_0}\,\hat{z}\ .$$

- For the e^{+jkz} solution,

$$\mathbf{S} = -\frac{B^2}{2Z_0}\,\hat{z}\ .$$

4.3 Solutions for Perfect Dielectrics

- A material is classified as a perfect dielectric if $\sigma = 0$.
- In this case,

$$\alpha = 0 \qquad \beta = \omega\sqrt{\mu\epsilon} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}\ .$$

- Implications:
  - waves do not suffer attenuation.
  - $\mathbf{E}$ and $\mathbf{H}$ are in phase.
  - $v = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{\mu\epsilon}}$.
  - $\mathbf{E}$ and $\mathbf{H}$ are everywhere perpendicular to each other.

4.4 Solutions for Partially Conducting Media

- For a region which is slightly conductive, one can assume an electric field of the form:

$$\mathbf{E} = E_0\, e^{-\gamma z}\, \hat{x}\ .$$

- Because

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

it can be shown that

$$\mathbf{H} = \sqrt{\frac{\sigma + j\omega\epsilon}{j\omega\mu}}\, E_0\, e^{-\gamma z}\, \hat{y}\ .$$

- Here, the ratio E_x/H_y gives the intrinsic impedance of the medium.

- In this case, the intrinsic impedance is complex and can be expressed in the form $|\eta|\angle\theta$.

- We can express the electric and magnetic fields in the form

$$\begin{aligned}\mathbf{E}(z,t) &= E_0\, e^{-\alpha z} e^{j(\omega t - \beta z)}\hat{x} \\ \mathbf{H}(z,t) &= \frac{E_0}{|\eta|}\, e^{-\alpha z} e^{j(\omega t - \beta z - \theta)}\hat{y}\end{aligned}$$

- Exercise: What is the physical significance of the parameters α and θ?

- Exercise: What are the velocity of propagation and wavelength in terms of μ, ω, ϵ, and σ?

4.5 Solutions for Good Conductors; Skin Depth

- A material is classified as a good conductor if $\sigma >> \omega\epsilon$.
- Exercise: What is the significance of the ratio $\sigma/\omega\epsilon$?
- In this case,

$$\begin{aligned} \gamma &= \alpha + j\beta \\ \alpha = \beta &= \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma} \\ \eta &= \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \end{aligned}$$

- We can express the fields in the form

$$\begin{aligned} \mathbf{E}(z,t) &= E_0\, e^{-\alpha z} e^{j(\omega t - \beta z)} \hat{x} \\ \mathbf{H}(z,t) &= \frac{E_0}{|\eta|} e^{-\alpha z} e^{j(\omega t - \beta z - \pi/4)} \hat{y} \end{aligned}$$

- Exercise: What are the velocity of propagation and wavelength in terms of μ, ω, ϵ, and σ?
- What is the significance of the skin depth, $\delta = \dfrac{1}{\sqrt{\pi f \mu\sigma}}$?
- *Ans.* Skin depth is the the distance over which the fields decay by $1/e$. At 5δ, the magnitude is 0.67% of its initial value, or essentially nil.
- Exercise: How else is skin depth significant?

4.6 Spherical Waves

- Plane waves are idealizations that cannot exist in practice. Why?
- Practical antennas radiate spherical waves.
- In spherical coordinates, the Helmholtz equation is given by:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial^2\psi}{\partial\phi^2}\right)+k^2\psi=0.$$

- Solve using the method of separation of variables. We begin by assuming that the solutions takes the form

$$\psi = R(r)\Theta(\theta)\Phi(\phi)\ .$$

- We can show that:
  - The R equation gives rise to spherical Bessel functions.
  - The Θ equation gives rise to associated Legendre functions.
  - The Φ equation is the familiar harmonic equation.
- In the simplest case, we assume spherical symmetry and ignore spherical harmonics.
- The result is satisfied by solutions of the form

$$\Psi(r) = C\,\frac{e^{-j\beta r}}{r} \qquad \text{and} \qquad \Psi(r) = D\,\frac{e^{j\beta r}}{r}\ .$$

- The first solution corresponds to an outward travelling wave while the second corresponds to an inward travelling wave (of no practical interest).

5.Wave Propagation and Boundary Conditions

- What happens to the field strength and flux density when fields cross the interface between two dielectrics?
- In a source-free region, we can show that:
  - the normal component of flux density is continuous across the interface;
  - the tangential component of field strength is continuous across the interface.
- This can be proven fairly easily by assuming static conditions. The results apply equally well to time varying fields.

5.1 Boundary Conditions for Electric Fields

- In a charge-free region
$$D_{n1} = D_{n2} \, .$$
- If there is surface charge,
$$(\mathbf{D}_1 - \mathbf{D}_2) \cdot \hat{n}_{12} = -\rho_s \, .$$
- In all cases,
$$E_{t1} = E_{t2} \, .$$
- In a charge-free region,
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r2}}{\epsilon_{r1}} \, .$$
- Can you prove this?

5.2 Boundary Conditions for Magnetic Fields

- In all cases
$$B_{n1} = B_{n2}\,.$$
- If the surface is current-free,
$$H_{t1} = H_{t2}\,.$$
- If there is a current sheet,
$$(\mathbf{H}_1 - \mathbf{H}_2) \cdot \hat{n}_{12} = \mathbf{K}\,.$$
- If there is no current,
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r2}}{\mu_{r1}}\,.$$
- Can you prove this?

5.3 Plane Waves at Boundaries

1. Normal Incidence
2. Oblique Incidence and Snell's Laws
3. Perpendicular or TE Polarization
4. Parallel or TM Polarization

5.3.1 Normal Incidence

- When a propagating wave reaches an interface between two different regions, it is partly reflected and partly transmitted.
- Application of the appropriate boundary conditions allows us to determine the transmission and reflection coefficients.
- At the boundary,

$$E_0^i + E_0^r = E_0^t. \quad H_0^i + H_0^r = H_0^t\,.$$

- Furthermore,

$$\frac{E_0^i}{H_0^i} = \eta_1, \quad \frac{E_0^r}{H_0^r} = -\eta_1. \quad \frac{E_0^t}{H_0^t} = \eta_2\,.$$

- These equations can be combined to yield:

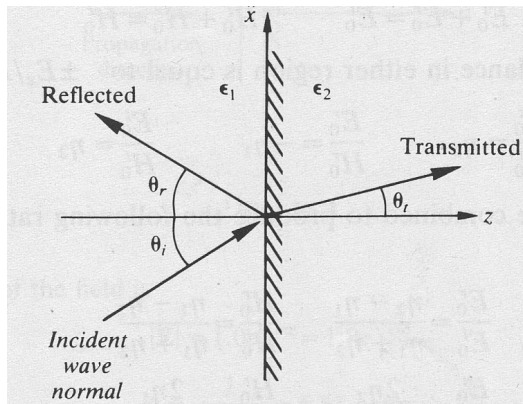
$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}, \quad \frac{H_0^r}{H_0^i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}, \quad \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_1 + \eta_2}, \quad \frac{H_0^r}{H_0^i} = \frac{2\eta_1}{\eta_1 + \eta_2}\,.$$

- As shown in §4, the intrinsic impedances for various types of media are:

free space	$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\Omega$
perfect dielectric	$\eta = \sqrt{\frac{\mu}{\epsilon}}$
partially conducting medium	$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$
conducting medium	$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ$

5.3.2 Oblique Incidence and Snell's Laws

- A wave incident upon a plane interface between two media will lead to a reflected wave in the first medium and a transmitted wave in the second.



- The *plane of incidence* contains the normal to the interface and the normal to the directions of incident, reflected, and transmitted wave propagation.
- Snell's law of reflection:

$$\theta_i = \theta_r \ .$$

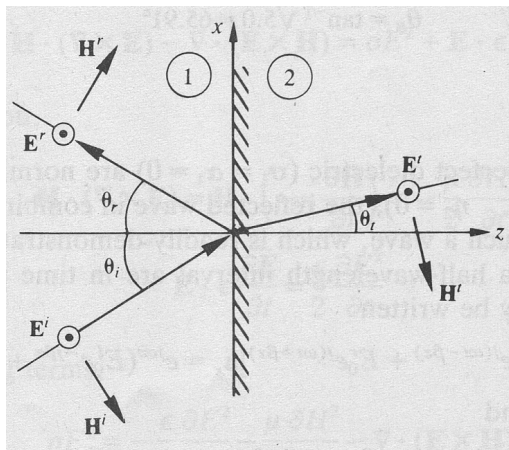
- Snell's law of refraction:

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}} \ .$$

- What is the *critical angle of incidence* observed when a wave propagates from the denser medium into the less dense medium?

5.3.3 Perpendicular or TE Polarization

- For perpendicular or TE polarization, $\mathbf{E}$ is perpendicular to the plane of incidence.



- At the interface between two regions,

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}\,.$$

and

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}\,..$$

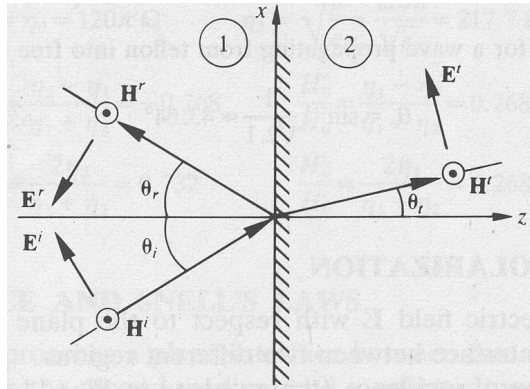
- For normal incidence, $\theta_i = \theta_t = 0^\circ$ and the expressions reduce to those of the previous section.

- Does the reflected wave ever drop to zero for TE polarization?

- *Ans.* No. Prove this.

5.3.4 Parallel or TM Polarization

- For parallel or TM polarization, $\mathbf{E}$ is entirely within the plane of incidence.



- At the interface,

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

and

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}. .$$

- If $\mu_1 = \mu_2$, will there be a particular angle of incidence for which there is no reflected wave?

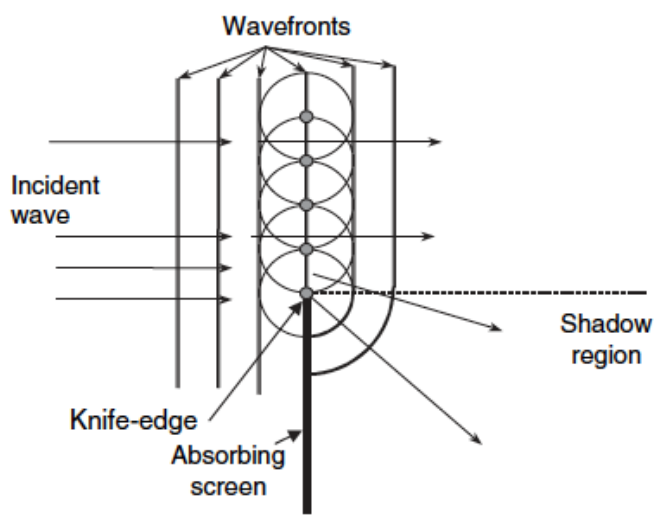
- *Ans.* Yes, this angle is given by the Brewster angle,

$$\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_2}} \ .$$

6. Diffraction

- When a plane wave is partially blocked by an obstruction, geometric optics predicts that there is an infinitely sharp transition between the shadow region and the illuimated region.
- In practice, some energy *diffracts* into the shadow region.
- The effect becomes more significant as wavelength increases relative to the dimensions of the obstruction.
- According to Huygens' principle, each element of a wavefront may be regarded as the centre of a secondary disturbance that gives rise to spherical wavelets.
- This approach can be used to model diffraction of a plane wave by a perfectly absorbing half screen.

6.1 Diffraction by a Knife Edge



- Diffraction by a perfectly absorbing half screen is an important canonical problem in wave propagation.
- The contributions from an infinite number of secondary sources above the edge are summed in a manner that accounts for their relative amplitudes and phases.
- We'll leave the detailed formulation and solution of the integral for a later date.
- The final result can be expressed as excess path loss L_{ke} in dB in terms of a diffraction parameter ν that depends on the problem geometry,

$$L_{ke}(\nu) = -20\log\left(\left|\frac{E_d}{E_i}\right|\right) = -20\log|F(\nu)|.$$

- $F(\nu)$ is given by

$$F(\nu) = \frac{1+j}{2}\int_{\nu}^{\infty}\exp\left(-\frac{j\pi t^2}{2}\right)dt$$

which can be reduced to

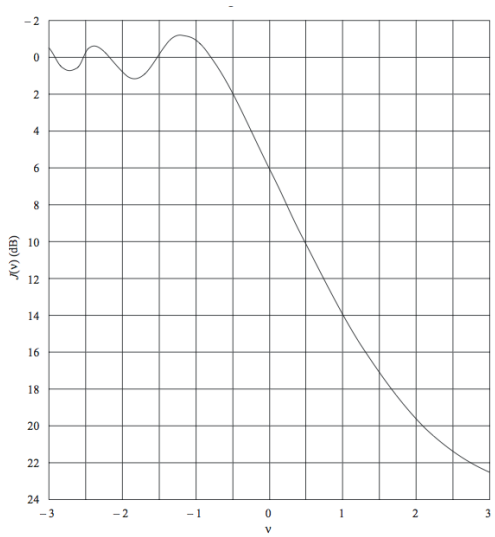
$$|F(\nu)| = \frac{1}{2}\left(\frac{1}{2} + C^2(\nu) - C(\nu) + S^2(\nu) - S(\nu)\right)$$

where $C(\nu)$ and $S(\nu)$ are the well-known Fresnel sine and cosine integrals.

- For $\nu > 1$, L_{ke} can be well-approximated by

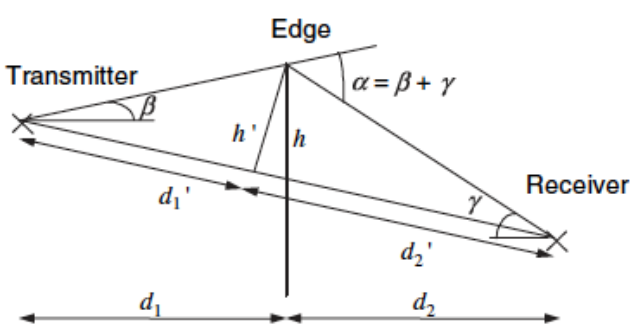
$$L_{ke}(\nu) \approx -20\log\frac{1}{\pi\nu\sqrt{2}} \approx -20log\frac{0.225}{\nu}\,.$$

Knife Edge Diffraction Attenuation as a Function of ν



Knife Edge Diffraction Geometry

- The geometrical parameters that define the Fresnel diffraction parameter ν are given by:



- The exact value of ν is given by

$$\nu = h'\sqrt{\frac{2(d_1' + d_2')}{\lambda d_1' d_2'}} = \alpha\sqrt{\frac{2d_1' d_2'}{\lambda(d_1' + d_2')}}\,.$$

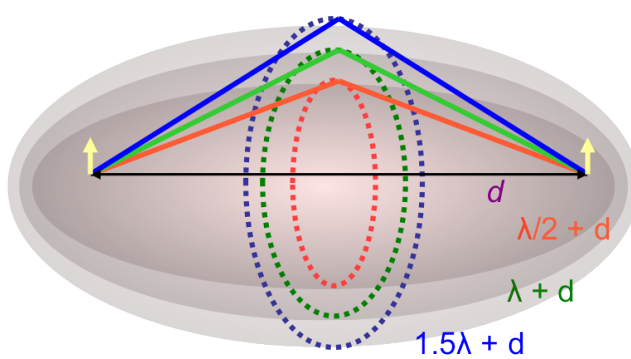
- For $d_1, d_2 >> h$, ν is well approximated by the more convenient expression

$$\nu \approx h\sqrt{\frac{2(d_1 + d_2}{\lambda d_1 d_2}} = \alpha\sqrt{\frac{2d_1 d_2}{\lambda(d_1 + d_2)}}\,.$$

- Why is the latter expression more convenient?

Fresnel Zones

- The nth Fresnel zone is the region inside an ellipsoid defined by the locus of points where the distance $(a + b)$ is larger than the direct path between transmitter and receiver $(d_1 + d_2)$ by n half-wavelengths.



- The radius of the nth zone r_n is given by applying the condition

$$a + b = d_1 + d_2 + \frac{n\lambda}{2} \,.$$

- If we assume $r_n << d_1, d_2$, then

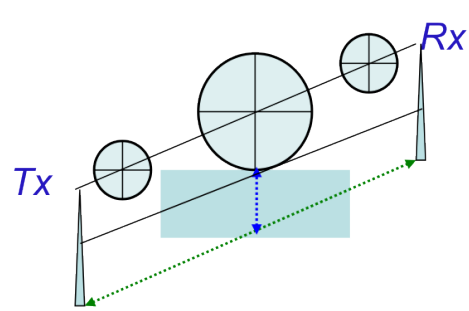
$$r_n \approx \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}} \,.$$

- The diffraction parameter ν can be determined from the *Fresnel zone clearance* h/r_n through

$$\nu \approx h\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = \frac{h}{r_n}\sqrt{2n} \,.$$

Fresnel Zone Clearance

- If the inner 60% of the first Fresnel zone is penetrated by an obstacle, the link will experience excess attenuation.
- If a half-plane penetrates to the line-of-sight, the signal will be attenuated by 6 dB.



Summary

We have reviewed (or, in some cases, introduced):

- The history of electromagnetics, including key milestomes.
- Electromagnetic quantities and their physical significance.
- The scalar electric and vector magnetic potentials.
- Maxwell's equations in various forms.
- Plane wave propagation in free space, partially conducting media, and good conductors.
- Spherical wave propagation in free space.
- Boundary conditions at the interface between two different materials.

- Wave propagation across the interface between two different media.
- Huygens' principle and diffraction by a half screen.
- Fresnel zones and Fresnel zone clearance.

References

[1] R.F. Harrington, *Introduction to Electromagnetic Engineering.* McGraw-Hill, 1958.

[2] R.F. Harrington, *Time Harmonic Electromagnetic Fields.* McGraw-Hill, 1961.

[3] J.A. Edminister, *Electromagnetics,* 2nd. ed., Schaum's Outline series, New York: McGraw-Hill, 1993.

[4] J. Kraus and R. Marhefka, *Antennas: For All Applications,* 3rd Ed., McGraw-Hill, 2002.

[5] W. Hayt and J. Buck, *Engineering Electromagnetics,* McGraw-Hill, 2006.

[6] S. R. Saunders and A. Aragon-Zavala, *Antennas and Propagation for Wireless Communication Systems,* 2nd ed., Wiley, Chap. 3, 2007.